Homework #6

(50 points)

Due Date: November 15th, 2024, at the beginning of class

Directions

1. **This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**
2. Start early! Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via piazza.com and our office hours.
3. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.
4. When your PDF is ready to be turned in, do so on gradescope.com. Be sure to assign pages to problems after you upload your PDF. Need help? See "Submitting an Assignment" on https://help.gradescope.com/.
5. **Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**

Topic: Relations

1. (6 points) A bit–string is a string of zeros and ones. For example, '1', '00', and '01001' are all bit–strings. Determine whether or not each of the following relations of bit–strings are equivalence relations, and explain how you know your answer to be correct. *Hint:* Think about "through length 3" and "... 4" before trying 10.
   (a) $R = \{(x, y) \mid x$ and y have the same first two bits$\}$ on the set of all bit–strings of length two through length 10.
   (b) $R = \{(x, y) \mid x$ and y may differ only in their first three bits$\}$ on the set of all bit–strings of length three through length 10.
2. (4 points) Consider the equivalence relation $E = \{(6,6),(6,8),(6,9),(7,7),(8,6),(8,8),(8,9),(9,6),(9,8),(9,9)\}$ on $\{6,7,8,9\}$. Which elements are in each of these equivalence class sets?
   (a) $[8]$
   (b) $[9]$
3. (6 points) Determine which variety of partial order (weak, strict, or neither) correctly describes each of the following relations.
   (a) $\{(a,a),(b,b),(c,c)\}$ on $\{a,b,c\}$
   (b) $\{(1,1),(1,2),(2,1),(1,3),(3,1),(3,3)\}$ on $\{1,2,3\}$
   (c) $\{(a,a),(a,c),(a,d),(b,b),(b,c),(b,d),(c,c),(d,d)\}$ on $\{a,b,c,d\}$
4. (2 points) Let $R = \{(a,b) \mid$ "$a \mid b$" is true$\}$ on $F = \{3,4,5,6,7,8,9,10,11,12\}$. Ignoring the (a,a) pairs, which pairs of elements of F are comparable in R?
5. (4 points) Each of the following relations is a weak partial order. Are they also total orders? If yes, explain why. If not, what must be added to the relation to make it a total order?
   (a) $\{(1,1),(1,4),(2,2),(2,3),(3,3),(4,4)\}$ on $\{1,2,3,4\}$
   (b) $\{(d,d),(d,c),(d,b),(c,c),(b,c),(b,b),(a,d),(a,c),(a,b),(a,a)\}$ on $\{a,b,c,d\}$

(Continued on the back . . .)

Topic: Functions

6. (4 points) For each of the following, is it a function from $\mathbb{Z}$ to $\mathbb{R}$? If the answer is 'No,' explain why.

   (a) $g(x) = \pm x$ *("±4" is a short–cut way of saying "4 and −4")*

   (b) $n(x) = \sqrt{|\ x\ |}$ *(that is, the square root of the absolute value of x)*

7. (4 points) For each of these functions, what are the domains and the ranges? (To find the domains, figure out what is legal input to the function as described.)

   (a) The function that returns the integer defined by the rightmost pair of digits of an integer. For example, $f(-701) = 1$.

   (b) The function that returns the quantity of digits in the decimal representation of a positive integer. For example, $f(6718) = 4$.

8. (4 points) Evaluate each of these functions.

   (a) $\lceil 4.999 \rceil$

   (b) $\lfloor -0.12 \rfloor$

   (c) $\lfloor \lceil -\frac{5}{2} \rceil \rfloor$

   (d) $\lfloor \frac{7}{5} - \lceil \frac{7}{5} \rceil + \lfloor \frac{7}{5} \rfloor \rfloor$

9. (4 points) By hand, draw a graph of each of the following functions. (We know that it is tempting to use a function plotting app or website to do these, but you won't have access to one on quizzes or on exams, so it's best to do these yourself. Also, those tools are known to handle less–common functions incorrectly.)

   (a) $f(x) = x^2 - 2x$ on the domain of integers $\{-2 \ldots 2\}$, inclusive.

   (b) $f(x) = \lceil |\ x\ | \rceil$ on the domain of reals $\{-4 \ldots 4\}$, inclusive.

10. (12 points) Python Programming: Magic Squares

    Write a complete Python 3 program that finds a 3×3 magic square containing the integers 1 through $3 * 3 = 9$ using the approach described below. For our purposes, a 3×3 matrix's content is "magic" because the sums of all 3 rows, all 3 columns, and both diagonals of the matrix are the same. A 3×3 magic square is shown to the right.

8	1	6
3	5	7
4	9	2

    The sums are all 15, which is known as the square's *magic constant*. The magic constant can be computed in advance of the square's construction; it is $\frac{n^3+n}{2}$ for an $n \times n$ magic square.

    There are algorithms for computing the content of magic squares, but for this program we're having you brute–force it: Make a list of integers 1 through 9. Mix them up (see below for how!), test the result to see if it forms a magic square, and repeat the mixing and testing until a magic square is formed. When one is found, display the content of the square and also display the quantity of mixed–up lists your program had to create and test to find that magic square. (The output format of the square is up to you, but it must look square–ish. The minimally acceptable 'square' is three lines of output, with one space between the numbers on each line, but fancier output is fine.)

    For example, if your program produces the mixed–up list 8, 1, 6, 3, 5, 7, 4, 9, 2, the output would be the square shown above. That is, fill the matrix row by row with the values, starting at the top.

    Here's how to "mix–up" a list of values: For each index i in the list (that is, 0 through 8), generate a random integer r in the range 0 .. 8 and swap the value at index i with the value at index r. Easy! (This is called forming a *permutation* of the list's values.)

    When your program is working, copy/paste a screenshot of your code and a screenshot of the output from one execution of your program into your homework PDF as the answer to this question.

    (Got some time on your hands? Try increasing the size of the magic square from 3×3 to 4×4. ☺)