

Homework #3

(50 points)

Due Date: February 9th, 2024, at the beginning of class

Directions

- This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**
 - Start early!** Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via piiazza.com and our office hours.
 - Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.
 - When your PDF is ready to be turned in, do so on gradescope.com. Be sure to assign pages to problems after you upload your PDF. Need help? See “Submitting an Assignment” on <https://help.gradescope.com/>.
 - Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**
-

Topic: Propositional Logic

- (4 points) Let p be “Vernon got a tetanus booster,” let q be “Vernon has a sore arm,” and let r be “Vernon’s mom is relieved.” Convert each of the following logical expressions into a conversational English sentence.
(a) $\neg q \rightarrow \neg p$ (b) $r \rightarrow (q \rightarrow p)$
- (6 points) Let w be “The wind is stiff,” let t be “The trash cans are upright,” and let n be “The neighborhood is littered with trash.” Express each of the following English sentences as equivalent logical expressions in terms of w , t , and the logical operators \wedge , \vee , \oplus , negation, and \rightarrow , as appropriate.
(a) The neighborhood being littered with trash follows from the trash cans not being upright and the wind being stiff.
(b) Only if the trash cans are on their sides is the wind is stiff.
(c) The trash cans are upright unless the neighborhood is littered with trash or the wind is stiff.
- (6 points) For each of the English statements in Question 2, construct its contrapositive in English as an “if – then” sentence. For example, if the statement is “The toy is shiny if it is clean,” the contrapositive in “if – then” form is “If the toy is dull, then it is dirty.”
- (4 points) Rewrite each of the following English expressions as logically equivalent English expressions that correctly use the “only if” wording to express implication.
(a) A student entering their password correctly and authenticating using Duo on their phone is sufficient for that student to log into D2L.
(b) A student may log into D2L provided that they entered their password and either authenticated using Duo or had their laptop computer remember last week’s authentication with Duo.

(Continued on the back ...)

5. (6 points) The parts of this question refer to this logical expression: $(a \oplus b) \leftrightarrow \neg((a \rightarrow b) \wedge (b \rightarrow a))$

First, build a complete truth table for that expression. When constructing a truth table, be sure to follow the table construction rules explain in class on Wednesday Jan 24 – order the rows by the pattern I explained, build up to the desired proposition one operator at a time, and use T and F for true and false.

Second, answer this question: What does this truth table tell us, if anything, about the expressions $(a \oplus b)$ and $\neg((a \rightarrow b) \wedge (b \rightarrow a))$?

6. (10 points) Table III Line (h) of the “POLE” claims that $\neg(p \rightarrow q) \equiv p \wedge \neg q$. Verify the correctness of that claim ...

- (a) ... by using a truth table. That is, construct the table, **and** explain how the table shows that the equivalence holds.
- (b) ... by using a sequence of logical equivalences (that does **not** use Table III line (h) to show the truth of itself!). All of the other equivalences in the POLE are fair game. Be sure to both show and justify all of your steps.

7. (14 points) Show that $(y \wedge z) \rightarrow (y \rightarrow z)$ is a tautology by ...

- (a) ... using a sequence of logical equivalences. Be sure to both show and justify all of your steps.
- (b) ... using reasoning (that is, by choosing either y or z and showing that the expression is a tautology both when that term is true as well as when that term is false).