CSc 345: Analysis of Discrete Structures
Fall 2018 (Lewis)

HW 1
Initial version (3 copies) due at the beginning of lecture: Tue 11 Sep 2018
Peer feedback due at 5pm, Fri 14 Sep 2018
Updated version due (1 copy) due at the beginning of lecture: Tue 18 Sep 2018

If you don’t want to lose points, remember:

• Write your name and your NetID (your university username) on the front page of your homework.
• Staple your homework pages together. (But don’t staple your three Peer Review copies to each other!)

Peer Review

We’re going to be trying peer review on this homework (and probably, on at least one more this semester). The objective of peer review is two-fold:

• To help you find weaknesses in your own solutions, and fix them
• To expose you to a variety of possible solutions

Here’s how it will work: everyone will bring three copies of their homework to class on the first due date. (That is, bring your original and two duplicates.) The TAs will collect all of these copies, and keep one of each to grade.

We will then randomly select two people to give your other copies to. The TAs will organize these into piles, and distribute them to you as you leave class that day.

Later that week (on Friday, for this assignment), you will have to submit a Peer Review document for each of the two homeworks; you will turn this in online, to D2L. We will collect these, and then email them out to everybody; you will read your reviews, and update your solutions. Then, you will turn in a new hardcopy the next week, in class.

We will divide up your grade as follows:

• 50% - correctness of your original homework
• 10% - you turned in two Peer Review documents which fulfilled all of the requirements
• 40% - correctness of your updated homework

Peer Review Format

For simplicity, we are requiring that everyone turn in their Peer Review document as a PDF. Please do not use any other file format! (There’s nothing better or worse about PDF - but it ensures that we don’t have any trouble reading your files.)

Unless the homework you’re looking at is extremely good, expect that each Peer Review will be around half a page.

You should name the peer review files by the NetID of the student. For instance, if you are reviewing my homework, you would turn in (to D2L) a file named russell.pdf
Peer Review Document Requirements

The purpose of peer review is to help your fellow students succeed. Thus, a good Peer Review document will have all of the following characteristics:

- **Correctness**
  Your Peer Review must give useful information about the homework, which helps the other student write a better solution to their problems. Sometimes, you may not find any errors in the other student’s solution; in that case, it’s OK to write that you didn’t find anything. But if the TAs disagree, and find obvious errors, you’ll lose points on your Review.

- **Politeness**
  It is critical that your document be polite. We won’t tolerate any mockery (subtle or overt) or strident criticism. We want constructive criticism - which means information which is clear and useful, but which is not degrading.
  (This is one of the reasons we are having you turn in the Peer Review document to us, so we can watch out for this.)

- **Not a Solution**
  While you must give clear and useful feedback about errors you see, you must not simply write the solution in your feedback. Instead, you should focus on the weak points which need to be strengthened. Is the algebra in error? Point out the step which is wrong, but not how to fix it. Does the inductive step fail to make use of the Inductive Hypothesis? Point that out, but don’t tell them how you solved it.

Corner Cases

- **If I don’t turn in a homework on the first due date, can I still turn in one on the second due date?**
  No, if you don’t turn it in the first time, you can’t turn in a second version. Likewise, if you don’t give a reasonable attempt to a problem the first time, you can’t get credit for it in the second version.

- **If I turn in a first version, and decide not to update it, is that OK?**
  Yes, if we don’t receive a 2nd version, we will give you an identical score - the same you got for the 1st.

- **If I am not at class, can I still get Peer Review?**
  Sorry, no. Although you can still come to Office Hours for some feedback.

- **If I don’t turn in a homework, can I still get my 10% for doing some Peer Review?**
  No.

- **What if I don’t get Peer Review?**
  Unfortunately, I don’t control what the other students do. I can only send you what I’ve been given. However, you can come to Office Hours for some feedback.

- **Can I see my score from the 1st version before I turn in the 2nd?**
  Sorry, no. This is considered a single assignment, not two, so it is not likely that we will return the grades for the 1st assignment before the 2nd is due.
1 Evaluating Quantifications

Evaluate each of the quantifications below. State whether they are true or false, and explain why this is true.

You should assume that all variables are in the domain $\mathbb{Z}$ (that is, the integers), unless explicitly stated. (A few quantifications will also limit the domain to only some integers.)

(a) $\forall x \exists y, \ x + y \geq 0$
(b) $\exists x \forall y, \ x + y \geq 0$
(c) $\forall x \exists y, \ x^2 = y$
(d) $\forall (y \in \mathbb{Z}) \exists (x \in \mathbb{R}), \ x^2 = y$
(e) $\forall (n > 0) \forall (x \neq 0), \ x^n < x^{n+1}$
(f) $\exists (n > 0) \forall (x \neq 0), \ x^n < x^{n+1}$
(g) $\forall (c > 0) \ \exists n_0 \ \forall (n > n_0), \ n^2 \geq cn$

2 Quantifications and Code

In this problem, you will rewrite some Java code as a quantification; you will then explain your answer.

Suppose that I have the function $p(x,y)$, defined as follows:

```java
boolean p(int x, int y);
```

and I have written the following Java function:

```java
boolean mystery(int[] domain)
{
    for (int i: domain)
    {
        boolean found = false;
        for (int j: domain)
            if (p(i,j))
                found = true;
        if (!found)
            return false;
    }
    return true;
}
```

Rewrite the function `mystery()` as a quantification over the predicate $p(x,y)$. Explain how you came up with your answer; simply writing the predicate will be insufficient.
3 Induction

Prove each of the following conjectures using induction. (You will lose a lot of points if you give a non-inductive proof.)
(Thanks to the class TAs for several great induction problem ideas!)
(a) **Conjecture:**
   \[ 1 + 3 + 5 + 7 + \ldots + (2n - 1) = n^2, \quad n \in \mathbb{Z}^+ \]
(b) **Conjecture:**
   \[ 1^3 + 2^3 + 3^3 + \ldots n^3 = (1 + 2 + \ldots + n)^2, \quad n \in \mathbb{Z}^+ \]
   **HINT:** Can you simplify the things inside the parentheses on the right-hand side?
(c) **Conjecture:**
   \[ 3^{2n} - 1 \text{ is a multiple of } 8, \quad n \in \mathbb{N} \]

4 Structural Induction (with example)

Consider the following theorem (and its proof). After you have read this proof, prove the same conjecture again - but this time, use the **root plus subtrees** strategy for structural induction.

**Conjecture:**
A non-empty k-ary tree with \( n \) nodes has \( n - 1 \) edges.

**Base Case: 1 node**
A tree with only a single node has no edges between the nodes, and thus the conjecture holds trivially.

**Inductive:**
Assume that the conjecture holds for any tree which has exactly \( n \) nodes. We will prove that it also holds for any tree which has exactly \( n + 1 \) nodes.

Any tree with \( n + 1 \) nodes is a tree with \( n \) nodes, plus a single new leaf, added as a child of a certain node. The new, larger tree has one more edge than the previous one - but it also has one more node.

By the I.H., any tree with \( n \) nodes has \( n - 1 \) edges; thus, the tree with \( n + 1 \) nodes certainly had \( n \) edges.
Thus, the inductive step holds.

**Summary**
Thus, the conjecture holds for all non-empty trees.

*Note that you can’t assume that it’s a binary tree!"