# Topic 1:

**Course Background**

(or: Why You're Here, and What You Learned to Get Here)



# What Is Discrete Math?

**Definition: Discrete Mathematics**

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Contrast this with 'the calculus,' which was developed by Newton and Leibniz to study objects in motion. As a result:

- 'The Calculus' tends to focus on real values
- Discrete Mathematics tends to focus on integer values

# Sample Discrete Math Topics

Topics that fall under the umbrella of discrete math include:

- Integral Functions and Relations
- Matrix Operations and Representations
- Sets
- Sequences and Summations
- And everything in CSc 244, and CSc 345, and ...

To understand those, you also need:

- First-Order Logic
- Logical Arguments
- Proof Techniques
- ...and a fair amount of pre–calculus mathematics



# "But Why Do I Have To Take Discrete Math?"

Discrete Structures is an ACM/IEEE core curriculum topic

- See:

  https://www.acm.org/binaries/content/assets/education/cs2013_web_final.pdf

DM topics underlie much of Computer Science, including:

- Logic $\longrightarrow$ Knowledge Representation, Reasoning,
  Natural Language Processing, Computer Architecture
- Proof Techniques $\longrightarrow$ Algorithm Design, Code Verification
- Relations $\longrightarrow$ Database Systems, Sorting
- Functions $\longrightarrow$ Hashing, Programming Languages
- Recurrence Relations $\longrightarrow$ Recursive Algorithm Analysis
- Probability $\longrightarrow$ Algorithm Design, Simulation

# Topics You May Need To Review

- Mathematical concepts, including, but not limited to:
  - Fractions
  - Rational Numbers
  - Basics of Sets
  - Associative, Commutative, Distributive, and Transitive Laws
  - Properties of Inequalities
  - Summation and Product Notation
  - Integer Division (Modulo, Divides, and Congruences)
  - Even and Odd Integers
  - Logarithms and Exponents

  *The Math Review appendix (available from the class web page) can also help you review these topics.*

- Fundamental programming skills in Python or Java
  *We trust that you can review this on your own!*



# Notations for Sets of Values

| | | |
|---|---|---|
| $\mathbb{Z}$ | All integers | $\{\ldots,-2,-1,0,1,2,\ldots\}$ |
| $\mathbb{Z}^+, \mathbb{N}^+$ | All positive integers | $\{1,2,3,\ldots\}$ |
| $\mathbb{Z}^*, \mathbb{N}_0$ | The non–negative integers | $\{0,1,2,3,\ldots\}$ |
| $\mathbb{Z}^{\text{even}}$ | Even integers | $\{\ldots,-4,-2,0,2,4,\ldots\}$ |
| $\mathbb{Z}^{\text{odd}}$ | Odd integers | $\{\ldots,-3,-1,1,3,\ldots\}$ |
| $\mathbb{Q}$ | Rational numbers | ${}^{a}/_{b},\ a,b \in \mathbb{Z}, b \neq 0$ |
| $\overline{\mathbb{Q}}$ | Irrational Numbers | $\{i \mid i \notin Q\}$ |
| $\mathbb{R}$ | The real values | $\{\mathbb{Q} \cup \overline{\mathbb{Q}}\}$ |

**Note:** Avoid the term “natural numbers” and the plain $\mathbb{N}$.

# Commutativity

Assume that ▲ is a binary operator on a set of values $S$.

If $x \blacktriangle y = y \blacktriangle x$ for any elements $x$ and $y$ in $S$,

then ▲ is a *commutative* operator.

**Example(s):**

Addition is commutative on $\mathbb{R}$:

Subtraction is not commutative on $\mathbb{R}$:



# Associativity

Assume that ▲ is a binary operator on a set of values $S$.

If $(x \blacktriangle y) \blacktriangle z = x \blacktriangle (y \blacktriangle z)$ for any $x, y, z$ in $S$,

then ▲ is an *associative* operator.

**Example(s):**

Multiplication is associative on $\mathbb{Z}$:

Subtraction is not associative on $\mathbb{Z}$:

# Distributivity (1 / 2)

Assume that $\blacktriangle$ and $\blacksquare$ are binary operators on a set $S$, and that $a, b, c$ are all values of $S$.

$\blacktriangle$ is *left–distributive* over $\blacksquare$ when $a \blacktriangle (b \blacksquare c) = (a \blacktriangle b) \blacksquare (a \blacktriangle c)$

$\blacktriangle$ is *right–distributive* over $\blacksquare$ when $(b \blacksquare c) \blacktriangle a = (b \blacktriangle a) \blacksquare (c \blacktriangle a)$



# Distributivity (2 / 2)

**Example(s):**

Multiplication distributes over addition:

This knowledge can help you do larger products by hand:

# Transitivity

Assume that $\diamond$ defines a relationship on values from $S$.

For any $x, y, z$ in $S$, $\diamond$ is *transitive* if whenever $x \diamond y$ and $y \diamond z$, then $x \diamond z$.

**Example(s):**

"Greater than" is transitive on $\mathbb{R}$:

In sports, "defeats" is not transitive on a set of teams:



# Three Fraction Reminders

① The product of fractions is the ratio of the products of the numerators over the products of the denominators:

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$$

② One fraction divided by another equals the product of the numerator fraction and the reciprocal of the denominator:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$$

③ Computing the sum of two fractions requires a common denominator, then we add the numerators:

$$\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{b}{b} \cdot \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad+bc}{bd}$$

# Rational and Irrational Numbers (1 / 2)

**Definition: Rational Number**

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**Example(s):**

A real number that is not rational is **irrational**.

