10: Abstract Data Types
Stacks and Queues
Abstract Data Types
Abstract Data Types

An abstract data type (ADT) describes a set of data values and associated operations that are specified independent of any particular implementation.

An ADT is a logical description of how we view the data and the operations allowed on that data.

- describes what the data represents
- not how is the data represented

The data is encapsulated.
Abstract Data Types

Because the data is *encapsulated* we can change the underlying implementation without affecting the *logical* way the ADT behaves.

- the logical description remains the same
- the operations remain the same

Example:

- lists
  - Python built-in lists
  - linked lists
Abstract Data Types

Consider the ADT definition of a list.

Lists:

- logical description
  - linear ordering of elements
  - elements can be inserted or deleted from any location

- operations
  - len, indexing, slicing, in, concatenation, insert, delete, ...
linear data structures
Linear data structures

A *linear* data structure is a collection of objects with a straight-line ordering among them

– each object in the collection has a *position*
– for each object in the collection, there is a notion of the object *before* it or *after* it
### Data structures we've seen

<table>
<thead>
<tr>
<th>Linear</th>
<th>Not linear</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Python lists</td>
<td>• Dictionaries</td>
</tr>
<tr>
<td>• Linked lists</td>
<td>• Sets</td>
</tr>
</tbody>
</table>
Today's topic

Linear

• Python lists
• Linked lists
• Stacks
• Queues
• Dequeues

Not linear

• Dictionaries
• Sets

Key property: the way in which objects are added to, and removed from, the collection
stacks
The Stack ADT

A stack is a linear data structure where objects are inserted or removed only at one end

- all insertions and deletions happen at one particular end of the data structure
- this end is called the top of the stack
- the other end is called the bottom of the stack

insertions and deletions happen at one end
Stacks: insertion of values

Insertion of a sequence of values into a stack:

5  17  33  9  43
Stacks: insertion of values

Insertion of a sequence of values into a stack:

5 17 33 9 43
Stacks: insertion of values

Insertion of a sequence of values into a stack:

5 17 33 9 43
Stacks: insertion of values

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Stacks: insertion of values

Insertion of a sequence of values into a stack:

5  17  33  9  43
Stacks: insertion of values

Insertion of a sequence of values into a stack:

| 5 | 17 | 33 | 9 | 43 |

Stack diagram:

- Stack top
- Stack bottom

Values:

```
[43]
[33]
[17]
[5]
```
Stacks: insertion of values

5 17 33 9 43

order in which values were inserted
Stacks: removal of values

Removing values from the stack:

order in which values were inserted
Stacks: removal of values

5  17  33  9  43

*order in which values were inserted*

Removing values from the stack:

43
Stacks: removal of values

\[5\, 17\, 33\, 9\, 43\]

*order in which values were inserted*

Removing values from the stack:

\[43\, 9\]
Stacks: removal of values

5  17  33  9  43

*order in which values were inserted*

Removing values from the stack:

43  9  33
Stacks: removal of values

Removing values from the stack:

5  17  33  9  43

order in which values were inserted

43  9  33  17

stack top
stack bottom
Stacks: removal of values

Removing values from the stack:

5  17  33  9  43

order in which values were inserted

43  9  33  17  5
Stacks: removal of values

5 17 33 9 43

*order in which values were inserted*

Removing values from the stack:

43 9 33 17 5

*order in which values were removed*
Stacks: LIFO property

order in which values were inserted

Removing values from the stack:

order in which values were removed

values are removed in reverse order from the order of insertion

"LIFO order"
Last in, First out
Methods for a Stack class

• Stack() : creates a new empty stack

• push(*item*) : adds *item* to the top of the stack
  – returns nothing
  – modifies the stack

• pop() : removes the top item from the stack
  – returns the removed item
  – modifies the stack

• is_empty() : checks whether the stack is empty
  – returns a Boolean
Implementing a Stack class

class Stack:
    # create a Stack
    def __init__(self):
        self._items = ?

