THE TEACHING OF MATHEMATICS TO TEXTILE STUDENTS.

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I. Before commencing his purely Textile Course, the student has had, at least, a two years' course consisting of Practical Mathematics and Drawing, English, and Laboratory Work in Mechanics and Physics. The syllabus for Mathematics includes: Mensuration of the rectangle, parallelogram, triangle and circle; surface areas and volumes of the cylinder, cone and sphere; measurements are made from everyday objects and models, and symbolical representation of the results leads to formulae and substitution of numbers in these formulae. From this we pass to simple equations. Squared paper is used to plot results of measurements made (either in the Mathematics lessons or the Laboratory), such as comparing:—circumferences and diameters of circles; areas of triangles of equal altitudes and different bases; extensions and loads for a spring; corresponding readings of temperatures in C.° and F.°, and from these we pass to proportionality, or variation. Powers and roots are dealt with, and the use of Logarithms taught. The Drawing, which is taught alongside the Mathematics, consists of: Hand-sketches, plans, elevations and sections of models and parts of machinery used by the students.

The examples usually given are from the Engineering and Building trades, and these are not at all interesting to a student working in a factory and intending later to study Weaving and Designing, Spinning, or Scribbling Engineering. Some 6 or 7 years ago, students of this type, who, being interested in Mathematics, were attending my Mathematics Class for Building Students in addition to their Textile Classes, asked me the question: “How does all this work help us in our textile calculations?”

I suggested that they should bring their textile problems to the Mathematics class. I found that they had note-books full of rules and formulae: one for each type of problem. They were using these mechanically, and they were at once at sea if they met with a problem to which the rule could not be applied directly.

Allow me to illustrate this as clearly and briefly as possible:

(A) There are several systems of numbering or counting yarns, and it is frequently necessary to change from one system to another system.

Suppose we wish to find the cotton counts equivalent to 1/24s worsted counts.

Worsted counts are based on a hank of 500 yards, and the number of these hanks weighing 1 lb. is the count.

The cotton hank is 840 yds. long, and the number of these hanks per lb. is the count.

Then 24s worsted means 24 worsted hanks weigh 1 lb. or (24 x 560) yards of yarn weigh 1 lb.

\[
\text{Number of cotton hanks (840 yds.)} = \frac{24 \times 560}{840};
\]

\[
\therefore \text{equivalent cotton counts} = \frac{24 \times 560}{840} = 16s.
\]

To work this and similar calculations various text-books give various rules, thus:

(i) One book gives the following:

\[
\text{If } l = \text{ yds. in 1 lb. of 1s worsted; } l_i = \text{ yds. in 1 lb. of } i \text{a cotton.}
\]

\[
c = \text{counts of worsted; } c_i = \text{counts of cotton.}
\]

\[
y = \text{yds. in 1 lb. of yarn; } y_i = \text{yds. in 1 lb. of yarn.}
\]
Then \[ y = l \times e; \quad y_1 = l_1 \times e_1. \]

But \[ y_1 = y; \quad l_1 \times e_1 = l \times e. \]

(ii) Another book gives the rule: "Reduce the given counts to yards and divide by the standard length of the required system."

(iii) Yet another gives the rule:

\[
\text{Required counts} = \frac{\text{given counts} \times \text{yds. per hank}}{\text{yds. per hank in required denomination}}.
\]

(B) When two or more threads are twisted or folded together it is necessary to find the counts of the twisted or folded thread.

Suppose we wish to find the counts of a compound twist thread composed of one thread of 1/60s worsted, one of 60/2 spun silk and one of 1/30s worsted.

The first step is to find the equivalent worsted counts of the 60/2 spun silk.

Now 60/2 spun silk means that \((60 \times 840)\) yards weigh 1 lb.

The worsted counts are based on a hank of 560 yards.

Hence \(\text{Equivalent worsted counts} = \frac{60 \times 840}{560} = 90s.\)

For purposes of calculation we may choose to twist any number of hanks of each thread \((e.g. 1, 2, 30, 60, \text{or } x)\).

Let us twist 180 hanks \((560 \text{ yds. per hank})\) of each thread, and if there is no "take up" we shall have 180 hanks of twisted thread.

\[
\begin{align*}
\text{Then} & \quad 180 \text{ hanks of 60s worsted weigh 3 lbs.} \\
\text{180} \quad & \quad 90s \text{ worsted or } 60\% \text{ spun silk weigh } 2 \text{ lbs.} \\
\text{180} \quad & \quad 30s \text{ worsted weigh } 6 \text{ lbs.} \\
\therefore & \quad 180 \quad \text{twisted thread weigh 11 lbs.} \\
\text{or } & \quad \frac{1,080}{11} \quad \text{"} \\
\therefore & \quad \text{counts of the twisted yarn } = \frac{1,080}{11} = 98\frac{2}{11}\text{ s worsted.}
\end{align*}
\]

To deal with calculations like this the student is given

(i) a formula: \[\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}, \quad \text{or},\]

(ii) a rule: "Select the highest count and divide it by itself and each of the given counts; the quotient in each case will then represent the relative weight of each thread in lbs.; then divide the selected count by the sum of these weights and the answer will equal the resultant counts."

