Topic 2:

Logic

What Is Logic?

Definition: Philosophical Logic

Definition: Mathematical Logic

Propositional Logic

Propositional Logic is part of Mathematical Logic. Versions include:

- *First Order Logic* (FOL, a.k.a. *First Order Predicate Calculus* (FOPC)) includes simple term variables and quantifications.

- *Second Order Logic* allows its variables to represent more complex structures (in particular, predicates).

- *Modal Logic* adds support for modalities; that is, concepts such as possibility and necessity.

Well-Formed Formulae

Definition: Well-Formed Formula (wff)

. . . .

Example(s):

Why Are We Studying Logic?

A few of the many reasons:

- Logic is the foundation for computer operation.
- Logical conditions are common in programs:
    - Selection:

        ```
        if (score <= max) { ... }
        ```

    - Iteration:

        ```
        while (i<limit && list[i]!=sentinel) ...
        ```

- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).

    - Examples: Trees, Graphs, Recursive Algorithms, ...

- Even programs can be proven correct!
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).

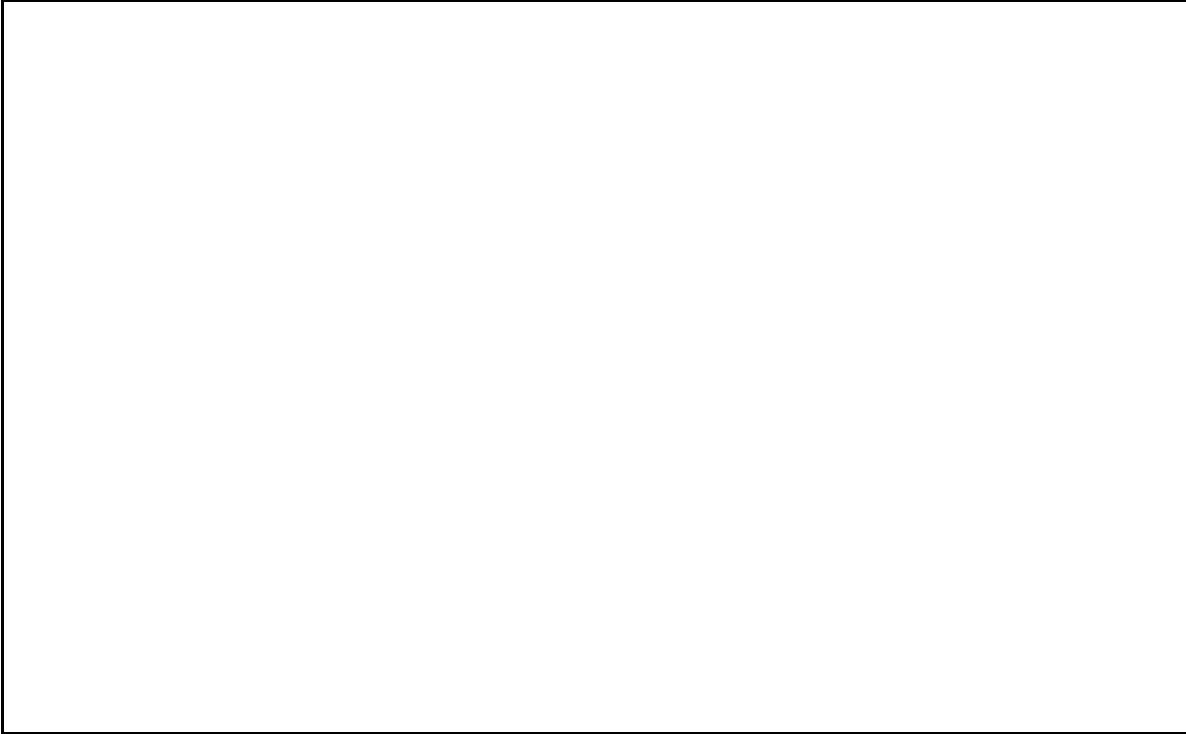# Simple Propositions (1 / 2)

Definition: Proposition

. . . .