    # adds item to the "top"
    def push(self, item):
        ?

    # removes the last item from the Stack
    def pop(self):
        ?
Implementing a Stack class

class Stack:

    # the top of the stack is the last item in the list
    def __init__(self):
        self._items = []

    def push(self, item):
        self._items.append(item)

    def pop(self):
        return self._items.pop()
EXERCISE

```python
>>> s = Stack()
>>> s.push(4)
>>> s.push(17)
>>> s.push(5)
>>> x = s.pop()
>>> y = s.pop()
```

← what does the stack `s` look like here? what are the values of `x` and `y`?
EXERCISE

```python
>>> s = Stack()
>>> s.push(4)
>>> s.push(17)
>>> s.push(5)
```
stacks: applications
An application: balancing parens

IDLE (the Python shell) matches up left and right parens ( ), brackets [ ], and braces { }

```python
>>> x = [1, 2, [3, 4, [5], 7], 8]
```

How does it figure out how far back to highlight?
An application: balancing parens

Basic idea: Match each ] with corresponding [
- similarly for ( ... ) and { ... } pairs

- Idea:
  - maintain a stack
  - on seeing '[' : push
  - on seeing ']' : pop the matching symbol

Example: [ 1, 2, [ 3, [ 4 ], 5, [ 7 ] ] ]

Stack (empty)
An application: balancing parens

Basic idea: Match each ] with corresponding [
  – similarly for ( ... ) and { ... } pairs

– Idea:
  o maintain a stack
  o on seeing '[' : push
  o on seeing ']' : pop the matching symbol

Example: [1, 2, [3, [4], 5, [7]]]
An application: balancing parens

Basic idea: Match each ] with corresponding [
- similarly for ( ... ) and { ... } pairs

- Idea:
  - maintain a stack
  - on seeing '[' : push
  - on seeing ']' : pop the matching symbol

Example: [ 1, 2, [ 3, [ 4 ], 5, [ 7 ] ] ]
An application: balancing parens

Basic idea: Match each ] with corresponding [
  – similarly for ( ... ) and { ... } pairs

– Idea:
  o maintain a stack
  o on seeing ']' : push
  o on seeing ']' : pop the matching symbol

Example:  [ 1, 2, [ 3, [ 4 ], 5 , [ 7 ] ] ]

Stack
An application: balancing parens

Basic idea: Match each ] with corresponding [
  – similarly for ( ... ) and { ... } pairs

  – Idea:
    o maintain a stack
    o on seeing '[' : push
    o on seeing ']' : pop the matching symbol

Example: [ 1, 2, [ 3, [ 4 ], 5, [ 7 ] ] ]
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Example: [ 1, 2, [ 3, [ 4 ], 5 , [ 7 ] ] ]
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    o on seeing '[' : push
    o on seeing ']' : pop the matching symbol

Example: [ 1, 2, [ 3, [ 4 ], 5, [ 7 ] ] ]
An application: balancing parens

Basic idea: Match each ] with corresponding [
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- Idea:
  - maintain a stack
  - on seeing '[' : push
  - on seeing ']' : pop the matching symbol

Example: [ 1, 2, [ 3, [ 4 ], 5 , [ 7 ] ] ]
An application: balancing parens

Basic idea: Match each ] with corresponding [
  - similarly for ( ... ) and { ... } pairs

- Idea:
  - maintain a stack
  - on seeing '[' : push
  - on seeing ']' : pop the matching symbol

Example: [ 1, 2, [ 3, [ 4 ], 5 , [ 7 ] ] ]
An application: balancing parens

Basic idea: Match each ] with corresponding [
  - similarly for ( ... ) and { ... } pairs

- Idea:
  - maintain a stack
  - on seeing '[: push
  - on seeing ']': pop the matching symbol

Example: [ 1, 2, [ 3, [ 4 ], 5, [ 7 ] ] ]

Elaboration: Have each stack element keep track of the position of its [ ]
Given the Stack class, write a function `balanced(s)` that returns True if the string `s` is balanced with respect to `[` and `]` and False otherwise.

class Stack:
    def __init__(self):
        self._items = []

    def push(self, item):
        self._items.append(item)

    def pop(self):
        return self._items.pop()

    def is_empty(self):
        return self._items == []
CSc 120: Phylogenetic Trees

This problem brings together many different programming concepts, including trees. It is one of the most technically challenging problems we have analyzed in the past few weeks.

Background

An evolutionary tree (also called a phylogenetic tree) is a.

This program involves writing code to construct phylogenetic trees. For example, since programs are sequences of characters, we.

Expected Behavior

Write a Python program, in a file phylo.py, that behaves.

