CSc 120
Introduction to Computer Programming II

Adapted from slides by Dr. Saumya Debray

08: Python lists
some performance puzzles
Example 1: insert vs append

**insert:** adds an element into the middle of a list

```
>>> list0 = [1,2,3,4]
>>> list0
[1, 2, 3, 4]
>>> list0.insert(2, 'aaa')
>>> list0
[1, 2, 'aaa', 3, 4]
>>> list0.insert(3, 'bbb')
>>> list0
[1, 2, 'aaa', 'bbb', 3, 4]
```

**append:** adds an element at the end of a list

```
>>> list0 = [1,2,3,4]
>>> list0
[1, 2, 3, 4]
>>> list0.append('aaa')
>>> list0
[1, 2, 3, 4, 'aaa']
>>> list0.append('bbb')
>>> list0
[1, 2, 3, 4, 'aaa', 'bbb']
```
Example 1: insert vs append

**insert**: adds an element into the middle of a list

```python
list0 = mklist(n)  # length of list0 == n
list0.insert(n//2, 0)  # insert at midpoint
```

**append**: adds an element at the end of a list

```python
list0 = mklist(n)
list0.append(0)  # add at the end
```

Why this difference?
Example 2: mk_nxn_list

# given a number n > 0, return an n x n list-of-lists of n

**Version 1**

```python
def mk_nxn_list(n):
    return [[n]*n]*n
```

```python
>>> mk_nxn_list(2)
[[2, 2], [2, 2]]
>>> mk_nxn_list(3)
[[3, 3, 3], [3, 3, 3], [3, 3, 3]]
>>> mk_nxn_list(4)
[[4, 4, 4, 4], [4, 4, 4, 4], [4, 4, 4, 4], [4, 4, 4, 4]]
```

**Version 2**

```python
def mk_nxn_list(n):
    outlist = []
    for i in range(n):
        row_i = []
        for j in range(n):
            row_i.append(n)
        outlist.append(row_i)
    return outlist
```

```python
>>> mk_nxn_list(2)
[[2, 2], [2, 2]]
>>> mk_nxn_list(3)
[[3, 3, 3], [3, 3, 3], [3, 3, 3]]
>>> mk_nxn_list(4)
[[4, 4, 4, 4], [4, 4, 4, 4], [4, 4, 4, 4], [4, 4, 4, 4]]
```
mk_nxn_list(n) version 1
mk_nxn_list(n) version 2

![Graph showing the relationship between run time and input size n for mk_nxn_list(n) version 2. The graph demonstrates a positive correlation where run time increases as input size n increases.]
**mk_nxn_list(n) both versions**

Why this difference?
data organization in memory
Data organization in memory

**insert**: adds an element into the middle of a list

```python
>>> list0 = [1,2,3,4]
>>> list0
[1, 2, 3, 4]
>>> list0.insert(2, 'aaa')
>>> list0
[1, 2, 'aaa', 3, 4]
>>> list0.insert(3, 'bbb')
>>> list0
[1, 2, 'aaa', 'bbb', 3, 4]
```

**append**: adds an element at the end of a list

```python
>>> list0 = [1,2,3,4]
>>> list0
[1, 2, 3, 4]
>>> list0.append('aaa')
>>> list0
[1, 2, 3, 4, 'aaa']
>>> list0.append('bbb')
>>> list0
[1, 2, 3, 4, 'aaa', 'bbb']
```

- The organization of lists in memory affects the implementation of primitive operations
Data organization in memory

Consider list insertion

```python
>>> list0 = [1,2,3,4]
>>> list0
[1, 2, 3, 4]
>>> list0.insert(2, 'aaa')
>>> list0
[1, 2, 'aaa', 3, 4]
>>> list0.insert(3, 'bbb')
>>> list0
[1, 2, 'aaa', 'bbb', 3, 4]
>>> |
```

- The organization of this data in memory affects the primitive operations
- This affects the complexity of algorithms that work on the data
Data organization in memory

- Computer memory is organized as a sequence of *locations*
  - each location is identified by its *address* (a number)
  - a location typically consists of 8 bits (a "byte")
  - bytes are often grouped into "words" (32 or 64 bits)

⇒ A location (or word) can only hold a limited amount of data

<table>
<thead>
<tr>
<th>address</th>
<th>Memory locations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td></td>
</tr>
<tr>
<td>104</td>
<td></td>
</tr>
<tr>
<td>108</td>
<td></td>
</tr>
<tr>
<td>...</td>
<td></td>
</tr>
</tbody>
</table>
Data organization in memory

• A memory location can hold only a limited amount of data
• An object typically spans multiple memory locations
• Data are organized as follows:
  – objects and values are placed where memory is available
  – the object's memory address is used as a reference to it

E.g.: for \( L = ['aaa', 'bbb'] \)
Data organization in memory

We typically write these data structures in a way that abstracts away actual address values.