Referring to the statement (C) above, it is quite a simple matter to go a step farther, and

\[(a)\] if given the price per lb. of the various threads we may find the cost of the folded thread, or,

\[(b)\] we may find the weight of each thread used in producing a certain weight of folded thread.

The rules and formulæ are modified to apply to these cases. The same remark holds for problems dealing with "set" systems or the different methods in vogue of indicating the spacing of the warp threads across the loom. I will not take up your time by giving examples. The point, with regard to all this, I wish to make this morning is, that a knowledge of the various methods of counting yarns and of the various set systems in use is all that is really required by the student. Clear thinking and a visualisation of the conditions of the problem leave no need for rules or formulæ.

II. I will now briefly indicate how I have attempted to adapt the Mathematics Syllabus to meet the needs of these students.

(i) Problems dealing with \(a\) the counting of single and folded yarns, \(b\) testing yarns for counts, \(c\) the setting of warps and counting of healds.
(d) the weight and cost of warp and weft in woven fabrics, (e) wool blending, are purely arithmetical, mainly simple rule of three.

(ii) In finding the weight and cost of the warp and weft required for the making of a piece of cloth to given particulars, expressions like the following require simplifying:

\[
\frac{68 \times 384 \times 229}{40 \times 840} \text{ lbs; } \frac{54 \times 40 \times 64 \times 65}{24 \times 560 \times 100 \times 240} \text{ lbs.}
\]

The student arrives at these expressions from his knowledge of technical terms and simple arithmetic, and a further knowledge of Logarithms and the Slide-Rule is most useful, especially as in actual practice the fractions obtained when calculating for a series of similar cloths have a constant factor. Then the log. of this factor may be found or the slide-rule set to this value.

(iii) Many problems, such as finding the counts of a thread required to be twisted with one or more threads to produce a twist thread of a given count and cost, are easily worked by simple equations. This is preferable to trying to learn rules or formulae. Some problems actually require two unknowns.

(iv) A large variety of textile problems require a knowledge of Proportionality or Variation.

(a) In the production of a well-balanced cloth, one of the chief factors to consider is the diameter of the yarn employed. An empirical rule is employed for calculating the diameter of a yarn. This is:

\[
\text{Diameter of a yarn (in inches)} = \frac{1}{k \sqrt{\text{yards per lb.}}}
\]

where \(k\) is a constant determined by experience.

The commonly accepted values for \(k\) are: 0.85 for woollen yarns, 0.90 for worsted, 0.92 for cotton and linen yarns.

The student is interested to see how this formula has been derived, by considering yarn as a cylinder and assuming the specific gravity of the yarn. (The sp. gr. of yarn is difficult to determine accurately, hence the constant will differ from the value found by experience.)

From the above formula it is easily shown that: “the diameters of yarns of the same material are inversely proportional to the square roots of their counts.”

Also the “sett” (or the number of ends which are laid over a standard width) is directly proportional to the square root of the counts.

(Note.—Logarithms are extremely useful in these calculations in which square roots are involved.)

(b) Establish from first principles the fact that:

“The speeds of pulleys driven by a belt or the speeds of gear wheels are inversely proportional to the diameters or numbers of teeth, respectively,” and with the aid of a diagram the student will be able to solve the problems dealing with the variation of (1) the speed of the loom; (2) the speed of the “tappets” for raising the healds; (3) the “take-up” motion of the loom, i.e. winding the woven piece on to the beam at a given rate, so as to obtain the correct number of picks per inch; (4) the speeds of the various rollers in the “carding engine,” the function of which is to separate the wool fibres one from another and arrange them to form one continuous sliver, all parts of which are equal in weight and thickness; (5) the speeds of the various rollers in the drafting or drawing out of the threads and the speeds of the spindles, etc., in the spinning machines. In all these calculations, constant factors occur, and change wheels are used to vary the speeds.

As an example, let us consider the “take-up” motion.

When the going part moves towards the fabric it presses the catch \(C\), by means of the lever \(L\), to which it is attached by the pin \(G\), forward, thus imparting motion to the ratchet wheel \(R\). Working on the same stud as \(R\)
is the change-wheel $W$. This gears with the intermediate $E$, while through the pinion wheel $D$, imparts motion to the friction-beam roller-whh
$B$. The friction-beam roller $A$ turns the piece beam solely by friction.