Definition: Simple Proposition

. . . .

Simple Propositions (2 / 2)

Example(s):

Proposition Labels

To save writing, it is traditional to label propositions with lower-case letters called *proposition labels* or *statement letters*.

Example(s):

Compound Propositions

Definition: Compound Proposition

> . . . .

And with what do we combine them?

Conjunctions (1 / 2)

Heard of ABC's "Schoolhouse Rock" education series?

"Conjunction Junction" (1973)

(Music/Lyrics by Bob Dorough; Performed by Jack Sheldon)
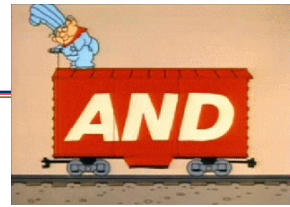
Conjunctions (2 / 2)

Conjunctions are:

- compound propositions formed in English with "and" & "but",
- formed in logic with the caret symbol (" $\wedge$ "), and
- true only when both participating propositions are true.

Example(s):

Disjunctions (1 / 3)

Consider this compound proposition:

Under which circumstances is that claim true? Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If all three are acceptable, the disjunction is

_____ (   ).

Disjunctions (2 / 3)

Consider the same example and possibilities:

3 is the number of sides of a triangle or the number of times this class meets per week.

Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If the third possibility is <u>not</u> acceptable, the disjunction is

_____ (   ).

Disjunctions (3 / 3)

Example(s):

Negation

Negating a proposition simply flips its value.

Common negation notations: $\neg x \qquad \overline{x} \qquad \sim x \qquad x'$

Example(s):

Notes:

Truth Tables (1 / 2)

Truth tables aid in the evaluation of compound propositions.

Structure of a Truth Table:

p	q	$p \wedge q$	$(p \wedge q) \vee p$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

Truth Tables (2 / 2)

Truth Tables of $\wedge$, $\vee$, $\oplus$, and $\neg$:

NOT ($\neg$)

p	$\neg p$

AND ($\wedge$)

p	q	$p \wedge q$

OR ($\vee$)

p	q	$p \vee q$

XOR ($\oplus$)

p	q	$p \oplus q$

Precedence of Logical Operators

Total agreement is hard to come by:

Precedence	Rosen 8/e p. 11	Gersting 5/e p. 6	Hein 2/e p. 351	Epp 1/e p. 24
Highest	$\neg$	$'$	$\neg$	$\sim$
	$\wedge$	$\wedge, \vee$	$\wedge$	$\wedge, \vee$
$\updownarrow$	$\vee$	$\rightarrow$	$\vee$	$\rightarrow, \leftrightarrow$
	$\rightarrow$	$\leftrightarrow$	$\rightarrow$	
Lowest	$\leftrightarrow$			

(**Note:** We'll cover $\rightarrow$ and $\leftrightarrow$ soon.)

In this class:

Operator Associativity

Consider evaluating: $a = b = -2 * 3 * 7;$ in Python

Example(s):

Equivalence of Propositions

Definition: Logically Equivalent

Example(s):

Natural Language Stmts → Propositions (1 / 4)

Review: Is There isn't a cloud in the sky a proposition?

Question: Is the following sentence a proposition?

Natural Language Stmts → Propositions (2 / 4)

Step 1: Identify the simple propositions.

> Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 2: Assign easy-to-remember statement labels.

Natural Language Stmts → Propositions (3 / 4)

Step 3: Identify the logical operators.

> Either Walter deposits his mortgage payment or else he
> will lose his house and move in with Donna.

Step 4: Construct the matching logical expression.

Natural Language Stmts → Propositions (4 / 4)

So . . . what's the point? Three examples:

- Expressing Program Conditions

- Natural Language Understanding

- Proof Setup

Three Categories of Propositions (1 / 2)

Definition: Tautology

Definition: Contradiction

Definition: Contingency

. . . .

Three Categories of Propositions (2 / 2)

Example(s): Which of those is $d \oplus (\neg k \wedge m)$?