1. Read in the input parameters:
   - Read in the name of an input file using input
   - Read an integer value $N$ using input

2. Read in the input file. The file format is specified un
Related: Displaying web pages

Web page

Display considerations

main header: large font, bold

secondary header: medium font, bold

bold font

italics font

Question: how does the web browser figure out how much a given display format should include? E.g., which text is in boldface, how much is in italics, etc.
CSc 120: Phylogenetic Trees

This problem brings together many different programming constructs we covered over the course of the semester including: manipulation, (Python) lists, dictionaries, tuples, classes, list comprehensions, and trees. It is one of the most technically challenging programs assigned in this class by many, think it's also one of the most interesting.

An evolutionary tree (also called a phylogenetic tree) is a

This program involves writing code to construct phylogenetic trees, since programs are sequences of characters, we

Expected Behavior

Write a Python program, in a file phylo.py, that behaves

1. Read in the input parameters:
   ○ Read in the name of an input file using input
   ○ Read in an integer value N using input('n- 
2. Read in the input file. The file format is specified un
Related: Displaying web pages

Web page

CSc 120: Phylogenetic Trees

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**Background**

An evolutionary tree (also called a phylogenetic tree) is a tree that expresses evolutionary relationships between a set of organisms.

**Expected Behavior**

Write a Python program, in a file `phylo.py`, that behaves:

1. Read in the input parameters:
   - Read in the name of an input file using `input`
   - Read in an integer value `N` using `input` (`n-gram`)
2. Read in the input file. The file format is specified un

HTML source

```html
<html>
  <head>
    <title>CSc 120: Phylogenetic Trees</title>
  </head>
  <body bgcolor="white">
    <p>This problem brings together many different programming constructs we covered over the course of the semester including manipulation, (Python) lists, dictionaries, tuples, classes, list comprehensions, and trees. It is one of the most technically challenging programs assigned in this class and I think it’s also one of the most interesting.</p>

  <h2>Background</h2>
  An evolutionary tree (also called a phylogenetic tree) is a tree that expresses evolutionary relationships between a set of organisms.

  <h2>Expected Behavior</h2>
  Write a Python program, in a file `phylo.py`, that behaves:

  1. Read in the input parameters:
     - Read in the name of an input file using `input`
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  2. Read in the input file. The file format is specified un
```
Related: Displaying web pages

Web page

CSc 120: Phylogenetic Trees

This problem brings together many different programming constructs we covered over the course of the semester including: manipulation. (Python) lists, dictionaries, tuples, classes, list comprehensions, and trees. It is one of the most technically challenging programs assigned in this class this semester. I think it's also one of the most interesting.

Expected Behavior

Write a Python program, in a file phylo.py, that behaves as specified below.

1. Read in the input parameters:
   - Read in the name of an input file using input
   - Read in an integer value N using input
2. Read in the input file. The file format is specified un

HTML source

```html
<head>
<body bgcolor="white">
<p>
<img src="../IMGS/uadc.png" alt="University of Arizona, Depa
</p>
<h1>CSc 120: Phylogenetic Trees</h1>

This problem brings together many different programming constructs we covered over the course of the semester including: manipulation, (Python) lists, dictionaries, tuples, classes, list comprehensions, and trees. It is one of the most technically challenging programs assigned in this class this semester. I think it's also one of the most interesting.

<h2>Background</h2>

An evolutionary tree (also called a phylogenetic tree) is a

This program involves writing code to construct phylogenetic trees, since programs are sequences of characters, we could just apply this approach to sets of programs.

<h2>Expected Behavior</h2>

Write a Python program, in a file phylo.py, that behaves as specified below.

<ol>
  <li>Read in the input parameters</li>
</ol>
```
Related: Displaying web pages

"tags"

\[ \text{\textless h1 \rangle : "open header 1"} \]
\[ \text{\textless /h1 \rangle : "close header 1"} \]
\[ \text{\textless h2 \rangle : "open header 2"} \]
\[ \text{\textless /h2 \rangle : "close header 2"} \]
\[ \text{\textless i \rangle : "open italics"} \]
\[ \text{\textless /i \rangle : "close italics"} \]

...
Figuring out how to display different parts of the web page requires matching up “open-” and “close-” HTML tags. This is essentially the same problem as balancing parens.
EXERCISE

```python
>>> s1 = Stack()
>>> s1.push(4)
>>> s1.push(17)
>>> s2 = Stack()
>>> s2.push(s1.pop())
>>> s2.push(s1.pop())
>>> s1.push(s2.pop())
>>> s1.push(s2.pop())
```

← what does the stack s1 look like here?
Hypothetical: Python 7 has just been released and built-in lists are inefficient. In fact, all operations are $O(n^2)$.

