Topic 9:

Functions

Functions as Relations (1 / 2)

Consider: $f(x) = x + 1, x \in \mathbb{Z}$

Definition: Function

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. . . .

Functions as Relations (2 / 2)

Example(s):

Function Terms (1 / 2)

Let $f : X \to Y$ be a function. $f(n) = p\ [\ (n, p) \in f\ ]$.

- X is the ____________ of f
- Y is the ______________ of f
- f ____________ X to Y
- p is the ____________ of n
- n is the ______________ of p
- f's __________ is the set of all images of X's elements

Note: A function's range need not equal its codomain.

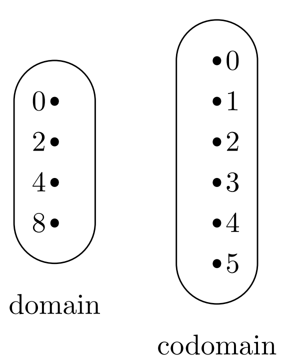
Function Terms (2 / 2)

Example(s):

Digraph Representation (1 / 2)

Example(s):

$$g = \{\ (a,b) \mid b = a/2\ \},\ a \in \{0,2,4,8\},$$
$$b \in \{0,1,2,3,4,5\}$$



Digraph Representation (2 / 2)

Example(s):

Two Functions You Need To Know (1 / 4)

1. Floor $(\lfloor x \rfloor)$

Definition: Floor Function

Example(s):

Two Functions You Need To Know (2 / 4)

1. Floor $(\lfloor x \rfloor)$ (cont.)

Using Floor for Rounding to the Nearest Integer

Two Functions You Need To Know (3 / 4)

2. Ceiling $(\lceil x \rceil)$

Definition: Ceiling Function

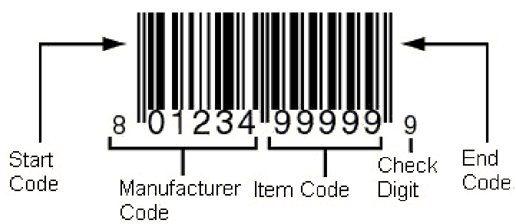
Example(s):

Two Functions You Need To Know (4 / 4)

2. Ceiling $(\lceil x \rceil)$ (cont.)

Example(s):

Example: Type A UPC Code Check Digits



The check digit equals the image of this function:

s = Sum of digits in positions 1, 3, 5, 7, 9, & 11

t = Sum of digits in positions 2, 4, 6, 8, & 10

$u = 3s + t$; the check digit is $(10 - u\%10)\%10$.

Using the above sample:

$s = 39$, $t = 24$, and $u = 3(39) + 24 = 141$.

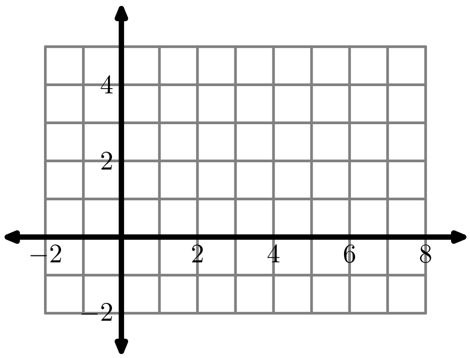
The check digit $= (10 - 141\%10)\,\%\,10 = 9$.

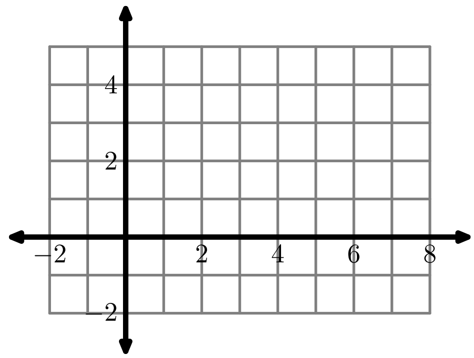
Image Credit: www.nasaexplores.com

Graphs Of Functions (1 / 2)

Important Distinction: *Continuous* vs. *Discontinuous* Functions

Consider: $f = \{(x, x+1) \mid x \in \ldots\}$

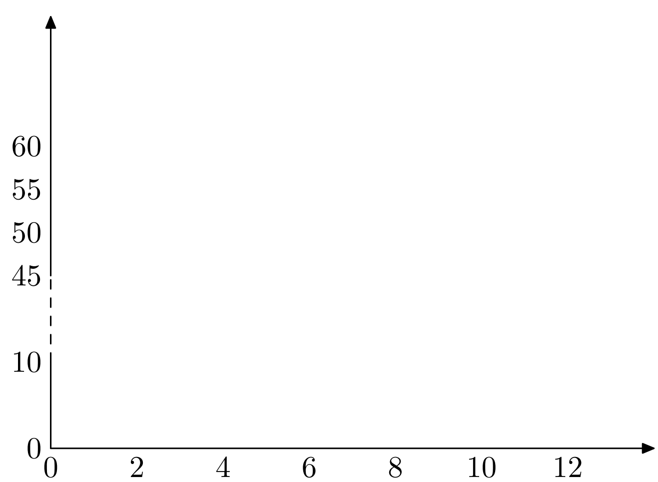




Graphs Of Functions (2 / 2)

How should the graph of our long-distance calling plan function look?

$$\text{Cost(length)} = \begin{cases} 50 \text{ cents} & \text{if length} \leq 10 \text{ minutes} \\ 50 + 5 \cdot \lceil \text{length} - 10 \rceil \text{ cents} & \text{Otherwise} \end{cases}$$



Categories of Functions: Injective

Definition: Injective Functions (a.k.a. One-to-one)

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Example(s):

Categories of Functions: Surjective

Definition: Surjective Functions (a.k.a. Onto)

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Example(s):

Categories of Functions: Bijective

Definition: Bijective Functions (a.k.a. One-to-one Correspondence)

Example(s):

Odds and Ends

Definition: Functional Composition

Let $f : Y \rightarrow Z$ and $g : X \rightarrow Y$. The composition of f and g, denoted $f \circ g$, is the function $h = f(g(x))$, where $h : X \rightarrow Z$.

Definition: Inverse Functions

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Beyond Unary Functions

Definition: Binary Functions

Example(s):