Example(s):

Aside: Logical Bit Operations in Python/Java

Operator	Name	Example (Dec.)	Example (Bin.)
$\sim$	Complement	$\sim 12 = -13$	$\sim 00001100 = 11110011$
$\&$	AND	$12 \& 10 = 8$	$\begin{array}{r} 1100 \\ \& \ 1010 \\ \hline 1000 \end{array}$
$\vert$	OR	$12 \vert 10 = 14$	$\begin{array}{r} 1100 \\ \vert \ 1010 \\ \hline 1110 \end{array}$
$\wedge$	XOR	$12 \wedge 10 = 6$	$\begin{array}{r} 1100 \\ \wedge \ 1010 \\ \hline 0110 \end{array}$
$>>$	Shift Right	$33 >> 1 = 16$	$00100001 >> 1 = 00010000$
$<<$	Shift Left	$33 << 2 = 132$	$00100001 << 2 = 10000100$

Example: Default Linux File Permissions

```
$ ls -l
-rw-rw-r-- 1 mccann mccann 3561 Oct 28 1929 stocktosell
```

Example:

Definition: Conditional Proposition

Conditional Propositions (2 / 3)

In "if p, then q", p and q are known by various names:

Common forms of "if p, then q" (Rosen 8/e, p. 7):

- ▷ if p, then q
- ▷ if p, q
- ▷ p implies q
- ▷ p only if q
- ▷ p is sufficient for q
- ▷ a necessary condition for p is q
- ▷ q unless $\neg p$

- ▷ q if p
- ▷ q when p
- ▷ q whenever p
- ▷ q follows from p
- ▷ q is necessary for p
- ▷ a sufficient condition for q is p
- ▷ q provided that p

Conditional Propositions (3 / 3)

Example(s):

Truth of Conditional Propositions (1 / 2)

When should this be considered 'true'?

If you make it through *voir dire*, you will serve on the jury.

The possibilities:

1. Antecedent true, Consequent true; statement is: _____ .

2. Antecedent true, Consequent false; statement is: _____ .

3. Antecedent false, Consequent true; statement is: _____ .

4. Antecedent false, Consequent false; statement is: _____ .

Truth of Conditional Propositions (2 / 2)

Not satisfied? Maybe this Python if statement will help:

```python
if y < x :
    temp = x
    x = y
    y = temp
```

Definition: Vacuously True

Inverse, Converse, and Contrapositive

Definition: Inverse

Definition: Converse

r	s	
T	T	
T	F	
F	T	
F	F	

Contraposition

Definition: Contrapositive

r	s	
T	T	
T	F	
F	T	
F	F	

Examples: English Translation (1 / 2)

Examples: English Translation (2 / 2)

Example: English $\rightarrow$ Logic

Another Example: English → Logic

Political Example: "Push" Polling

"What would you think of Elizabeth Colbert Busch
if she had done jail time?"

> — Asked in telephone calls by Survey Sampling
> International in the 2013 South Carolina 1st
> Congressional District special election

Biconditional Propositions and *iff* (1 / 2)

What is the meaning of:

A triangle is equilateral if and only if all three angles are equal.

Biconditional Propositions and *iff* (2 / 2)

Definition: Biconditional Proposition

r	s	
T	T	
T	F	
F	T	
F	F	

Biconditionals and Logical Equivalence

Definition: Logically Equivalent (2)

Example(s):

De Morgan's Laws

Example(s):

Example: De Morgan's Laws and Programming

Checking to see if a 0–100 numeric score is <u>not</u> a 'B':

Common Logical Equivalences (1 / 3)

<u>Table I</u>: Some Equivalences using AND ($\wedge$) and OR ($\vee$):

