

Topic 9:

Functions

Functions – CSc 144 v1.1 (McCann) – p. 1/19

Functions as Relations (1 / 2)

Consider: $f(x) = x + 1, x \in \mathbb{Z}$

Definition: Function

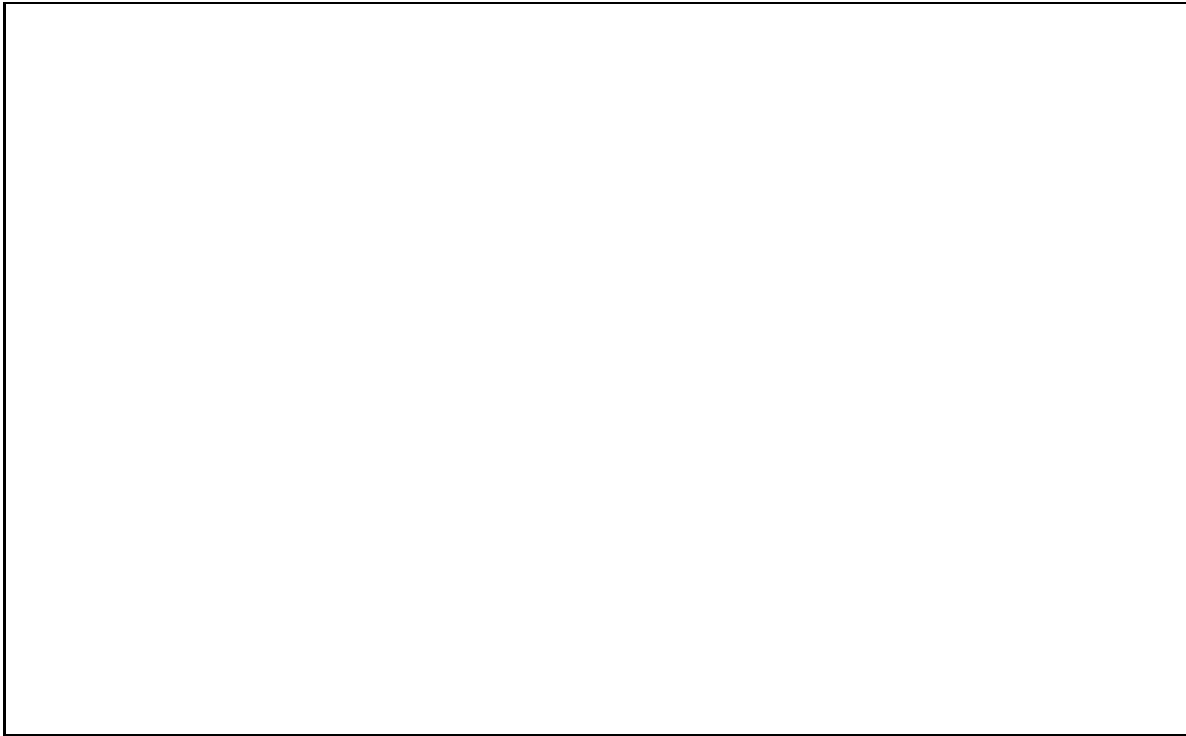
.....

.....

Functions – CSc 144 v1.1 (McCann) – p. 2/19

Functions as Relations (2 / 2)

Example(s):



Functions – CSc 144 v1.1 (McCann) – p. 3/19

Function Terms (1 / 2)

Let $f : X \rightarrow Y$ be a function. $f(n) = p$ [$(n, p) \in f$].

