CSc 144–002 — Discrete Mathematics for Computer Science I — Spring 2024 (McCann)
https://cs.arizona.edu/classes/cs144/spring24-002/

Homework #6

(50 points)

Due Date: April 5^{th}, 2024, at the beginning of class

Directions

1. **This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**
2. Start early! Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via piazza.com and our office hours.
3. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.
4. When your PDF is ready to be turned in, do so on gradescope.com. Be sure to assign pages to problems after you upload your PDF. Need help? See "Submitting an Assignment" on https://help.gradescope.com/.
5. **Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**

Topic: Matrices

1. (12 points) Consider the matrices $A = \begin{bmatrix} 0 & 0 & 4 \\ -1 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 5 & 5 & 1 & 2 \end{bmatrix}$, and $C = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 2 & 0 \\ -2 & 6 & 0 \\ 2 & 0 & -3 \end{bmatrix}$.
   (a) Compute the result of the scalar product $4C$.
   (b) Which pairs of these matrices can be multiplied together, and what is the size of the results of those matrix products? Note: We're **not** asking you to perform the multiplications. (Don't forget to consider multiplying matrices by themselves!)
   (c) Compute the matrix power A^3.
2. (8 points) Prove that adding two square numeric matrices produces the same answer regardless of the order of addition. That is, prove that $A + B = B + A$ when A and B are both $m \times m$ in size.
3. (6 points) Consider the zero–one matrices $D = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{bmatrix}$ and $E = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$.
   (a) Compute E join D.
   (b) Compute $D^{[2]}$.

(Continued on the back . . .)

Topic: Relations

4. (4 points) Consider the relation $O = \{(29, 23), (19, 23), (19, 19), (23, 29), (23, 19), (23, 23)\}$ on the set $\{19, 23, 29, 31\}$. Is O reflexive? Symmetric? Antisymmetric? Transitive? Show your reasoning/work! Answers without it will be considered incorrect.

5. (4 points) Consider the set of all dogs, and the relation $H = \{(x, y) \mid x \text{ weighs more than } y\}$ on that set. Is H reflexive? Symmetric? Antisymmetric? Transitive? Show your reasoning/work! Answers without it will be considered incorrect.

6. (4 points) Let $F = \{(\$, a), (\$, c), (\#, c), (\&, a), (\&, e)\}$ from $\{\#, \$, \&\}$ to $\{a, c, e\}$, and $G = \{((a, 4), (a, 8), (c, 2), (e, 8)\}$ from $\{a, c, e\}$ to $\{2, 4, 6, 8\}$.

   (a) Which of the two compositions $F \circ G$ and $G \circ F$ can be evaluated?

   (b) What is the result of the evaluation of your answer to part (a)?

7. (4 points) Let $J = \{(a, b) \mid a \leq b\}$ and $K = \{(a, b) \mid a \neq b\}$, where $a, b \in \mathbb{Z}^*$. Evaluate each of the following relational compositions and use set builder notation to describe the results.

   (a) $J \cup K$

   (b) $K - J$

8. (8 points) Referring back to the zero–one matrices in Question 3, for this question assume that they are representations of relations on the base set $\{1, 3, 5, 7\}$, and answer the following questions:

   (a) What is the representation of the matrix D as a set of ordered pairs, assuming that the rows and columns of the matrix are labeled in the same order as the base set is ordered?

   (b) Is the relation represented by matrix E an equivalence relation? If not, why not? Show your work for full credit for your answer!