Topic 12:

Sequences and Strings

Sequences

Definition: Sequence [1st Attempt]

Notation:

Example(s):

Rules

Recall: $$\sum_{i=1}^{n} 2i$$

Example(s):

Two Notations for Infinite Sequences:

Sequences and Functions

Definition: Sequence [Final Version]

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Example(s):

Arithmetic and Geometric Sequences

Definition: **Arithmetic Sequence (a.k.a. Arithmetic Progression)**

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Definition: **Geometric Sequence (a.k.a. Geometric Progression)**

. . . .

Example(s):

Arithmetic Series

The sum of the terms of an arithmetic sequence (a.k.a. arithmetic series): $\boxed{s_n = a_1 + \ldots + a_n = \frac{1}{2}n(a_1 + a_n)}$

Here's why: First, note that $a_n = a_1 + (n-1)d$.

Next, here are two expressions for s_n:

$$s_n = a_1 + (a_1 + d) + (a_1 + 2d) + \ldots + (a_1 + (n-1)d)$$
$$s_n = (a_n - (n-1)d) + (a_n - (n-2)d) + \ldots + (a_n - d) + a_n$$

Sum these expressions, and the d terms cancel, leaving:

$2s_n = na_1 + na_n$, or $s_n = \frac{1}{2}n(a_1 + a_n)$.

Geometric Series

The sum of the terms of a geometric sequence also has an expression: $s_n = g_1 + \ldots + g_n = \frac{g_1(1-r^n)}{1-r}$, assuming that $r \neq 1$. (If $r = 1$, $s_n = n \cdot g_1$.) Here's how to get it:

$$\begin{aligned}
s_n &= g_1 + g_1 r + g_1 r^2 + \ldots + g_1 r^{n-1} \\
r \cdot s_n &= g_1 r + g_1 r^2 + g_1 r^3 + \ldots + g_1 r^n \\
s_n - r \cdot s_n &= g_1 - g_1 r^n \\
s_n(1-r) &= g_1(1-r^n) \\
s_n &= \frac{g_1(1-r^n)}{1-r}
\end{aligned}$$

Increasing Sequences

Definition: Increasing Sequence

Definition: Non-Decreasing Sequence

Definition: Strictly Increasing Sequence

Decreasing Sequences

Definition: Decreasing Sequence

Definition: Non-Increasing Sequence

Definition: Strictly Decreasing Sequence

Examples: Increasing/Decreasing Sequences

Subsequences

Definition: Subsequence

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Example(s):

Need to Identify a Sequence?

A great resource for sequences:

The Online Encyclopedia of Integer Sequences

(http://oeis.org/)

Example(s):

Strings (1 / 2)

Somewhat beyond the programming language kind . . .

Definition: String

. . . .

Example(s):

Strings (2 / 2)

Notation:

- Lambda (λ) represents the empty (null) string
- xy means strings x and y are concatenated
- Superscripts denote repetition of concatenation
- $|x|$ represents the length of string x
- A^* is the set of strings that can be formed using elements of an alphabet A.
  - A^* is an infinite set
  - $\lambda \in A^*$

Set Cardinality Revisited (1 / 5)

An observation about set cardinality:

Definition: Finite

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Set Cardinality Revisited (2 / 5)

Definition: Countably Infinite (a.k.a. Denumerably Infinite)

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Definition: Countable

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Set Cardinality Revisited (3 / 5)

Example(s):

Set Cardinality Revisited (4 / 5)

Question: **Are the positive rational numbers countable?**

Set Cardinality Revisited (5 / 5)

Conjecture: A pairing function for $\mathbb{R}$ cannot exist.

Now You Can Understand More Cartoons! (1/2)

Background: Elephant jokes became popular form of absurdist humor in the U.S. in the 1960s. For example:

Q: How many elephants can fit in a Jeep?
A: Four – Two in the front and two in the back.

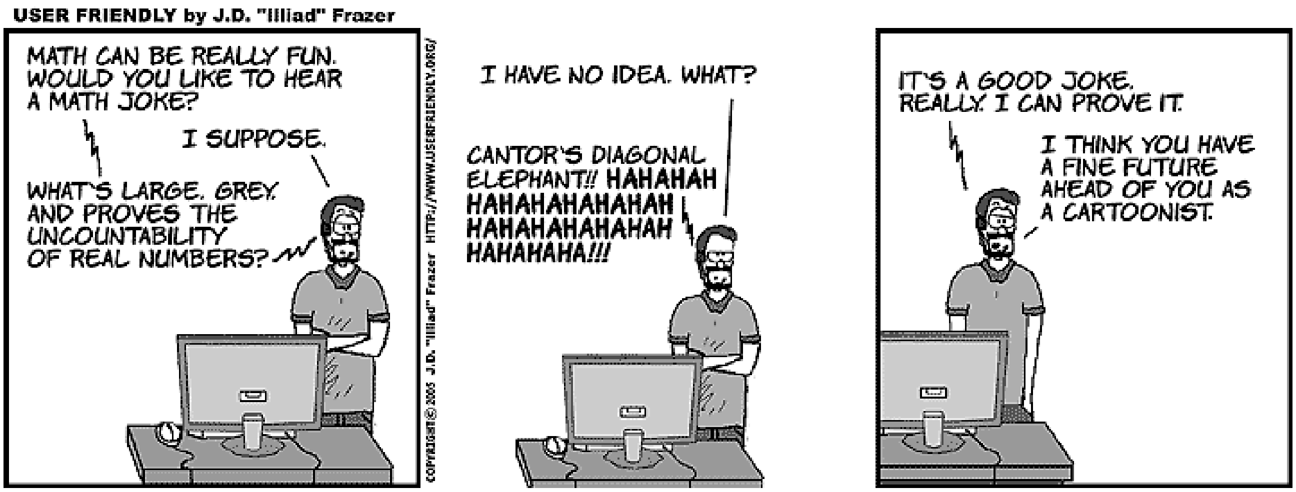
Q: How many bison can fit in a Jeep?
A: None – it's full of elephants.

Q: How do you know when there are two elephants in your closet?
A: You hear giggling when the door is closed.

Q: How do you know when there are three elephants in your closet?
A: You can't close the door.

Q: How do you know when there are four elephants in your closet?
A: There's an empty Jeep in the driveway.

Now You Can Understand More Cartoons! (2/2)



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