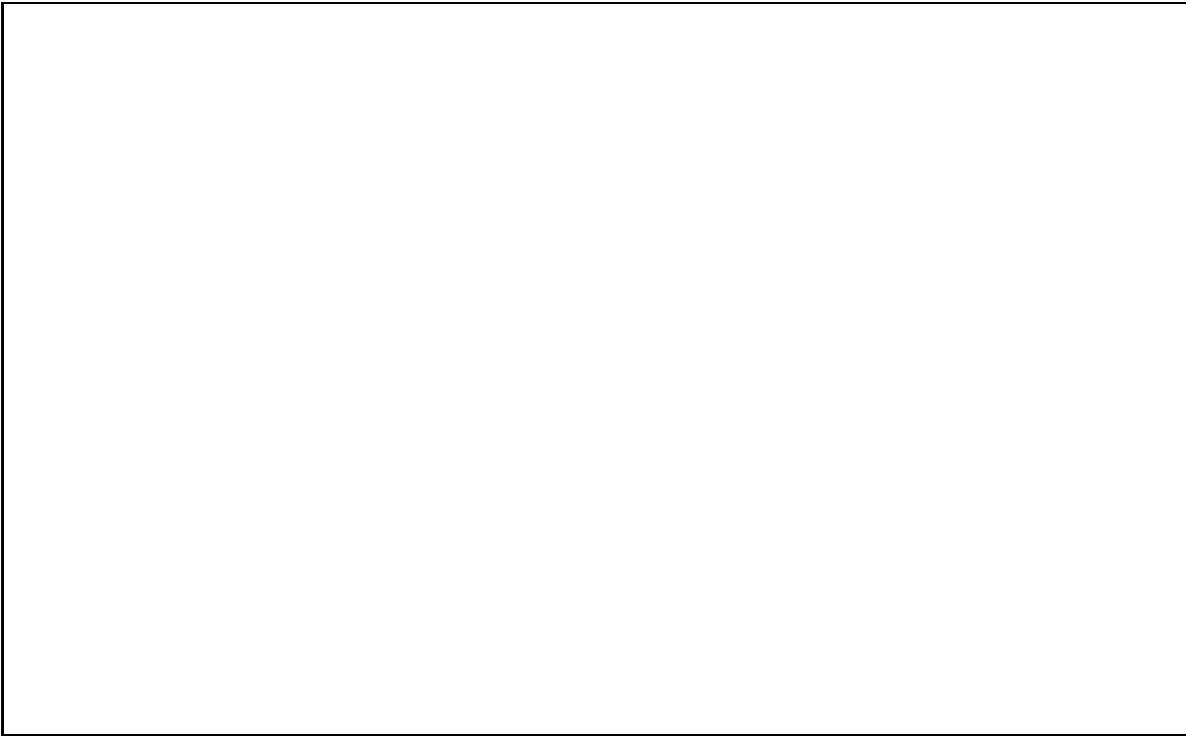
- X is the _____ of f
- Y is the _____ of f
- f _____ X to Y
- p is the _____ of n
- n is the _____ of p
- f 's _____ is the set of all images of X 's elements

Note: A function's range need not equal its codomain.

Functions – CSc 144 v1.1 (McCann) – p. 4/19

Function Terms (2 / 2)

Example(s):

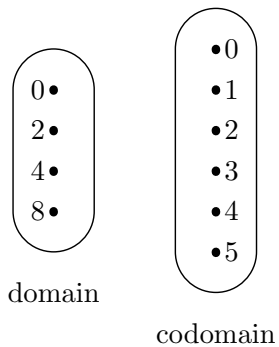


Functions – CSc 144 v1.1 (McCann) – p. 5/19

Digraph Representation (1 / 2)

Example(s):

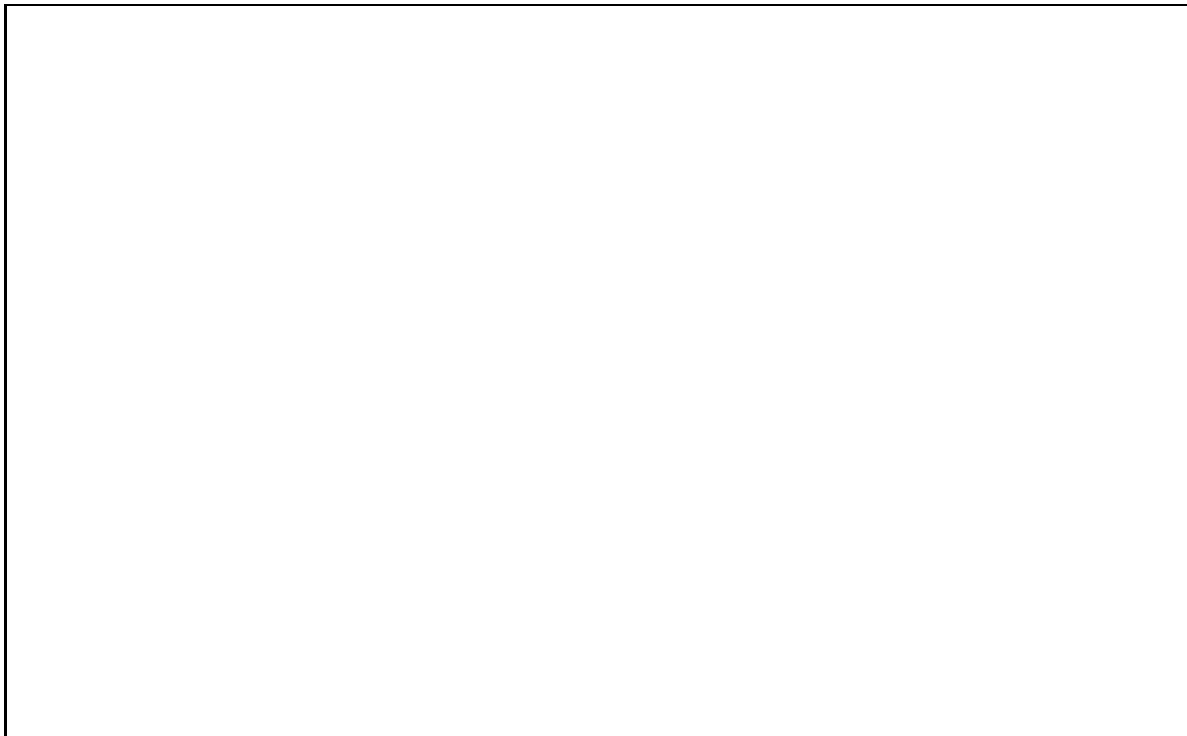
$$g = \{ (a, b) \mid b = a/2 \}, \quad a \in \{0, 2, 4, 8\}, \\ b \in \{0, 1, 2, 3, 4, 5\}$$



