CSc 144 — Discrete Structures for Computer Science I
Fall 2024 (McCann)

Collected Definitions for Exam #2

I can't recall the last time I didn't ask a definition question on an exam. To help you better prepare yourself for such questions, I've assembled this list. My pledge to you: If I ask you for a definition on the exam, the term will come from this list. Note that this is not a complete list of the definitions given in class. You should know the others, too, but I won't specifically ask you for their definitions on the exam.

Once in a while a student will express disappointment that I ask definition questions on exams. My justification is that I think it's important for you to know what the core terms mean so that you can use them correctly and effectively. At the same time, I don't require that you memorize the exact wording of the definitions you see here. If you provide a definition in your own words that captures all of the detail found here, without adding anything incorrect, that's fine.

Topic 3: Quantification

(Continued from the Exam #1 Topic 3 definition list. If I ask you to define a Topic 3 term on Exam #2, it will come from this handout.)

- The *Generalized De Morgan's Laws* are the equivalences $\overline{\forall x\, P(x)} \equiv \exists x\, \overline{P(x)}$ and $\overline{\exists x\, Q(x)} \equiv \forall x\, \overline{Q(x)}$

Topic 4: Arguments

- "An *argument* is a connected series of statements to establish a definite proposition." [Credit: Monty Python's Flying Circus, Series 3, Episode 3 ("The Money Programme"), "Argument Clinic."]
- An argument that moves from specific observations to a general conclusion is an *inductive argument.*
- An argument that uses accepted general principles to explain a specific situation is a *deductive argument.*
- Any deductive argument of the form $(p_1 \wedge p_2 \wedge \ldots \wedge p_n) \rightarrow q$ is *valid* if the conclusion must follow from the hypotheses.
- A valid argument that also has true hypotheses is a *sound* argument.
- A *fallacy* is an argument constructed with an improper inference.

Topic 5: Direct Proofs of $p \rightarrow q$

- A *conjecture* is a statement with an unknown truth value.
- A *theorem* is a conjecture that has been shown to be true.
- A sound argument that establishes the truth of a theorem is a *proof.*
- A *lemma* is a simple theorem whose truth is used to construct more complex theorems.
- A *corollary* is a theorem whose truth follows directly from another theorem.

(Continued . . .)

Topic 6: Additional Set Concepts

- A *set* is an unordered collection of unique objects.
- Set A is a *subset* of set B (written $A \subseteq B$) if every member of A can be found in B.
- A is a *proper subset* of B (written $A \subset B$) if $A \subseteq B$ and $A \neq B$.
- The *power set* of a set A (written $\mathcal{P}(A)$) is the set of all of A's subsets, including the empty set.
- Two sets are *disjoint* if their intersection is $\emptyset$ (the empty set).
- A *partition* of a set separates all of its members into disjoint subsets.
- An *ordered pair* is a group of two items (a, b) such that $(a, b) \neq (b, a)$ unless $a = b$.
- The *Cartesian Product* of two sets A and B (written $A \times B$) is the set of all ordered pairs (a, b) such that $a \in A$ and $b \in B$.

Topic 7: Matrices

- A *matrix* is an n-dimensional collection of values, $n \in \mathbb{Z}^+$.
- A *square* matrix is a two–dimensional matrix in which the # of rows equals the # of columns.
- Two matrices A and B are *equal* iff they share the same dimensions **and** each pair of corresponding elements is equal.
- The *transposition* of an $m \times n$ matrix A is the $n \times m$ matrix A^T in which the rows of A become the columns of A^T.
- A matrix A is *symmetric* iff $A = A^T$.
- The *matrix addition* (or *matrix sum*) of matrices A and B is the matrix containing the sums of the corresponding pairs of elements from A and B.
- A *scalar* is a real value. *(In the context of the next definition!)*
- The *scalar multiplication* of a scalar value d and an $m \times n$ matrix A is the $m \times n$ matrix B such that $b_{ij} = d \cdot a_{ij}$.
- The *matrix multiplication* of an $m \times n$ matrix A and an $n \times o$ matrix B is an $m \times o$ matrix $C = A \cdot B$ in which $c_{ij} = \sum_{k=1}^{n} (a_{ik} \cdot b_{kj})$.

Note: There are more definitions than these in Topic 7, but these are the definitions from the portion of this topic that will be covered by the exam. The others will be fair game for Exam #3 and the Final Exam.