

Homework #4

(50 points)

Due Date: March 1st, 2024, at the beginning of class

Directions

- This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**
 - Start early!** Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via piiazza.com and our office hours.
 - Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.
 - When your PDF is ready to be turned in, do so on gradescope.com. Be sure to assign pages to problems after you upload your PDF. Need help? See “Submitting an Assignment” on <https://help.gradescope.com/>.
 - Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**
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Topic: Quantifiers

- (4 points) For each nested quantification expression below, determine if it is True or is False for the given domain(s), and explain why you believe your answer to be correct.
 - $\forall a \exists b (a - b = b - a), a, b \in \mathbb{Z}$
 - $\forall c ((c \neq 0) \rightarrow \exists d (\frac{d}{c} = 2)), c, d \in \mathbb{Z}$
- (6 points) Using quantified predicates and appropriate logical as well as mathematical operators, express each of the following English statements in logic.
 - The square root of a positive real number is positive.
 - The mean of two negative real numbers is negative.
- (12 points) Using quantified predicates and appropriate logical operators, express each of the following English statements in logic.
 - Each apartment has exactly one assigned parking space.
 - Every problem has at least two solutions.
 - A run-off election has exactly two candidates.

(Continued on the back ...)

Topic: Arguments

4. (4 points) Using logical equivalences and rules of inference as appropriate, show how the hypotheses $a \vee c$ and $a \rightarrow b$ lead to the conclusion $b \vee c$.
5. (8 points) Using logical equivalences and rules of inference as appropriate, show how the following statements lead to the conclusion "The weather is sunny."

If the weather is not sunny or is not calm, we will fly kites and the kids will have fun.
If we fly kites, we will have kite string.
We do not have kite string.
6. (4 points) For each of the following arguments, determine if the argument is valid or is a fallacy. Clearly justify your answers.
 - (a) Rulan owns a laptop computer. Everyone who owns a laptop has gotten an error message. Therefore, Rulan has gotten an error message.
 - (b) "OK, so that car is really fast, you know. It's got a massive engine, and I made some modifications, but I can't drive it any more, man. I drove it downtown, and when I do that, you know I made some noise. That's when the cops pulled me over. I told them I was just showing off, but, man, there's that noise law, so I got a big ticket. Now I can't go to that concert. Dang! Now you know why I can't drive that car any more."

Topic: Direct Proofs

7. (6 points) Prove, using a direct proof: If x, y and z are odd integers, then xyz is odd.
8. (6 points) Prove, using a direct proof: If r, s and t are rational numbers, then $rt - st$ is a rational number.