CSc 144 — Discrete Structures for Computer Science I
Fall 2024 (McCann)

Collected Definitions for Exam #3

This is the 'official' collection of need–to–know definitions for Exam #3. I can't recall the last time I didn't ask a definition question on an exam. To help you better prepare yourself for definition questions, I've assembled this list. My pledge to you: If I ask you for a definition on the exam, the term will come from this list. Note that this is not a complete list of the definitions given in class. You should know the others, too, but I won't specifically ask you for their definitions on the exam.

Topic 7: Matrices

(Continued from the Exam #2 Topic 7 definition list. If we ask you to define a Topic 7 term on Exam #3, it will come from this list.)

- *Identity matrices*, denoted I_n, are $n \times n$ matrices populated with 1 down the main diagonal (upper-left to lower-right) and with 0 elsewhere.
- The n^{th} *matrix power* of an $m \times m$ matrix A, denoted A^n, is the matrix resulting from $n-1$ successive matrix products of A. $A^0 = I_m$.
- The *logical matrix product* of an $m \times n$ 0–1 matrix A and an $n \times l$ 0–1 matrix B is an $m \times l$ 0–1 matrix $C = A \odot B$ in which $c_{ij} = \bigvee_{k=1}^{n} (a_{ik} \wedge b_{kj})$.
- The r^{th} *logical matrix power* of an $m \times m$ 0–1 matrix A, denoted $A^{[r]}$, is the matrix resulting from $r-1$ successive logical matrix products of A. $A^{[0]} = I_m$.

Topic 8: Relations

- A *(binary) relation* from set X to set Y is a subset of the Cartesian Product of the domain X and the codomain Y.
- A relation R on set A is *reflexive* if $(a, a) \in R$, $\forall a \in A$.
- A relation R on set A is *symmetric* if, whenever $(a, b) \in R$, then $(b, a) \in R$, for $a, b \in A$.
- A relation R on set A is *antisymmetric* if $(x, y) \in R$ and $x \neq y$, then $(y, x) \notin R$, $\forall x, y \in A$.
- A relation R on set A is *transitive* if, whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for $a, b, c \in A$.
- The *inverse* of a relation R on set A, denoted R^{-1}, contains all of the ordered pairs of R with their components exchanged. (That is, $R^{-1} = \{(b, a) \mid (a, b) \in R\}$.)
- Let G be a relation from set A to set B, and let F be a relation from B to set C. The *composite* of F and G, denoted $F \circ G$, is the relation of ordered pairs $(a, c), a \in A, c \in C$, such that $b \in B$, $(a, b) \in G$, and $(b, c) \in F$.
- A relation R on set A is an *equivalence relation* if it is reflexive, symmetric, and transitive.
- The *equivalence class* of an equivalence relation R on set B, and an element $b \in B$, is $\{c \mid c \in B \wedge (b, c) \in R\}$ and is denoted $[b]$. That is, the equivalence class is the set of all elements of the base relation equivalent to a given element as defined by the relation.
- A relation R on set A is a *(reflexive/weak) partial order* if it is reflexive, antisymmetric, and transitive.
- A relation R on set A is *irreflexive* if, for all members of A, $(a, a) \notin R$.
- A relation R on set A is an *irreflexive* (or *strict*) *partial order* if it is irreflexive, antisymmetric, and transitive.
- Let R be a weak partial order on set A. a and b are said to be *comparable* if $a, b \in A$ and either $a \preceq b$ or $b \preceq a$ (that is, either $(a, b) \in R$ or $(b, a) \in R$).
- A weak partially-ordered relation R on set A is a *total order* if every pair of elements $a, b \in A$ are comparable.

(Continued ...)

Topic 9: Functions

- A *function* from set X to set Y, denoted $f : X \rightarrow Y$, is a relation from X to Y such that $f(x)$ is defined $\forall x \in X$ and, for each $x \in X$, there is exactly one $(x, y) \in f$.
- For each of the following, let $f : X \rightarrow Y$ be a function, and assume $f(n) = p$.
  - X is the *domain* of f; Y is the *codomain* of f.
  - f *maps* X to Y.
  - p is the *image* of n; n is the *pre-image* of p.
  - The *range* of f is the set of all images of elements of X. (Note that the range need not equal the codomain.)
- The *floor* of a value n, denoted $\lfloor n \rfloor$, is the largest integer $\leq n$.
- The *ceiling* of a value m, denoted $\lceil m \rceil$, is the smallest integer $\geq m$.
- A function $f : X \rightarrow Y$ is *injective* (a.k.a. *one-to-one*) if, for each $y \in Y$, $f(x) = y$ for at most one member of X.
- A function $f : X \rightarrow Y$ is *surjective* (a.k.a. *onto*) if f's range is Y (the range = the codomain).
- A *bijective* function (a.k.a. a *one-to-one correspondence*) is both injective and surjective.
- The *inverse* of a bijective function f, denoted f^{-1}, is the relation $\{(y, x) \mid (x, y) \in f\}$.
- Let $f : Y \rightarrow Z$ and $g : X \rightarrow Y$. The *composition* of f and g, denoted $f \circ g$, is the function $h = f(g(x))$, where $h : X \rightarrow Z$.
- A function $f : X \times Y \rightarrow Z$ (or $f(x, y) = z$) is a *binary* function.

Topic 10: Indirect ("Contra") Proofs of $p \rightarrow q$

There were no definitions in this topic!