CSc 144–002 — Discrete Mathematics for Computer Science I — Fall 2024 (McCann)
https://cs.arizona.edu/classes/cs144/fall24-002/

Homework #5

(50 points)

Due Date: November 8^{th}, 2024, at the beginning of class

Directions

1. **This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**
2. Start early! Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via piazza.com and our office hours.
3. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.
4. When your PDF is ready to be turned in, do so on gradescope.com. Be sure to assign pages to problems after you upload your PDF. Need help? See "Submitting an Assignment" on https://help.gradescope.com/.
5. **Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**

Topic: Matrices

(Yes, I'm backing up a bit into matrix topics that were on Exam #2. The scores on the matrix question suggest that some more practice on matrix operations wouldn't hurt.)

1. (12 points) Consider the matrices $D = \begin{bmatrix} 5 & -2 \\ 3 & 1 \\ 0 & 2 \end{bmatrix}$, $E = \begin{bmatrix} 4 & 2 & -4 \end{bmatrix}$, $F = \begin{bmatrix} 1 & 4 \\ -1 & 0 \\ -3 & -2 \end{bmatrix}$, and $G = \begin{bmatrix} 5 & 2 \\ 0 & 3 \end{bmatrix}$.
   (a) Compute: D^T.
   (b) Compute: $D + F$.
   (c) Which pairs of these matrices can be multiplied together, and what is the size of the results of those matrix products? Note: We're **not** asking you to perform the multiplications. (Don't forget to consider multiplying matrices by themselves!)
   (d) Compute: G^3.

2. (6 points) Zero–One Matrices. Let $H = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ and $J = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$.
   (a) Compute: H join J.
   (b) Compute: $J^{[3]}$.

3. (6 points) Prove (using a direct proof) that the 'meet' of a Zero–One matrix with itself equals itself. (That is, prove that $A \wedge A = A$.)

(Continued . . .)

Topic: Relations

4. (4 points) Using the 'set of ordered pairs' representation of relations, what is the relation that results from each of the following descriptions, where $X = \{-2, -1, 0, 1, 2\}$ and $Y = \{0, 1, 2, 3\}$?

   (a) $S = \{(x, y) \mid y - x > 0\}$

   (b) $T = \{(x, y) \mid x + y \text{ is even}\}$

5. (4 points) Consider the relation $R = \{(Q, M), (P, Q), (P, M), (P, P), (O, M), (O, N), (O, O), (O, P), (O, Q), (N, M), (N, N), (N, P)\}$ on the set $\{M, N, O, P, Q\}$. Is R reflexive? Symmetric? Antisymmetric? Transitive? Show your reasoning/work! Answers without it will be considered incorrect.

6. (4 points) Consider the set of all students at the U of Arizona, and the relation $S = \{(x, y) \mid x \text{ is a sister of } y\}$ on that set. Is S reflexive? Symmetric? Antisymmetric? Transitive? Show your reasoning/work! Answers without it will be considered incorrect.

7. (4 points) Let $Y = \{(-1, a), (-4, e), (-9, e), (-9, i), (-16, u)\}$ from $\{-1, -4, -9, -16\}$ to $\{a, e, i, o, u\}$, and $Z = \{((e, 19), (e, 17), (e, 13), (u, 17)\}$ from $\{a, e, i, o, u\}$ to $\{13, 15, 17, 19\}$. What is the result of the evaluation of $Z \circ Y$? If $Z \circ Y$ cannot be evaluated, explain why not.

8. (4 points) Consider the following zero–one matrix representation of a relation on the set $\{1, 3, 5, 7, 9\}$. Assuming that the rows and columns of the matrix are both labeled with those five values in the order given, what are both the 'set of ordered pairs' and the directed graph (digraph) representations of the same relation?

$$M = \begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

9. (6 points) Which of these relations are equivalence relations? Explain which of the equivalence relation definition's properties are possessed by the relation (if any) and which are not (again, if any).

   (a) $\{(d, d), (d, b), (c, c), (c, a), (b, d), (b, b), (a, c), (a, a)\}$ on $\{a, b, c, d\}$

   (b) $\{(g, h) \mid g \text{ and } h \text{ have played together}\}$ on the domain of 'puppies.'