Topic 6:

Additional Set Concepts

Set Concepts Already Covered

You may wish to review these basic set concepts, previously covered in the Math Review appendix, before starting this topic:

- Properties of sets (e.g., duplicate members are not allowed)
- Set notation (membership, set builder notation, etc.)
- Operators (union, intersection, difference, complement, cardinality)
- Venn diagrams

Why Are We Learning More About Sets?

Sets are foundational in many areas of Computer Science.

For example:

Subsets

Definition: Subset

Definition: Proper Subset

Example(s):

Set Equality

Definition: Set Equality

Example(s):

Power Sets

Definition: Power Set

Example(s):

Generalized Forms of $\cup$ and $\cap$

Remember summation and product notations? E.g.:

$$\sum_{n=0}^{9}(2n+1)$$

Similar notation is used to generalize the union and intersection operators.

Assuming that $A_1 \ldots A_m$ and $B_1 \ldots B_n$ are sets, then:

Two More Set Properties

Definition: Disjoint

Definition: Partition

Example(s):

Examples of Set Identities

Look familiar?

Associativity $(A \cap B) \cap C = A \cap (B \cap C)$

$(A \cup B) \cup C = A \cup (B \cup C)$

Commutativity $A \cap B = B \cap A$

$A \cup B = B \cup A$

Distributivity $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

De Morgan $\overline{A \cup B} = \overline{A} \cap \overline{B}$

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

Note: As with logical identities, you need not memorize set identities.

Set Builder Notation

Often, set contents are easier to describe than to list:

Describe: $10 \leq x \leq 99, x \in \mathbb{Z}$

List: $10, 11, 12, 13, \ldots, 98, 99$

To describe a set's content, we use *set builder notation:*

Form: $\{n \mid$ description of the legal values of $n\}$

Example(s):

Expressing Set Operations in Logic

We've seen the first two already.

$$X \subseteq Y \equiv \forall z\,(z \in X \rightarrow z \in Y)$$

$$X \subset Y \equiv \forall z\,(z \in X \rightarrow z \in Y) \wedge \exists w\,(w \notin X \wedge w \in Y)$$

For those that return sets, Set Builder notation is a good choice:

Proving Set Identities (1 / 4)

To prove that set expressions S and T are equal, we can:

1. Prove that $S \subseteq T$ and $T \subseteq S$, or
2. Convert the equality to logic, prove it, and convert back

Example(s):

Proving Set Identities (2 / 4)

Conjecture: $S \cup \mathcal{U} = \mathcal{U}$

Proving Set Identities (3 / 4)

Proving Set Identities (4 / 4)

Conjecture: $S \cup \mathcal{U} = \mathcal{U}$

Final Set Operator: Cartesian Product (1 / 2)

Definition: Ordered Pair

Example(s):

Final Set Operator: Cartesian Product (2 / 2)

Definition: Cartesian Product

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Example(s):

Notes:

Example: Computer Representation of Sets