Functions – CSc 144 v1.1 (McCann) – p. 6/19

Digraph Representation (2 / 2)

Example(s):




Functions – CSc 144 v1.1 (McCann) – p. 7/19

Two Functions You Need To Know (1 / 4)

1. Floor ($\lfloor x \rfloor$)

Definition: Floor Function



Example(s):



Functions – CSc 144 v1.1 (McCann) – p. 8/19

Two Functions You Need To Know (2 / 4)

1. Floor ($\lfloor x \rfloor$) (cont.)

Using Floor for Rounding to the Nearest Integer

Functions – CSc 144 v1.1 (McCann) – p. 9/19

Two Functions You Need To Know (3 / 4)

2. Ceiling ($\lceil x \rceil$)

Definition: Ceiling Function

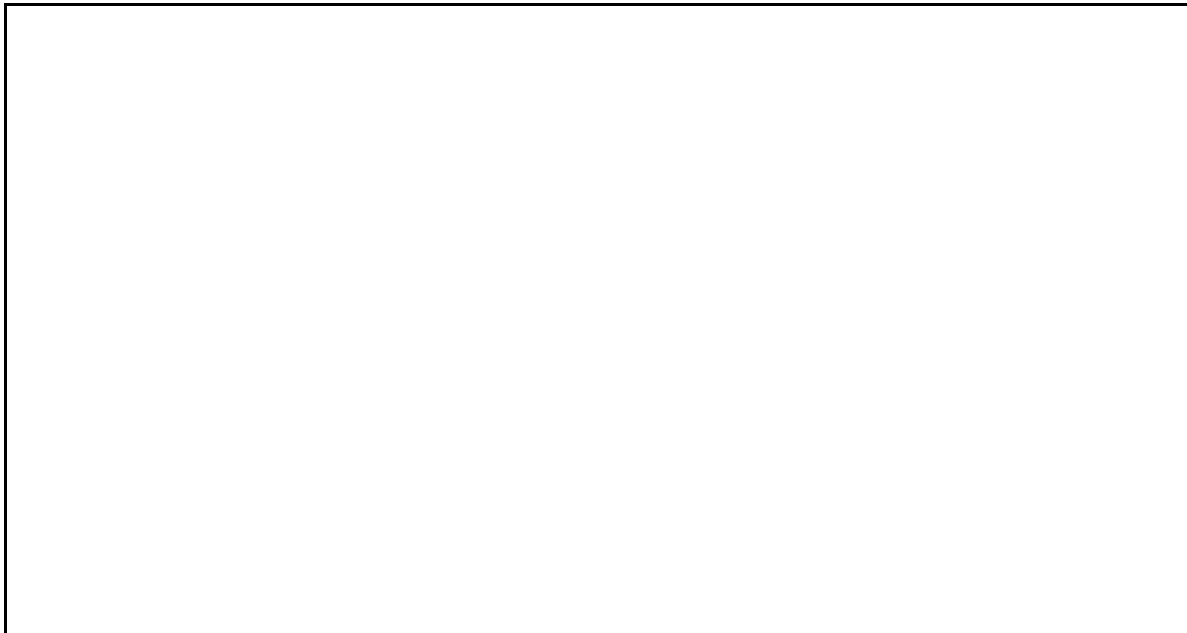
Example(s):