Avoid these inefficiencies by implementing the Stack class using LinkedLists.
queues
A Queue ADT

A *queue* is a linear data structure where insertions and deletions happen at different ends

- insertions happen at one end (the queue's "back“, or “tail”)
- deletions happen at the other end (the queue's "front“, or “head”)

![Diagram of a queue with insertions and deletions at different ends]
Queues: insertion of values

Insertion of a sequence of values into a queue:

| queue back | None | queue front | None | 5 | 17 | 33 | 9 | 43 |
Queues: insertion of values

Insertion of a sequence of values into a queue: 5 17 33 9 43
Queues: insertion of values

Insertion of a sequence of values into a queue:

5  17  33  9  43
Queues: insertion of values

Insertion of a sequence of values into a queue:

5 17 33 9 43
Insertion of a sequence of values into a queue:

5 17 33 9 43

Queue: insertion of values

| 9 | 33 | 17 | 5 |

Queue back

Queue front
Queues: insertion of values

Insertion of a sequence of values into a queue:
Queues: insertion of values

order of insertion → 5 17 33 9 43

queue back

queue front
Queues: removal of values

*order of insertion* → 5 17 33 9 43

Removing values from this queue:

```
43  9  33  17  5
```

queue back

queue front
Queues: removal of values

**order of insertion** → 5 17 33 9 43

Removing values from this queue:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>9</td>
<td>33</td>
<td>17</td>
<td>5</td>
</tr>
</tbody>
</table>

queue back

queue front
Queues: removal of values

*order of insertion* → 5 17 33 9 43

Removing values from this queue: 5 17

<table>
<thead>
<tr>
<th>43</th>
<th>9</th>
<th>33</th>
<th>17</th>
<th>5</th>
</tr>
</thead>
</table>

queue back

queue front
Queues: removal of values

*order of insertion* → 5 17 33 9 43

Removing values from this queue: 5 17 33

| 43 | 9 | 33 | 17 | 5 |

queue back

queue front
Queues: removal of values

order of insertion → 5 17 33 9 43

Removing values from this queue:

5 17 33 9

queue back

queue front
Queues: removal of values

order of insertion → 5  17  33  9  43

Removing values from this queue: 5  17  33  9  43

<table>
<thead>
<tr>
<th></th>
<th>queue back</th>
<th>None</th>
<th>queue front</th>
<th>None</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>9</td>
<td>33</td>
<td>17</td>
<td>5</td>
</tr>
</tbody>
</table>
Queues: removal of values

order of insertion  →  5  17  33  9  43

order of removal   →  5  17  33  9  43
Queues: FIFO property

Order of insertion

5  17  33  9  43

Order of removal

5  17  33  9  43

Values are removed in order in which they are inserted

"FIFO order"
First in, First out
Methods for a queue class

- **Queue()**: creates a new empty queue
- **enqueue(item)**: adds *item* to the back of the queue
  - modifies the queue
  - returns nothing
- **dequeue()**: removes and returns the item at the front of the queue
  - returns the removed item
  - modifies the queue
- **is_empty()**: checks whether the queue is empty
  - returns a Boolean
- **size()**: returns the size of the queue
  - returns an integer
Implementing a queue class

• Use a built-in list for the internal representation
  – Python lists can be added to from the front or the end

• First implementation:
  – the head is the 0\textsuperscript{th} element
  – the tail is the \textit{n}th element

• Second implementation
  – the head is the \textit{n}th element
  – the tail is the 0\textsuperscript{th} element
Implementing a Queue class I

class Queue:

    # the front of the queue is the first item in the list

def __init__(self):
    self._items = []

def enqueue(self, item):
    self._items.append(item)

def dequeue(self):
    return self._items.pop(0)
EXERCISE

```python
>>> q = Queue()
>>> q.enqueue(4)
>>> q.enqueue(17)
>>> x = q.dequeue()
>>> q.enqueue(5)
>>> y = q.dequeue()
```