**Example(s):**



# Rational and Irrational Numbers (2 / 2)

**Example(s):**

Express $0.41\overline{6}$ as a reduced fraction.

## Set Notations

- Sets are named with upper–case letters and their elements are listed within curly brackets.
  - For example, let $S = \{0, 5, 10, 15\}$
- The **universe** $(\mathcal{U})$ is the set of all available elements
- The symbol $\in$ says that an element is a member of a set
  - $15 \in S$ is true
- The symbol $\notin$ says that an element is not a member of a set
  - $8 \notin S$ is true
- The **cardinality** of a set is the quantity of elements within it
  - $|\,S\,| \;=\; 4$
- The **empty set** ($\varnothing$ or $\{\}$) contains no elements, and so $|\varnothing| = 0$.



## Basic Set Operators (1 / 2)

1. **Union** $(\cup)$**:** $A \cup B$ contains all elements of both set $A$ and set $B$
2. **Intersection** $(\cap)$**:** $C \cap D$ contains only the elements present in both sets $C$ and $D$
3. **Difference** $(-)$**:** $E - F$ contains only the elements of set $E$ that are **not** also in set $F$

   (**Note:** Take out the “not,” and you’ve got a definition for $\cap$ )
4. **Complement** $(\overline{\square})$**:** Given a set $G$, $\overline{G} = \mathcal{U} - G$, the set of available items. where $\mathcal{U}$ is the *universe*.

**Note:** $X - Y = X \cap \overline{Y}$

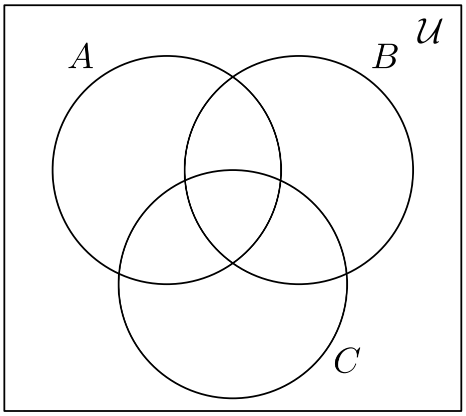
# Basic Set Operators (2 / 2)

**Example(s):**

$\mathcal{U} = \{0, 1, 2, 3, 4, 6, 7, 8, 9\}$

$A = \{1, 2, 4, 9\}$ $\quad C = \{2, 4, 7\}$

$B = \{0, 2, 6, 8\}$






# Summation and Product Notation (1 / 2)

$$\sum_{i=1}^{5} 2i = 2(1) + 2(2) + 2(3) + 2(4) + 2(5) = 30$$

where:

- $\Sigma$ is the ____________________________.
- $i$ is the ____________________________.
- $1$ is the ____________________________.
- $5$ is the ____________________________.
- $2i$ is the ____________________________.

# Summation and Product Notation (2 / 2)

Switch $\Sigma$ to $\Pi$ (capital Pi) for multiplication:

**Example(s):**

Use parentheses to eliminate confusion:

**Example(s):**



# Nested Summations and Products

Much like nested `FOR` loops.

**Example(s):**

# Modulo and Divides (1 / 2)

Integer Division $(\backslash)$ produces quotients;

Modulo $(\%)$ produces remainders

**Example(s):**



# Modulo and Divides (2 / 2)

**Definition: Divides**

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**Example(s):**

# Congruences

Another set symbol: "$a \in B$" means $a$ is an element of set $B$.

**Definition: Congruent Modulo $m$**

(*b* is called the *base*, $r$ is the *residue* or *remainder*, and $m$ is the *modulus*)

**Example(s):**



# Laws of Exponents and Logarithms (1 / 3)

1. $w^{x+y} = w^x w^y$
2. $(w^x)^y = w^{xy}$
3. $v^x w^x = (vw)^x$
4. $\frac{w^x}{w^y} = w^{x-y}$
5. $\frac{v^x}{w^x} = \left(\frac{v}{w}\right)^x$

# Laws of Exponents and Logarithms (2 / 3)

The connection between exponents and logarithms:

If $b^y = x$, then $\log_b x = y$.

For each of the following laws, $a, b > 0$ and $a, b \neq 1$:

1. $\log_a x = \frac{\log_b x}{\log_b a}$
2. If $m > n > 0$, then $\log_b m > \log_b n$
3. $b^{\log_b x} = x$
4. $\log_b(x^y) = y \cdot \log_b x$
5. $\log_b(xy) = \log_b x + \log_b y$
6. $\log_b(\frac{x}{y}) = \log_b x - \log_b y$



# Laws of Exponents and Logarithms (3 / 3)

**Example(s):**

Fully evaluate: $\log_2 \left(2^3\right)^5$

# Remember!

The math review topics are used in this class, and direct questions about them will be asked on Quiz #1, Exam #1, and the Final Exam.

If you are not confident in your knowledge of them:

- Read Appendix A in “Kneel Before $\mathbb{Z}^{\text{odd}}$,”
- Attend TA office hours, **and**
- Review and self–test the topics on your own!