Topic 8:

Relations

Background

Having collections of data: Good.

Knowing the connections between collections: Better!

Example(s):

Relations (1 / 2)

Definition: (Binary) Relation

. . . .

Example(s):

Relations (2 / 2)

Definition: Related

Example(s):

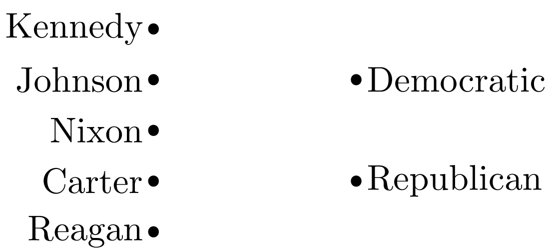
Graph Representations of Relations (1 / 2)

Example #1: Presidents–Parties

Recall: $A = \{$Kennedy, Johnson, Nixon, Carter, Reagan$\}$

$B = \{$Dem, Rep$\}$

$R = \{$(Kennedy, Dem), (Johnson, Dem),
(Nixon,Rep), (Carter, Dem), (Reagan, Rep) $\}$

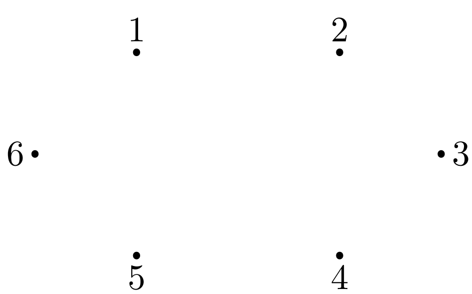


Graph Representations of Relations (2 / 2)

Example #2: $x \,\%\, y = 0, x \neq y$

Recall: $H = \{1, 2, 3, 4, 5, 6\}$

$R = \{(2,1), (3,1), (4,1), (5,1), (6,1), (4,2), (6,2), (6,3)\}$



Properties of Relations: Reflexivity

Definition: Reflexivity

. . . .

Example(s):

Properties of Relations: Symmetry (1 / 2)

Definition: Symmetry

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Example(s):

Properties of Relations: Symmetry (2 / 2)

Example(s): Graph Representations & Symmetry

Properties of Relations: Antisymmetry (1 / 2)

Definition: Antisymmetry

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Example(s):

Properties of Relations: Antisymmetry (2 / 2)

Example(s): Graph Representations & Antisymmetry

Properties of Relations: Transitivity (1 / 2)

Definition: Transitivity

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Example(s):

Properties of Relations: Transitivity (2 / 2)

Example(s):

Relational Composition Examples (1 / 4)

Three examples of creating relations from relations.

Example #1: Set Operators

Relational Composition Examples (2 / 4)

Example #2: Swapping content of ordered pairs

Definition: Inverse

Relational Composition Examples (3 / 4)

Example #3: Composites

Definition: Composite

Example(s):

Relational Composition Examples (4 / 4)

Example #3: Composites (cont.)

Example(s):

Definition: Complement

Matrix Representation of Relations (1 / 4)

(Assumption: Relations are on just one set.)

The 0-1 matrix representation of relation R on set A is $|A| \times |A|$, with both dimensions labeled identically. When $(a, b) \in R$, then `matrix[a][b]=1`. Else, `matrix[a][b]=0`.

Example(s):

Matrix Representation of Relations (2 / 4)

Observation #1: Detecting Reflexivity

$\Rightarrow$ A relation is reflexive when its corresponding matrix representation has all 1's along the main diagonal

Example(s):

Matrix Representation of Relations (3 / 4)

Observation #2: Detecting Symmetry

$\Rightarrow$ Let matrix M represent relation R. R is symmetric when $m_{ij} = 1$ *iff* $m_{ji} = 1$ is true

Example(s):

Matrix Representation of Relations (4 / 4)

Observation #3: Detecting Transitivity

⇒ Let matrix M represent relation R. R is transitive when no zero in M becomes non–zero in M^2 (or in $M^{[2]}$).

Example(s):

Equivalence Relations (1 / 4)

You may have already implemented one in Java...

Definition: Equivalence Relation

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Equivalence Relations (2 / 4)

Example(s):

Equivalence Relations (3 / 4)

So . . . why are these called *equivalence* relations?

Recall:

$$R = \{\ (0,0),\ (1,1),(1,-1),(-1,1),(-1,-1),\ (2,2),(2,-2),(-2,2),(-2,-2)\ \}$$

Equivalence Relations (4 / 4)

Definition: Equivalence Class

Example(s):

Partial Orders (1 / 3)

Consider scheduling the construction of a house.

Definition: Reflexive (a.k.a. Weak) Partial Order

Partial Orders (2 / 3)

Example(s):

Partial Orders (3 / 3)

Definition: Irreflexivity (of Relations)

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Definition: Irreflexive (a.k.a. Strict) Partial Order

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Total Orders (1 / 2)

Definition: Comparable

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Definition: Total Order

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Total Orders (2 / 2)

Example(s):