If the circumference of the beam roller $A$ is $13''$, the ratchet wheel contains 60 teeth; the intermediate $E$ has 120 teeth; the pinion $D$ has teeth and the friction-beam wheel $B$ has 120 teeth; it is required to find the number of teeth in the change-wheel $W$ for weaving 56 picks per inch.

![Fig. 1.](image)

When $A$ makes 1 revolution, 13 ins. of cloth are drawn on to the beam, as this contains $(13 \times 56)$ picks; hence

in 1 rev. of $A$ $(56 \times 13)$ teeth of $R$ move forward;

\[ \therefore \quad R \text{ makes } \frac{56 \times 13}{60} \text{ revs.} \]

Then (i)

\[ \text{speed of } A = \frac{60}{x \times 20} = \frac{56 \times 13}{60 \times 120} \]

Assume that the change-wheel $W$ has $x$ teeth, and find the velocity ratio of $A$ and $R$ from dimensions of the gear-wheels, and equate the two values.

Thus (ii)

\[ \text{speed of } A = \frac{x \times 20}{120 \times 120} = \frac{90}{56 \times 13}; \quad \therefore \quad x = 59. \]

This is a much more preferable method to that of setting the student work by a rule, thus:

"First, multiply the picks per inch by the circumference of the friction beam in inches, and divide by the number of teeth in the ratchet wheel; second, multiply the teeth in the friction-beam wheel by the teeth in the intermediate, and divide by the first result multiplied by the teeth in the pinion."

(v) Squared-paper Work.

(a) We may draw graphs to convert the counts of a yarn in one system to the corresponding counts in another system, or to convert settings of warp in one system to the corresponding sett in another system, etc.

(b) Use the graphical method in variation problems.

(c) Use it in discussing the motion of the "going-part" of the loom. The "going-part" serves a twofold purpose. It beats up the weft when in front position and it provides the medium on which the shuttle may travel from one side to the other when in its back position. The motion communicated to the driving shaft is a continuous circular one, and by means of a cran
and connecting arm, the "going-part" is caused to move backwards and forwards along what is approximately a straight line. This motion must not be regular. At the time the shuttle is passing across the loom, as much rest as possible must be given to the going-part, whilst in bringing it up to the cloth a smart blow must be delivered. Give actual dimensions of the crank radius and length of connecting arm, and let the students make a diagram to scale showing the position of the connecting-pin for each 10° of revolution of the crank. From these measurements draw a displacement-time curve and a velocity-time curve. These show clearly that the velocity of the "going-part" is less as the crank approaches and leaves the back centre, than its velocity as the crank approaches and leaves the front centre. This gives (a) the "dwell" necessary to allow the shuttle to cross and (b) a smart blow to the cloth by the reed at the time of the "beat-up."

(d) Use squared paper in discussing the motion of the healds. The healds raise and lower the warp threads, thus making the "shed" through which the weft thread passes. They must be raised gradually at first, with increasing velocity to a maximum, and then decreasing as they reach their highest position. In the tappet loom this motion is imparted by means of tappets or cams which are designed to give harmonic motion. The students construct displacement-time and velocity-time graphs for harmonic motion and also draw designs of tappets for various weaves.

(vi) Similar Triangles and Trigonometrical Ratios.

(a) The distances through which the warp threads are raised and lowered in making the "shed" for the shuttle depend upon the relative distances moved by various lever-arms. The fact that the corresponding sides of equiangular triangles are proportional helps the tuner of the loom to make the necessary adjustments in the connecting rods.

(b) In the simplest weave, each weft thread passes over a warp thread and under the next, and so on, and each warp thread passes over a weft thread and under the next. If the threads lay perfectly straight and the diameter of each warp and weft thread were \( \frac{1}{2} \) in, then a warp thread and a weft intersection would occupy \( \frac{1}{2} \) in, and the warp would be "set" 30 ends per inch. Similarly there would be 30 picks of weft per inch. But the warp and weft threads, when interlacing, bend under and over each other, and the curvature will allow a closer setting of the warp threads and a greater number of picks per inch. The angle due to the curvature affects this.

The figure shows that two diameters occupying \( \frac{1}{2} \) in, and represented by \( AB \), only occupy a distance represented by \( AC \) measured across the cloth, and

\[ AC = AB \cos BAC. \]

There is ample room for coordination between the Mathematical and Textile Departments, along the lines I have suggested to you. Let the Textile teacher dispense with the majority of his formulae and rules and the Mathematics teacher choose examples from the textile trade. In this way we can prevent the student thinking that he has entered a new world when he comes to his particular trade calculations, and we may lead him to see that, the builder when estimating quantities and costs of materials; the surveyor when measuring his land; the colliery manager when calculating his ventilation pressure; the bank-clerk when estimating interests and discounts; the textile designer when calculating the weight and cost of cloth, are all simply applying principles learnt in the mathematics class at school.

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