QuadTrees:
A data simple data structure for geometric objects (e.g., points, houses, an image, 3D scene)

Support efficiently a very wide variety of queries.

QuadTrees
Assume we are given a red/green picture defined a $2^h \times 2^h$ grid. E.g. pixels. Each pixel is either green or red.

(more general and interesting examples – soon)

Need to represent the shape “compactly”

Need a data structure that could answers multiple types of queries. For example:
1. For a given point $q$, is $q$ red or green?
2. For a given query disk $D$, are there any green points in $D$?
3. How many green points are there in $D$?
4. Etc etc

Alg constructQT for a shape $S$:

• Input – a node $v \in T$, and a shape $S$.

• Output – a Quadtree $T$, representing the shape of $S$ within $R(v)$.

• Assume we are given a red/green picture defined on a $2^h \times 2^h$ grid of pixels.
• Each pixel has a unique color (Green or Red)
• Every node $v \in T$ is associated with a geometric region $R(v)$

• If $S$ is fully green in $R(v)$, or $S$ is fully red in $R(v)$ – then
• $v$ is a leaf, labeled Green or Red. Return
• Otherwise, divide $R(v)$ into 4 equal-sized quadrants, corresponding to nodes
• Call constructQT recursively for each quadrant.
QuadTrees

Consider a picture stored on an $2^h \times 2^h$ grid. Each pixel is either red or green.

We can represent the shape “compactly” using a QT.

Height – at most $h$.
Point location operation – given a point $q$, is it black or white
– takes time $O(h)$
- could it be much smaller?

Many other operations are very simple to implement.

QuadTree for a set of points

Now consider a set of points (red) but on a $2^h \times 2^h$ grid.

Splitting policy: Split until each quadrant contains $\leq 1$ point.

Build a similar QT, but we stop splitting a quadrant when it contain $\leq 1$ point (or some other small constant)
Point location operation – given a point $q$, is it black or white
– takes time $O(h)$ (and less in practice)

Many other splitting policies are very simple to implement.
(e.g. A leaf could contain contains $\leq 17$ points)

Regions of nodes

In general, every node $v$ is associated with a region $R(v)$ in the plane

$R(root)$ is the whole region

The smallest area of $R(v)$ is a single pixel.

Let $NW(v)$ denote the North West child of $v$ (similarly $NE, SW, SE$)

$R(v) = \text{is the union of } R(NW(v)), R(NE(v)), R(SW(v)), R(SE(v))$
QuadTrees for a set of points

Report(Q,v)
// Q – a query disk
*/report all the points in stored
at the subtree rooted at v, which
are also inside Q. */

1. If v is NULL – return.
2. If R(v) is disjoint from Q – return
3. If R(v) is fully contained in Q –
   report all points in the subtree
   rooted at v.
4. If v is a leaf – check each point
   in R(v) if inside Q
5. Else
   ◆ Report(Q, NW(v))
   ◆ Report(Q, NE(v))
   ◆ Report(Q, SW(v))
   ◆ Report(Q, SE(v))

QuadTrees for shape

Input: Set S of triangles
S={t₁, t₂, …, tₙ}

Splitting policy: Split
quadrant if it intersects
more than 1 triangle of S.

Note – a triangle might be stored in multiple leaves.
Some leaves might store no triangles.

Finding all triangles inside a query region Q –
essentially same Report Report(Q,v) as before
(minor modifications)

Terrain representations

Raw data – a grid of points (Xᵢ, Yᵢ, Zᵢ)
For every grid point i,j
Triangulated terrain
(TIN – Triangulated irregular network)

Each triangle approximately fits the surface below it.

How to find good triangulation?

Input – a very large set of points $S = \{(x_i, y_j, z_{ij})\}$. $z_{ij}$ is the elevation at point $(x_i, y_j)$.

Want to create a surface, consists of triangles, where each triangle interpolates the data points underneath it.

Idea: Build a QT $T$ for the 2D points.

If want triangles: Each quadrant is split into 2 triangles.

Assign to each vertex the height of the terrain above it.

The approximated elevation of the terrain at any point is the linear interpolation of its elevated vertices.

QT Split Policy: Splitting a quadrant into 4 sub-quadrants:

- split a node $v$ if for some data point $(x_i, y_j) \in R(v)$, the elevation of $z_{ij}$ is too far from the corresponding triangle.
- That is, it is too far from the interpolated elevation.
- Note: A quadrant might contain a huge number of points, but they behave smoothly. E.g. all a the slop of a mountain, but this slope is more or less linear.
Level Of Details

- Idea – the same object is stored several times, but with a different level of details
- Coarser representations for distant objects
- Decision which level to use is accepted ‘on the fly’ (e.g., in graphics applications, if we are far away from a terrain, we could tolerate usually large error)

69,451 polys  2,502 polys  251 polys  76 polys