CSc 144–002 — Discrete Mathematics for Computer Science I — Spring 2024 (McCann)
https://cs.arizona.edu/classes/cs144/spring24-002/

Homework #7

(50 points)

Due Date: April 12th, 2024, at the beginning of class

Directions

1. **This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**
2. Start early! Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via piazza.com and our office hours.
3. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.
4. When your PDF is ready to be turned in, do so on gradescope.com. Be sure to assign pages to problems after you upload your PDF. Need help? See "Submitting an Assignment" on https://help.gradescope.com/.
5. **Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**

Topic: Relations

1. (8 points) Determine whether or not each of the following relations is a weak partial order, a strict partial order, both, or neither. Explain your reasoning for full credit.
   (a) $\{(0,0)\}$ on $\{0\}$.
   (b) $\{(a,b) \mid a \subset b\}$, where a and b are elements of the power set of a set A. (Remember that a and b are themselves sets, because elements of $\mathcal{P}(A)$ are subsets of A.)
2. (4 points) The relation represented by the zero–one matrix below is on the set $\{3,4,5,6\}$. Is the relation a total order? Explain your reasoning for full credit.

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

3. (2 points) Consider the relation $\{(a,b) \mid a \,\%\, b = 0\}$ on $\{2,4,6,8\}$. Which pairs of values from the base set, if any, are not comparable in this relation?

Topic: Functions

4. (4 points) For each of the following expressions, is it a function? Assume a domain of $\mathbb{Z}$ and a codomain of $\mathbb{R}$. Justify your answer.
   (a) $f(n) = \frac{2}{n^3-8}$
   (b) $f(n) = \pm(n^2)$ (The symbol "$\pm$" means positive or negative. For example, ± 4 means "4 or -4.")
5. (4 points) What are the domain and the range of each of these functions?
   (a) The function whose pre–image is a real number and whose image is the integer formed from the first three digits to the right of the decimal point. For example, $f(54.23) = 230$.
   (b) The function whose pre–image is an ordered pair of integers and whose image is the absolute value of the difference of those integers.

(Continued on the back . . .)

6. (4 points) For each of the following functions from $\mathbb{Z}$ to $\mathbb{Z}$, are they injective, surjective, both, or neither? Justify your answers for full credit.

   (a) $a(x) = -x$

   (b) $b(x) = \lfloor \frac{x}{5} \rfloor$

7. (2 points) Provide an example function on $\mathbb{Z}^*$ that is surjective but not injective. Clearly explain how your function satisfies the constraints.

8. (2 points) Is $\{(-2,1),(-1,2),(0,0),(1,-2),(2,-1)\}$ a bijective function on the set $\{-2,-1,0,1,2\}$? Justify your answer.

9. (4 points) Plot (a.k.a. graph) the function $f(x) = \lfloor x \rfloor - \lceil 2x \rceil$ on the domain of real numbers -2 to 2, inclusive.

Program: How Often are Random Relations Transitive?

10. (16 points) In class we recently covered both transitivity of relations and matrix representation of relations. From the examples we covered, it may seem that a lot has to go right for a relation to be transitive, which raises the question: How often are relations transitive? Let's find out!

    Write a complete Python 3 or Java 16 program (your choice!) that generates n random $m \times m$ zero–one matrices and tests them to see if they represent transitive relations.

    We learned in class that we can detect transitivity fairly easily be comparing the relation's matrix representation (call it M) to the result of M^2. If you've forgotten, review the slides from March 29th. By 'random matrices,' we mean that each value in the matrix has an equal chance of being either 0 or 1.

    Measuring the time that code execution requires depends on the language. In Python 3, import the counter (`from time import perf_counter`), then record the time both before and after you call the approach to be measured. The difference is the elapsed time in seconds. Java 16's method of choice is `System.nanoTime()`, which reports the current time in billionths of a second, so a division is needed to convert to seconds. Here are examples, with Python on the left and Java on the right:

```
start = perf_counter()
mySubprogram(args)
stop = perf_counter()
seconds = stop - start
```

```
start = System.nanoTime();
mySubprogram(args);
stop = System.nanoTime();
seconds = (stop - start) / 1000000000.0;
```

    Input: The user is to provide two values: m, the matrix dimension, and n, the quantity of random relation matrix representations to be created and checked for transitivity. You may prompt the user for m and n, or get them from the command line, or both.

    Output: Your program is to report the number of relation matrix representations created (and their size), the quantity of those relations that were transitive, and how long it took your program to test those matrices. The output format that we expect is:

```
[time] seconds were needed to find that [qty] transitive relations
were found in [n] randomly generated [m]x[m] zero-one matrices.
```

    where `[time]` is to be replaced with the number of seconds (as a real number; fancy formatting is optional) that it took your program to do the work, `[qty]` is the number of transitive relations discovered, and `[n]` and `[m]` are the number of matrices generated and size of those matrices, respectively.

    Hand In: To 'answer' this question in your PDF, include photos / screenshots of the following items within the PDF:

    (a) Your complete source code, including what you feel is adequate documentation.

    (b) Your program's output when run on these three pairs of inputs:
    (i) $m = 2, n = 1{,}000{,}000$; (ii) $m = 5; n = 1{,}000{,}000$; and (iii) $m = 10; n = 1{,}000{,}000$.