### Topic 8:

Relations

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#### Background

9 11
Having collections of data: Good.
Knowing the connections between collections: Better!
Example(s):

# Relations (1 / 2)

Definition: (Binary) Relation	
Example(s):	
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Relations (2 / 2)	
Definition: Related	1
xample(s):	

#### Graph Representations of Relations (1 / 2)

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Example #1: Presidents—Parties Recall: A = \{Kennedy, Johnson, Nixon, Carter, Reagan\}
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 $B = \{ Dem, Rep \}$ 

 $R = \{ (\text{Kennedy, Dem}), (\text{Johnson, Dem}), (\text{Senson, Dem}), (\text$ 

(Nixon,Rep), (Carter, Dem), (Reagan, Rep) }

 $\operatorname{Kennedy} \bullet$ 

 $\operatorname{Johnson} \bullet$ 

• Democratic

Nixon•

Carter•

 $\bullet$ Republican

 $\operatorname{Reagan} \bullet$ 

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#### Graph Representations of Relations (2 / 2)

Example #2:  $x \% y = 0, x \neq y$ 

Recall:  $H = \{1, 2, 3, 4, 5, 6\}$ 

$$R = \{(2,1), (3,1), (4,1), (5,1), (6,1), (4,2), (6,2), (6,3)\}$$

1

2

6.

• 3

5

4

### Properties of Relations: Reflexivity

Definition: Reflexivity		
example(s):		
		Relations – CSc 144 v1.1 (McCann) – p. 7/30
roperties of Relation	ns: Symmetry	y (1 / 2)
efinition: Symmetry		
example(s):		

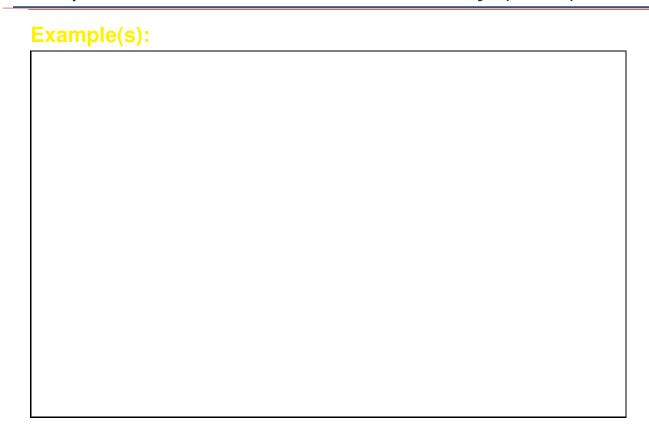
### Properties of Relations: Symmetry (2 / 2)

Example(s): C	Graph Representations & Symmetry	
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Properties	of Relations: Antisymmetry	v (1 / 2)
Properties Operation: Ant	<u>*</u>	v (1 / 2)
	<u>*</u>	v (1 / 2)
	<u>*</u>	v (1 / 2)
	<u>*</u>	(1/2)
Definition: Ant	<u>*</u>	(1 / 2)
	<u>*</u>	v (1 / 2)
Definition: Ant	<u>*</u>	(1/2)

## Properties of Relations: Antisymmetry (2 / 2)

Example(s)	Graph Re	presentation	ons & Antisy	mmetry	
				Relations – CSc 144 vi	.1 (McCann) – p. 11/30
-				Ticidations Coc 144 Vi	, , p
 Propertie	s of Rel	ations:	Transitiv		
Propertie			Transitiv		
Propertie			Transitiv		
			Transitiv		
			Transitiv		
Definition:	Transitivity		Transitiv		
	Transitivity		Transitiv		
Definition:	Transitivity		Transitiv		
Definition:	Transitivity		Transitiv		
Definition:	Transitivity		Transitiv		
Definition:	Transitivity		Transitiv		

#### Properties of Relations: Transitivity (2 / 2)



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#### Relational Composition Examples (1 / 4)

Three examples of creating relations from relations.

Example #1: Set Operators

### Relational Composition Examples (2 / 4)

Example #2: Swapping content of ordered pairs	
Definition: Inverse	
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Relational Composition Examples (3 / 4)	
Example #3: Composites	
Definition: Composite	
•	
Example(s):	

#### Relational Composition Examples (4 / 4)

ample #3: Composites (cont.)	
ample(s):	
efinition: Complement	

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#### Matrix Representation of Relations (1 / 4)

(Assumption: Relations are on just one set.)

The 0-1 matrix representation of relation R on set A is  $|A| \times |A|$ , with both dimensions labeled identically. When  $(a,b) \in R$ , then matrix[a][b]=1. Else, matrix[a][b]=0.

$(a,b)\in R$ , then matrix[a][b]=1. Else, matrix[a][b]=0.
Example(s):

#### Matrix Representation of Relations (2 / 4)

Observation #1: Detecting Reflexivity

⇒ A relation is reflexive when its corresponding matrix representation has all 1's along the main diagonal

Example(s):

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#### Matrix Representation of Relations (3 / 4)

Observation #2: Detecting Symmetry

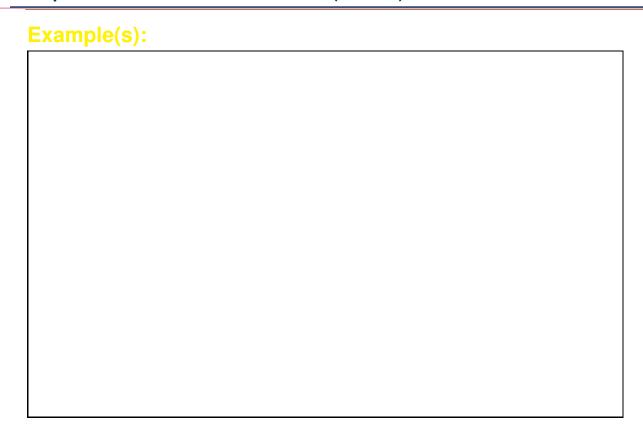
 $\Rightarrow$  Let matrix M represent relation R. R is symmetric when  $m_{ij}=1$  iff  $m_{ji}=1$  is true

#### Example(s):

### Matrix Representation of Relations (4 / 4)

Observation #3: Detecting Transitivity
$\Rightarrow$ Let matrix $M$ represent relation $R.\ R$ is transitive
when no zero in $M$ becomes non–zero in $M^2$ (or in $M^{[2]}$ ).
Example(s):
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Equivalence Relations (1 / 4)
Equivalence Relations (1 / 4)  You may have already implemented one in Java
Equivalence Relations (1 / 4)  You may have already implemented one in Java

### Equivalence Relations (2 / 4)



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#### Equivalence Relations (3 / 4)

So ... why are these called equivalence relations?

Recall:

$$R = \{ (0,0),$$

$$(1,1), (1,-1), (-1,1), (-1,-1),$$

$$(2,2), (2,-2), (-2,2), (-2,-2) \}$$

### Equivalence Relations (4 / 4)

Definition: Equivalence Class	
Example(s):	
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Partial Orders (1 / 3)	
Consider scheduling the construction of a house.	
Definition: Reflexive (a.k.a. Weak) Partial Order	

## Partial Orders (2 / 3)

Example(s):	
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Partial Orders (3 / 3)	
Definition: Irreflexivity (of Relations)	
	-
Definition: Irreflexive (a.k.a. Strict) Partial Order	
	1
	-

## Total Orders (1 / 2)

Definition: Comparable
Definition: Total Order
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Total Orders (2 / 2)
Example(s):