CSc 144–002 — Discrete Mathematics for Computer Science I — Fall 2024 (McCann)
https://cs.arizona.edu/classes/cs144/fall24-002/

Homework #3

(50 points)

Due Date: October 11th, 2024, at the beginning of class

Directions

1. **This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**
2. Start early! Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via piazza.com and our office hours.
3. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.
4. When your PDF is ready to be turned in, do so on gradescope.com. Be sure to assign pages to problems after you upload your PDF. Need help? See "Submitting an Assignment" on https://help.gradescope.com/.
5. **Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**

Topic: Quantification

1. (12 points) For each of the following English statements, do the following: (i) Express the statement as an equivalent quantified logical expression using appropriate predicates and domains, (ii) Negate the logical expression from (i) and use Generalized De Morgan's Laws to move the negation inside of the quantifier(s), and (iii) Express the result of (ii) in conversational English (you may wish to perform additional operations to make this conversion easier).
   (a) Every Halloween is both fun and spooky.
   (b) Some bright white light bulbs are halogen.

2. (6 points) Using our logical constructions for "exactly one" and "exactly two" expressions, convert these English sentences to logic such that their meanings are preserved.
   (a) Exactly one woman was the first programmer.
   (b) A coin has exactly two sides.

3. (6 points) As discussed in class, "exactly two" statements can be viewed as the intersection of "at most two" and "at least two." Convert these English sentences to logic such that their meanings are preserved.
   (a) Ryan's motorcycle can carry at most two people.
   (b) At least two months have 31 days.

(Continued on the back . . .)

Topic: Arguments

4. (4 points) Which type of reasoning (inductive or deductive) does each of the following arguments best demonstrate, and why?

   (a) Ally's father bought her a balloon. She let it slip from her fingers and it rose into the sky. He bought her another, and it, too, flew away. If she somehow cries enough to get him to buy her a third, and she lets it loose on purpose, she hopes that it will also fly away.

   (b) When Wilhelm saw the crew unloading safety lines and harnesses, he knew they were going to be shingling the church's roof today, not sealing its parking lot.

5. (4 points) Each of the following arguments uses predicates and quantification. Identify the rule of inference for predicates used by each, and explain how you know that your choice is the appropriate rule for the situation.

   (a) "I've watched each ant in the anthill. Without exception, each one is hard–working. I conclude that every ant in the anthill is hard–working."

   (b) "One of the boxes of Cracker Jack on the shelf has a special secret toy surprise inside. Start opening them until you find which one it is."

6. (12 points) For each set of propositional hypotheses below, create two distinct, complete arguments that logically reach the provided conclusion proposition. By 'distinct,' we mean that they can share some steps, but they cannot share all steps. (Simply re–ordering some of the steps of the first argument does not create a second 'distinct' argument.) By 'complete,' we mean that each is to be written as a stand–alone argument. Be sure that you justify each step of each argument.

   (a) Hypotheses: $p \to q, r \to p, \neg q$. Conclusion: $\neg r$

   (b) Hypotheses: $a \wedge b, a \to \overline{c}, c \vee d$. Conclusion: d

7. (6 points) For the set of quantified hypotheses below, create an argument using rules of inference for predicates (and other logical principles as needed) that logically reaches the provided quantified conclusion.

   - Hypotheses: $\exists x A(x), \forall x(A(x) \to (B(x) \wedge C(x)))$ Conclusion: $\exists x(B(x) \wedge C(x))$