<table>
<thead>
<tr>
<th>address</th>
<th>info about the list</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100 (addr of &quot;aaa&quot;)</td>
</tr>
<tr>
<td></td>
<td>112 (addr of &quot;bbb&quot;)</td>
</tr>
<tr>
<td>100</td>
<td>&quot;aaa&quot;</td>
</tr>
<tr>
<td>104</td>
<td></td>
</tr>
<tr>
<td>108</td>
<td></td>
</tr>
<tr>
<td>112</td>
<td>&quot;bbb&quot;</td>
</tr>
</tbody>
</table>

≡

L

≡

L

"aaa" or "bbb"

or

L

"aaa"      "bbb"
insert vs. append
List (array) organization in Python

- (References to) the list elements are kept in a contiguous sequence of memory words
  - there is a little extra space at the end to give it some room to grow

- The following operations are O(1):
  - len()
    - read off length info from the header
  - accessing the i\textsuperscript{th} element of the list
    - compute its address using the value of i
    - access memory location at that address
Appending to a list \( O(1) \)

L

<table>
<thead>
<tr>
<th>info about the list</th>
</tr>
</thead>
<tbody>
<tr>
<td>“aaa”</td>
</tr>
<tr>
<td>“bbb”</td>
</tr>
<tr>
<td>“ccc”</td>
</tr>
<tr>
<td>“ddd”</td>
</tr>
<tr>
<td>“eee”</td>
</tr>
<tr>
<td>“fff”</td>
</tr>
<tr>
<td>“ggg”</td>
</tr>
<tr>
<td>“hhh”</td>
</tr>
</tbody>
</table>

extra space

list elements

L.append(‘qqq’)
Inserting into a list

```
L
+----------------+---------------+---------------+---------------+
| info about the |
| list           |
| “aaa”          |
| “bbb”          |
| “ccc”          |
| “ddd”          |
| “eee”          |
| “fff”          |
| “ggg”          |
| “hhh”          |
+----------------+---------------+---------------+---------------+
list elements
    extra space
```

```
L
+----------------+---------------+---------------+---------------+
| info about the |
| list           |
| “aaa”          |
| “bbb”          |
| “ccc”          |
| “ddd”          |
| “eee”          |
| “fff”          |
| “ggg”          |
| “hhh”          |
+----------------+---------------+---------------+---------------+
L.insert(2, ‘qqq’)  
```

```
L
+----------------+---------------+---------------+---------------+
| info about the |
| list           |
| “aaa”          |
| “bbb”          |
| “qqq”          |
| “ccc”          |
| “ddd”          |
| “eee”          |
| “fff”          |
| “ggg”          |
| “hhh”          |
+----------------+---------------+---------------+---------------+
```
## Inserting into a list $O(n)$

<table>
<thead>
<tr>
<th>info about the list</th>
</tr>
</thead>
<tbody>
<tr>
<td>“aaa”</td>
</tr>
<tr>
<td>“bbb”</td>
</tr>
<tr>
<td>“ccc”</td>
</tr>
<tr>
<td>“ddd”</td>
</tr>
<tr>
<td>“eee”</td>
</tr>
<tr>
<td>“fff”</td>
</tr>
<tr>
<td>“ggg”</td>
</tr>
<tr>
<td>“hhh”</td>
</tr>
</tbody>
</table>

*List elements have to be moved over by one position.*

<table>
<thead>
<tr>
<th>info about the list</th>
</tr>
</thead>
<tbody>
<tr>
<td>“aaa”</td>
</tr>
<tr>
<td>“bbb”</td>
</tr>
<tr>
<td>“ccc”</td>
</tr>
<tr>
<td>“ddd”</td>
</tr>
<tr>
<td>“eee”</td>
</tr>
<tr>
<td>“fff”</td>
</tr>
<tr>
<td>“ggg”</td>
</tr>
<tr>
<td>“hhh”</td>
</tr>
</tbody>
</table>