(a)	$p \wedge p \equiv p, \quad p \vee p \equiv p$	Idempotent Laws
(b)	$p \vee \mathbf{T} \equiv \mathbf{T}, \quad p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination Laws
(c)	$p \wedge \mathbf{T} \equiv p, \quad p \vee \mathbf{F} \equiv p$	Identity Laws
(d)	$p \wedge q \equiv q \wedge p$	Commutative Laws
	$p \vee q \equiv q \vee p$	
(e)	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative Laws
	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	
(f)	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive Laws
	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	
(g)	$p \wedge (p \vee q) \equiv p$	Absorption Laws
	$p \vee (p \wedge q) \equiv p$	

<u>Table II</u>: Some More Equivalences (adding $\neg$):

(a)	$\neg(\neg p) \equiv p$	Double Negation
(b)	$p \vee \neg p \equiv \mathbf{T}, \quad p \wedge \neg p \equiv \mathbf{F}$	Negation Laws
(c)	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	De Morgan's Laws
	$\neg(p \vee q) \equiv \neg p \wedge \neg q$	

Common Logical Equivalences (2 / 3)

<u>Table III</u>: Still More Equivalences (adding $\to$):

(a)	$p \to q \equiv \neg p \vee q$	Law of Implication
(b)	$p \to q \equiv \neg q \to \neg p$	Law of the Contrapositive
(c)	$\mathbf{T} \to p \equiv p$	"Law of the True Antecedent"
(d)	$p \to \mathbf{F} \equiv \neg p$	"Law of the False Consequent"
(e)	$p \to p \equiv \mathbf{T}$	Self-implication (a.k.a. Reflexivity)
(f)	$p \to q \equiv (p \wedge \neg q) \to \mathbf{F}$	Reductio Ad Absurdum
(g)	$\neg p \to q \equiv p \vee q$	
(h)	$\neg(p \to q) \equiv p \wedge \neg q$	
(i)	$\neg(p \to \neg q) \equiv p \wedge q$	
(j)	$(p \to q) \vee (q \to p) \equiv \mathbf{T}$	Totality
(k)	$(p \wedge q) \to r \equiv p \to (q \to r)$	Exportation Law (a.k.a. Currying)
(l)	$(p \wedge q) \to r \equiv (p \to r) \vee (q \to r)$	
(m)	$(p \vee q) \to r \equiv (p \to r) \wedge (q \to r)$	
(n)	$p \to (q \wedge r) \equiv (p \to q) \wedge (p \to r)$	
(o)	$p \to (q \vee r) \equiv (p \to q) \vee (p \to r)$	
(p)	$p \to (q \to r) \equiv q \to (p \to r)$	Commutativity of Antecedents

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Common Logical Equivalences (3 / 3)

<u>Table IV</u>: Yet More Equivalences (adding $\oplus$ and $\leftrightarrow$):

(a)	$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$	Definition of Biimplication
(b)	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	
(c)	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$	
(d)	$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$	Definition of Exclusive Or
(e)	$p \oplus q \equiv \neg(p \leftrightarrow q)$	
(f)	$p \oplus q \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q$	

Remember: You **do not** need to memorize these tables . . .

. . . But you **do** need to know how to use them!

Applications of Logical Equivalences (1 / 5)

Question: **Is $(p \wedge q) \rightarrow p$ a tautology? (1)**

By use of a Truth Table; we've seen this before:

p	q	$p \wedge q$	p	$(p \wedge q) \rightarrow p$
T	T	T	T	**T**
T	F	F	T	**T**
F	T	F	F	**T**
F	F	F	F	**T**

Because the expression evaluates to true for all possible arrangements of truth values, the expression is a tautology.

Applications of Logical Equivalences (2 / 5)

Question: **Is $(p \wedge q) \rightarrow p$ a tautology? (2)**

Question: Is $(p \wedge q) \rightarrow p$ **a tautology? (3)**

Example(s): Assume that `games` and `ties` are integers.

```
if (games <= 10 or ties > 2) and games >= 11 ...
```

Applications of Logical Equivalences (5 / 5)

Question: **Are** $(p \land q) \lor (p \land r)$ **and** $p \land \overline{(\overline{q} \land \overline{r})}$

logically equivalent?