CSc 573: Homework Assignment 1

Assigned: Tuesday Jan 17 2017
Due: 12:30pm, Tuesday January 31 2017

Clear, neat and concise solutions are required in order to receive full credit so revise your work carefully before submission, and consider how your work is presented. If you cannot solve a particular problem, state this clearly in your write-up, and write down only what you know to be correct. For involved proofs, first outline the argument and the delve into the details.

1. (a) Prove that the class of regular languages is closed under intersection. That is, if \( L_1 \) and \( L_2 \) are regular languages, then \( L_1 \cap L_2 \) is also a regular language. If your proof relies on finite automata, use only DFAs in the proof.

(b) Prove that the class of regular languages is closed under complement. That is, if \( L \) is a regular language, then \( L^c = \Sigma^* - L \) is a regular language. If your proof relies on finite automata, you can use DFAs or NFAs in the proof.

2. Construct a DFA for the languages:
   (a) \( L_1 = \{ w \in \{0, 1\}^* \text{ and } w\text{’s first and last symbol are different} \} \)
   (b) \( L_2 = \{ w \in \{0, 1\}^* \text{ such that the number of 0’s is odd and the number of 1’s is not equal to 2} \} \).

3. Convert the following regular expressions to finite automata (you can use the technique from Theorem 1.54). Recall that regular expressions evaluate to regular languages. Here “0++” means “one or more 0’s”:
   (a) \((01 \cup 11)^* 10\)
   (b) \(0^+ \cup (01)^+\)

4. For each of the languages bellow determine whether it is a Regular Language or not. If so, argue how a FA can be built for it. If not, prove using the Pumping Lemma.
   (a) \( L_1 = \{a^k | \text{ where } k \text{ is a multiple of n for a fixed value of } n \geq 1 \} \)
   (b) \( L_2 = \{w | \text{ where } w \text{ is a binary number that is a multiple of } n \text{ for a fixed value of } n \geq 1 \} \).

5. For each of the languages bellow determine whether it is a Regular Language or not. If so, argue how a FA can be built for it. If not, prove using the Pumping Lemma.
   (a) \( L_3 = \{ w | w \in \{0, 1\}^* \text{ and } w \text{ contains an equal number of occurrences of the substrings 01 and 10} \} \)
   (b) \( L_4 = \{ w | w \text{ is not a palindrome} \} \)

6. For each of the languages below determine whether it is a CFL or not. If so, construct a PDA for it. If not, prove using the pumping lemma.
   (a) \( L_1 = \{ wcx | w \text{ is a substring of } x, \text{ where } w, x \in \{a, b\}^* \} \)
   (b) \( L_2 = \{ w_1 # w_2 # \ldots # w_k | k \geq 0, w_i \in \{a, b\}^*, w_i = w_j^R \text{ for some } i \neq j \} \)

7. Design the simplest (highest-numbered) grammar for the languages below:
   (a) \( L_1 = \{ w | w \geq \#_a w \} \)
   (b) \( L_2 = \{ w | w \in \{a, b\}^* \text{ and } w \neq xx \text{ for any string } x \} \)
8. Design a grammar for $L = \{a^ib^jc^k|i = j \text{ or } j = k, \text{ where } i, j, k \geq 0\}$. Show that your grammar is ambiguous and argue that there cannot be a non-ambiguous grammar for this language.

9. Design a grammar for $L = \{w|w \in \{a, b\}^* \text{ such that the number of } a \text{ and } b\text{'s in } w \text{ are equal }\}$. Is your grammar ambiguous? If so, redesign it so that it is not ambiguous.

10. Let $G$ be a CFG in Chomsky Normal Form with $b$ variables.

   (a) Show that any string $s \in L(G)$ of length $n \geq 1$ requires exactly $2n - 1$ steps (rule applications) for any derivation of $s$.

   (b) Show that if $G$ generates some string with a derivation of at least $2^b$ steps, then $L(G)$ is infinite.

**Extra Credit**

True or false: if $p > 3$ is a prime number, then 24 divides $p^2 - 1$?