L.insert(2, ‘qqq’)

$L$
Q: Can we do insert in $O(1)$ time?
   (The complexity of other operations may change)
Data organization in memory

Given references, what happens when we copy a list?
shallow vs. deep copying
Shallow vs. deep copying

References require semantic choices for copy

For a compound object $A$ (i.e., an object that contains references to other objects, e.g., lists, dictionaries):

• shallow copying:
  – creates a copy $A'$ of $A$
  – inserts into $A'$ references to the objects in $A$

• deep copying:
  – creates a copy $A'$ of $A$
  – inserts into $A'$ references to deep copies of objects in $A$
References and “copy” choices

A

Copy references

Copy everything

Shallow

Deep
Shallow vs. deep copying

```
A

: copied
```

```
A

A'

: copied
```

```
A

A'

: copied
```

```
A

A'

: copied
```
Shallow vs. deep copying

Shallow: copied

Deep: copied
Shallow vs. deep copying

def mk_nxn_list(n):
    return [[n]*n]*n
Shallow vs. deep copying

def mk_nxn_list(n):
    return [[n]*n]*n

time: O(1)
def mk_nxn_list(n):
    return [[n]*n]*n

time: O(1)
Shallow vs. deep copying

```python
def mk_nxn_list(n):
    return [[n]*n]*n

time: O(n)
```
Shallow vs. deep copying

def mk_nxn_list(n):
    return [[n]*n]*n

time: O(1)
def mk_nxn_list(n):
    return [[n]*n]*n

time: O(n)
def mk_nxn_list(n):
    return [[n]*n]*n

time: O(n)
def mk_nxn_list(n):
    return [[n]*n]*n

Overall: O(n)
Shallow vs. **deep** copying

```python
def mk_nxn_list(n):
    outlist = []
    for i in range(n):
        row_i = []
        for j in range(n):
            row_i.append(n)
        outlist.append(row_i)
    return outlist
```

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def mk_nxn_list(n):
    outlist = []
    for i in range(n):
        row_i = []
        for j in range(n):
            row_i.append(n)
        outlist.append(row_i)
    return outlist

O(1)
def mk_nxn_list(n):
    outlist = []
    for i in range(n):
        row_i = []
        for j in range(n):
            row_i.append(n)
        outlist.append(row_i)
    return outlist

O(n)
def mk_nxn_list(n):
    outlist = []
    for i in range(n):
        row_i = []
        for j in range(n):
            row_i.append(n)
        outlist.append(row_i)
    return outlist

O(n^2)
def mk_nxn_list(n):
    outlist = []
    for i in range(n):
        row_i = []
        for j in range(n):
            row_i.append(n)
        outlist.append(row_i)
    return outlist

O(n)
Shallow vs. **deep** copying

def mk_nxn_list(n):
    outlist = []
    for i in range(n):
        row_i = []
        for j in range(n):
            row_i.append(n)
        outlist.append(row_i)
    return outlist

Overall: $O(n^2)$
Shallow vs. deep copying: howto

**Shallow copy**

```python
>>> import copy
>>> x = [[1,2,3],[4,5,6]]
>>> y = copy.copy(x)
>>> y
[[1, 2, 3], [4, 5, 6]]
>>> x[0].append('abc')
>>> y
[[1, 2, 3, 'abc'], [4, 5, 6]]
```

**Deep copy**

```python
>>> import copy
>>> x = [[1,2,3],[4,5,6]]
>>> y = copy.deepcopy(x)
>>> y
[[1, 2, 3], [4, 5, 6]]
>>> x[0].append('pqr')
>>> x
[[1, 2, 3, 'pqr'], [4, 5, 6]]
>>> y
[[1, 2, 3], [4, 5, 6]]
```
Summary

• Shallow vs. deep copying:
  – applies to any object, not just lists
  – how to:
    o import copy
    o copy.copy(obj), copy.deepcopy(obj)
  – shallow copying more efficient, but creates “aliases” that can cause weird behaviors
List organization in Python/Complexity of operations (Review)

• (References to) the list elements are kept in a contiguous sequence of memory words
  – there is a little extra space at the end to give it some room to grow

• Give the complexity of:
  – len()
  – accessing an element’s value: alist[i]
  – append()
  – insert()
Aliases

- Aliasing occurs when more than one variable contains the same reference to the same object