← *what are the values of* $x$ *and* $y$?
EXERCISE

```python
>>> q = Queue()
>>> q.enqueue(4)
>>> q.enqueue(17)
>>> x = q.dequeue()
>>> y = q.dequeue()
>>> q.enqueue(y)
>>> q.enqueue(x)
>>> q.enqueue(y)
```

← what does the queue q look like here?
EXERCISE-3

Implement a queue with a Python list. Make the front of the queue the last item in the list

class Queue:
    def __init__(self):
        
    def enqueue(self, item):
        
    def dequeue(self):

![Queue Diagram]
Answer: implementation II

class Queue:

    # the front of the queue is the last item in the list

def __init__(self):
    self._items = []

def enqueue(self, item):
    self._items.insert(0, item)

def dequeue(self):
    return self._items.pop()
queues: applications
Application 1: Simulation

• Typical applications simulate problems that require data to be managed in a FIFO manner
  – Hot potato
    o Kids stand in a circle and pass a “hot potato” around until told to stop. The person holding the potato is taken out of the circle. The process is repeated until only one person remains.

• Use a *simulation* to determine which element remains.
Application 1: Simulation

• Typical applications simulate problems that require data to be managed in a FIFO manner
  – Hot potato
    o Kids stand in a circle and pass a “hot potato” around until told to stop. The person holding the potato is taken out of the circle. The process is repeated until only one person remains.

• Use a *simulation* to determine which person remains after num "passes" or rounds
  – Person at front of queue "holds" the potato
  – To pass the potato: simulate by dequeue/enqueue
  – After num number of passes, the person at the front is removed: simulate by dequeue
EXERCISE

Write a function `hot_potato(q, num)` that takes a queue `q` and the number of rounds of simulation `num` and eliminates the correct element after `num` rounds.

What operations take an element from the front of the queue and place it at the back of the queue?
def hot_potato(q, num):
    for i in range(num):
        x = q.dequeue()
        q.enqueue(x)
    return q.dequeue()
Application 1: Simulation

• Typical applications simulate problems that require data to be managed in a FIFO manner
  – Hot potato
  – Generalized: Given n elements, eliminate every kth element repeatedly until only 1 element is left. What was the original position of the remaining element?
  – Known as the Josephus problem:
    o for any n and any k, what original starting position will not be eliminated?

• Use a queue and a *simulation* to determine which element remains.
EXERCISE

Problem: Given $n$, create a queue with $n$ elements, then repeatedly eliminate every $k$th element until only 1 element is left. What was the original position of the remaining element?

- use a queue to simulate the circle
- $n$ is the number of elements to put into the queue
- while there is more than one element in the queue
  - eliminate every $k$th element
General solution for k=2

• Given n elements, eliminate every kth element repeatedly until only 1 element is left. What was the original position of the remaining element?
• When k = 2, the original position can be derived from the binary representation of n.
  Take the first digit of the binary representation.
  Move it to the end
  The result is the original position.

Ex: n = 41, k=2

In binary
  n = 101001

Therefore, the original position (in binary) is
  010011

and 010011 = 2^4 + 2^1 + 2^0 = 19

• Let's check our simulation.

Application 2 : Simulation

• Suppose we are opening a grocery store. How many checkout lines should we put in?
  – too few ⇒ long wait times, unhappy customers
  – too many ⇒ wasted money, space

• Use simulations of the checkout process to guide the decision
  – study existing stores to figure out typical shopping and checkout times
  – estimate no. of customers expected at the new location
  – run simulations to determine customer wait time and checkout line utilization under different scenarios
Discrete event simulation

By varying the parameters of the simulation (arrival and departure rates, no. of servers) we can try out different scenarios.
Summary

• Stacks and queues are abstract data types (ADTs)
  – similar in that they are both linear data structures
  – items can be thought of as arranged in a line
  – each item has a position and a before/after relationship with the other items

• They differ in the way items are added and removed
  – stacks: items added and removed at one end
    o results in LIFO behavior
  – queues: items added at one end, removed at the other
    o results in FIFO behavior

• They find a wide range of applications in computer science
A Deque ADT

A *deque* is a linear data structure where insertions and deletions happen at both ends

insertions and deletions occur at this end (tail)

insertions and deletions occur at this end (head)