Topic 10:

Indirect (“Contra”) Proofs of $p \to q$

Review of Direct Proofs

To prove a conjecture of the form $p \to q$ by using a Direct Proof, we:

Assume that p is true, and

Show that q's truth logically follows.

Reminders:

- If p is *actually* true, the proof is a <u>sound</u> argument.
- If p is only *assumed* true, the argument is merely <u>valid</u>.

"Indirect" Proofs

We can replace $p \to q$ with logically equivalent forms to create additional "indirect" proof techniques.

Example(s):

Proof by Contraposition

(a.k.a. Proof of the Contrapositive)

Example #1: Proof by Contraposition

Conjecture: If $ac \leq bc$, then $c \leq 0$, when $a > b$.

Example #2: Proof by Contraposition

Conjecture: If n^2 is even, then n is even.

Proof by Contradiction

(a.k.a. Reductio ad Absurdum)

Recall the Law of Implication: $p \to q \equiv \neg p \vee q$

Example #1: Proof by Contradiction

Conjecture: If $3(n-6)$ is odd, then n is odd.

Example #2: Proof by Contradiction

Conjecture: If $a \in \overline{\mathbb{Q}}$ and $b \in \mathbb{Q}$, then $a + b \in \overline{\mathbb{Q}}$.

Example #3: Proof by Contradiction (1 / 2)

Conjecture: The sum of the squares of two odd integers is never a perfect square. (Or: If $n = a^2 + b^2$, then n is not a perfect square, where $a, b \in \mathbb{Z}^{odd}$.)

Example #3: Proof by Contradiction (2 / 2)

How To Prove Biconditional Expressions

(i.e., Conjectures Of The Form $p \leftrightarrow q$)

Example(s):