- A reference is a location in memory

- Aliases are created when lists (and compound objects) are copied

- Aliases are also created through assignment
Data organization in memory

we will use the simplified diagram of a list for the next example
Example: aliasing

What is the content of each list after the following assignment statements?

```python
list1 = [2, 3, 5]
list2 = [7, 11, 13]
list3 = list1
list4 = list1 + list2

list1[2] = -3
list3.append("foo")
list4[0] = None
```
```python
list1 = [2, 3, 5]
list2 = [7, 11, 13]
list3 = list1
list4 = list1 + list2

list1[2] = -3
list3.append("foo")
list4[0] = None
```

Remember, a list doesn't actually contain the values – it contains references to other objects.

But for simplicity, we often draw the object inside the list itself.
Example: aliasing

```python
list1 = [2, 3, 5]
list2 = [7, 11, 13]
list3 = list1
list4 = list1 + list2
list1[2] = -3
list3.append("foo")
list4[0] = None
```

```
2  3  5
7 11 13
```
Example: aliasing

```python
list1 = [2, 3, 5]
list2 = [7, 11, 13]
list3 = list1
list4 = list1 + list2

list1[2] = -3
list3.append("foo")
list4[0] = None
```

Assigning a list to a new variable creates an alias.
Example: aliasing

```python
list1 = [2, 3, 5]
list2 = [7, 11, 13]
list3 = list1
list4 = list1 + list2

list1[2] = -3
list3.append("foo")
list4[0] = None
```

List concatenation creates a brand-new list. It is a duplicate.
Example: aliasing

```
list1 = [2, 3, 5]
list2 = [7, 11, 13]
list3 = list1
list4 = list1 + list2

list1[2] = -3
list3.append("foo")
list4[0] = None
```

Changing a value inside a list changes one element.

**All of the aliases** of the list see the same change.
Example: aliasing

In the same way, .append() modifies the list (and thus all aliases of it). It does **NOT** create a new list.

So .append() and + are different.
Example: aliasing

There is not an alias to list4.
The assignment does not affect the other lists.
Summary

• Python lists: properties
  – `len`: $O(1)$
  – accessing an element by index: $O(1)$
  – append: $O(1)$
  – insert, delete: $O(n)$

• Shallow vs. deep copying:
  – applies to any object, not just lists
  – how to:
    – import copy
    – `copy.copy(obj), copy.deepcopy(obj)`
  – shallow copying more efficient, but creates “aliases” that can cause weird behaviors
  – assignment also create aliases
timing programs
import time

def mklist(n):
    outlist = []
    for i in range(n):
        outlist.append(n)
    return outlist

# estimate overhead of timing loop

def get_overhead(niters):
    start_time = time.time()
    for i in range(niters):
        pass
    stop_time = time.time()
    overhead = stop_time - start_time
    return overhead

# do the timing

def do_time(listsz_start, listsz_stop, niters, overhead):
    listsz = listsz_start
    insertion_pt = listsz/2
    delta = 100
    while listsz <= listsz_stop:
        x = mklist(listsz)
        start_time = time.time()
        for i in range(niters):
            x.insert(insertion_pt, 0)
        stop_time = time.time()
        ave_time = (stop_time-start_time-overhead)/niters
        print("{:d}, {:f}".format(listsz, ave_time))

        if listsz >= 10*delta:
            delta *= 10
            listsz += delta  # next list size to try

def main():
    niters = 10000
    overhead = get_overhead(niters)
    do_time(100, 10000000, niters, overhead)

main()
# do the timing

def do_time(listsz_start, listsz_stop, niters, overhead):
    listsz = listsz_start
    insertion_pt = listsz//2
    delta = 100
    while listsz <= listsz_stop:
        x = mklist(listsz)
        start_time = time.time()
        for i in range(niters):
            x.insert(insertion_pt, 0)
        stop_time = time.time()
        ave_time = (stop_time-start_time-overhead)/niters

        print("{:d}, {:f}".format(listsz, ave_time))

        if listsz >= 10*delta:
            delta *= 10
            listsz += delta  # next list size to try
# estimate overhead of timing loop

def get_overhead(niters):
    start_time = time.time()
    for i in range(niters):
        pass
    stop_time = time.time()
    overhead = stop_time - start_time
    return overhead

def main():
    niters = 10000
    overhead = get_overhead(niters)
    do_time(100, 100000000, niters, overhead)