Functions – CSc 144 v1.1 (McCann) – p. 10/19

Two Functions You Need To Know (4 / 4)

2. Ceiling ($\lceil x \rceil$) (cont.)

Example(s):



Functions – CSc 144 v1.1 (McCann) – p. 11/19

Example: Type A UPC Code Check Digits



The check digit equals the image of this function:

s = Sum of digits in positions 1, 3, 5, 7, 9, & 11

t = Sum of digits in positions 2, 4, 6, 8, & 10

$u = 3s + t$; the check digit is $(10 - u \% 10) \% 10$.

Using the above sample:

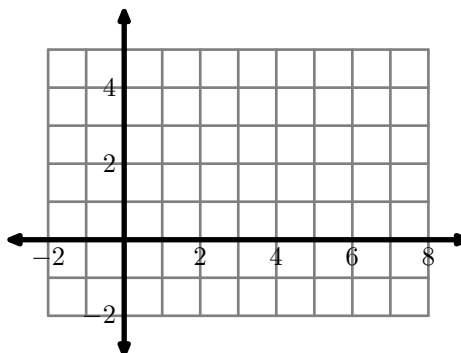
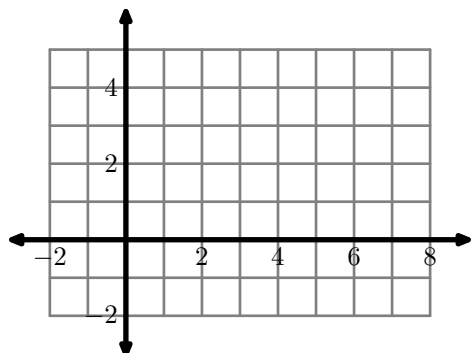
$s = 39$, $t = 24$, and $u = 3(39) + 24 = 141$.

The check digit = $(10 - 141 \% 10) \% 10 = 9$.

Graphs Of Functions (1 / 2)

Important Distinction: *Continuous* vs. *Discontinuous* Functions

Consider: $f = \{(x, x + 1) \mid x \in \dots\}$

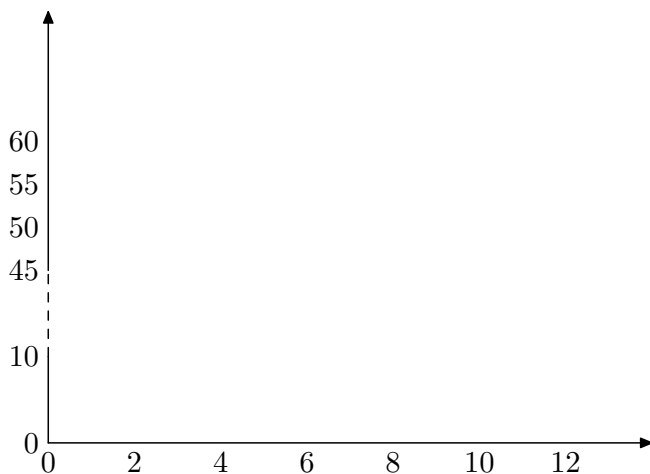


Functions – CSc 144 v1.1 (McCann) – p. 13/19

Graphs Of Functions (2 / 2)

How should the graph of our long-distance calling plan function look?

$$\text{Cost}(\text{length}) = \begin{cases} 50 \text{ cents} & \text{if length} \leq 10 \text{ minutes} \\ 50 + 5 \cdot \lceil \text{length} - 10 \rceil \text{ cents} & \text{Otherwise} \end{cases}$$



Functions – CSc 144 v1.1 (McCann) – p. 14/19

Categories of Functions: Injective

Definition: Injective Functions (a.k.a. One-to-one)

.....

Example(s):

Categories of Functions: Surjective

Definition: Surjective Functions (a.k.a. Onto)

.....

Example(s):

Categories of Functions: Bijective

Definition: Bijective Functions (a.k.a. One-to-one Correspondence)

Example(s):

Odds and Ends

Definition: Functional Composition

Let $f : Y \rightarrow Z$ and $g : X \rightarrow Y$. The composition of f and g , denoted $f \circ g$, is the function $h = f(g(x))$, where $h : X \rightarrow Z$.

Definition: Inverse Functions

.....

.....

Beyond Unary Functions

Definition: Binary Functions

Example(s):