CSc 144 — Discrete Mathematics for Computer Science I
Spring 2024 (McCann)

Collected Definitions for Exam #1

I can't recall the last time I didn't ask a definition question on a discrete math exam. To help you better prepare yourself for such questions, I've assembled this list. My pledge to you: If I ask you for a definition on the exam, the term will come from this list. Note that this is not a complete list of the definitions given in class. You should know the others, too, but I won't specifically ask you for their definitions on the exam.

Once in a while a student will express disappointment that I ask definition questions on exams. My justification is that I think it's important for you to know what the core terms mean so that you can use them correctly and effectively. At the same time, I don't require that you memorize the exact wording of the definitions you see here. If you provide a definition in your own words that captures all of the detail found here, that's fine.

The definitions are grouped by lecture topic, and should be in an order within each topic that is at least close to the order in which the definitions appeared in class.

Topic 1: Course Background

- *Discrete Mathematics* is the study of collections of distinct objects.
- Let $\triangle$ be a binary infix operator. If $x \triangle y = y \triangle x$, then $\triangle$ is a *commutative* operator.
- Let $\triangle$ be a binary infix operator. If we have multiple $\triangle$ operators in an expression, and we can group either operator with parentheses without changing the result (that is, $(a \triangle b) \triangle c = a \triangle (b \triangle c)$), then $\triangle$ is an *associative* operator.
- Let $\triangle$ and $\square$ be binary infix operators. If $a \triangle (b \square c) = (a \triangle b) \square (a \triangle c)$ and $(b \square c) \triangle a = (b \triangle a) \square (c \triangle a)$, then $\triangle$ *distributes* over $\square$.
- A value that can be expressed as the ratio of two integers is a *rational* number.
- x *divides* y (written "$x \mid y$") if $y \,\%\, x = 0$.
- Values b and r are said to be *congruent modulo* m (written $b \equiv r \pmod{m}$) iff $b \,\%\, m = r \,\%\, m$ (or, iff $m \mid (b - r)$).

Topic 2: Logic

- *Philosophical Logic* is the classical notion of 'logic': The study of thought and reasoning, including arguments and proof techniques.
- *Mathematical Logic* is the use of formal languages and grammars to represent the syntax and semantics of computation.
- A *Well-Formed Formula (wff)* is a correctly structured expression of a language.
- A *proposition* (a.k.a. *statement*) is a claim that is either true or false with respect to an associated context.
- A *simple proposition* is a proposition containing no logical operators.
- A claim that is a logical combination of multiple simple propositions is a *compound proposition.*

(Continued . . .)

$\boxed{\text{Topic 2: Logic}}$ (continued!)

- Two propositions p and q are *(Logically) Equivalent* (written $p \equiv q$) when both evaluate to the same result when presented with the same input. [Note: An alternate, equally–correct definition is given below.]
- A *Tautology* is a proposition that always evaluates to true.
- A *Contradiction* is a proposition that always evaluates to false.
- A *Contingency* is a proposition that is neither a tautology nor a contradiction.
- A conditional proposition whose antecedent is false is *vacuously true*.
- The *Inverse* of $p \to q$ is $\overline{p} \to \overline{q}$.
- The *Converse* of $p \to q$ is $q \to p$.
- The *Contraposition* of $p \to q$ is $\overline{q} \to \overline{p}$.
- p and q are *(Logically) Equivalent* (written $p \equiv q$) if $p \leftrightarrow q$ is a tautology. [Note: An alternate, equally–correct definition is given above.]

$\boxed{\text{Topic 3: Quantification}}$

- A statement that includes at least one variable and will evaluate to either true or false when the variable(s) are assigned value(s) is a *Predicate* (a.k.a. *Propositional Function*).
- The collection of values from which a variable's value is drawn is known as the *Domain of Discourse* (a.k.a. *Universe of Discourse*).
- A quantified variable in a predicate is a *Bound* variable.
- Unquantified variables are *Free* (a.k.a. *Unbound*) variables.
- The *Generalized De Morgan's Laws* are the equivalences $\overline{\forall x\, P(x)} \equiv \exists x\, \overline{P(x)}$ and $\overline{\exists x\, Q(x)} \equiv \forall x\, \overline{Q(x)}$