

Single Source Shortest Path (SSSP) Algorithms

Dijkstra's Algorithm

*Note: This algorithm is **not** guaranteed to work in the presence of negative edge weights.*

Legend:

source	the source (starting) vertex
$d(a, b)$	the path cost (distance) from vertex a to vertex b
$w(x, y)$	the weight of the edge connecting vertices x and y
Known	the set of vertices whose shortest paths from the source vertex are known
Fringe	the set of vertices that we know we can reach from the source

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1 Set  $d(\text{source}, \text{source}) = 0$  and  $d(\text{source}, x) = +\text{infinity}$ , for all other vertices  $x$ 
2 Initialize Known to contain source
3 Initialize Fringe to contain the vertices adjacent to source
4 So long as Fringe is not empty:
5     Find the fringe vertex  $f$  that has the smallest  $d(\text{source}, f)$ 
6     [ where  $d(\text{source}, f) = d(\text{source}, t) + w(t, f)$ , where  $t$  is Known ]
7     Move  $f$  from Fringe to Known
8     Add unKnown, unFringe vertices that are adjacent to  $f$  to the Fringe
9     Update Fringe data if necessary
```

Bellman-Ford Algorithm

Legend:

source	the source (starting) vertex
V	vertex set of the graph
$d(a, b)$	the path cost (distance) from vertex a to vertex b
$w(x, y)$	the weight of the edge connecting vertices x and y

```
1 Set  $d(\text{source}, \text{source}) = 0$  and  $d(\text{source}, x) = +\text{infinity}$ , for all other vertices  $x$ 
2 Loop ( $|V| - 1$ ) times:
3     For each edge  $(x, y)$  of the graph:
4         if  $d(\text{source}, x) + w(x, y) < d(\text{source}, y)$  then
5             Let  $d(\text{source}, y) \leftarrow d(\text{source}, x) + w(x, y)$ 
6 For each edge  $(x, y)$  of the graph:
7     if  $d(\text{source}, x) + w(x, y) < d(\text{source}, y)$  then
8         Indicate that algorithm failed due to a negative-weight cycle
9 Indicate that the algorithm completed successfully
```