Block Substitution, Part 1: Basic Concepts

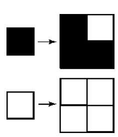
Intuition tells us that it takes a complex system to produce complex results. This intuition turns out to be incorrect: Very complex results can be produced by simple systems in which simple rules are applied repeatedly and in parallel [1].

This phenomenon is seen in drawdown automata [2] and L-Systems [3, 4]. This article describes another kind of system in which complex patterns can be produced by applying very simple rules over and over again. The technique is called *block substitution* [1].

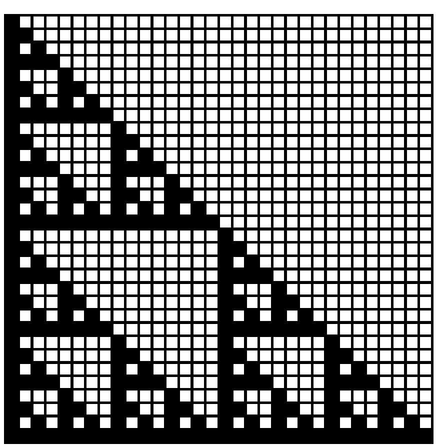
The Block Substitution Process

In block substitution systems, pattern are considered to be composed of cells of various colors. A (usually) small block of cells is associated with each color. For one iteration of block substitution, every cell in the pattern is replaced by its corresponding substitution block. This is where parallelism comes in. The process then is repeated. As the result of successive iterations, the original pattern grows larger and (usually) more complex.

Here is a simple example. There are two colors, black and white. The substitutions rules are

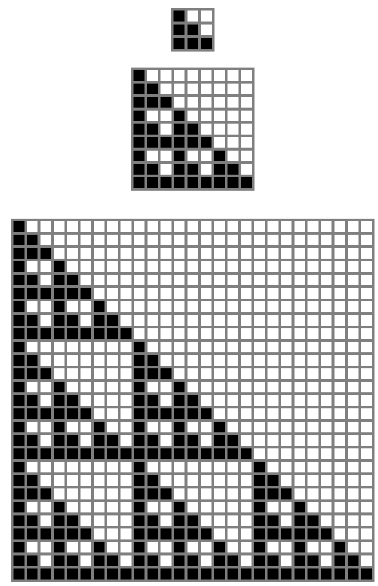


The process starts with a single black cell. Here is the pattern that results after five iterations:



This example dramatically illustrates that complex and surprising patterns can result from the repeated application of the simplest of rules.

Block substitution systems by their nature produce fractal patterns. For the example above, this can be seen by the development of the patterns through successive iterations:

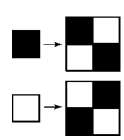


…

As block substitution rules are iterated, the pattern grows larger and larger — doubling in dimensions for 2 × 2 substitution blocks with each iteration: 2, 4, 8, 16, 32, … .

Another View

The process can be viewed in another way, in which the initial block is large and cells are subdivided on successive iterations. Here is an example that illustrates how patterns develop when viewed in this manner. The substitution rule is



and the process starts with a single white cell:

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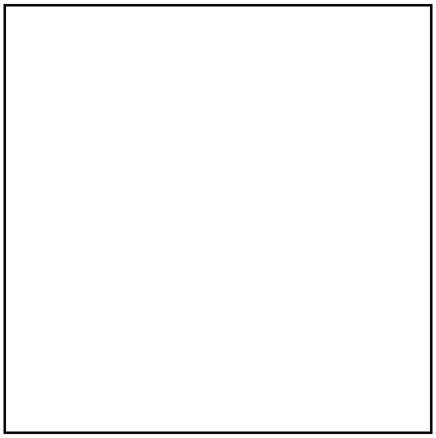
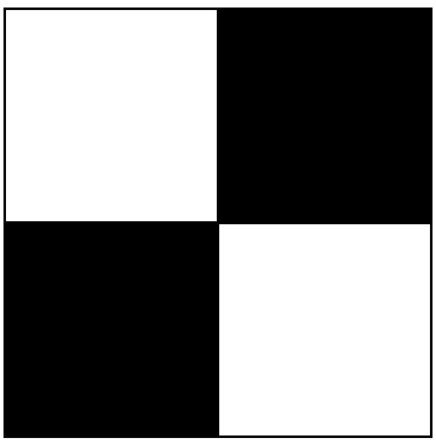
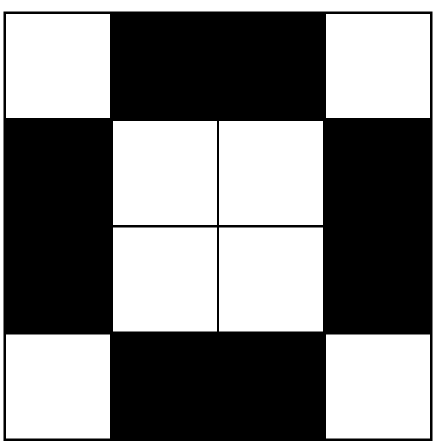


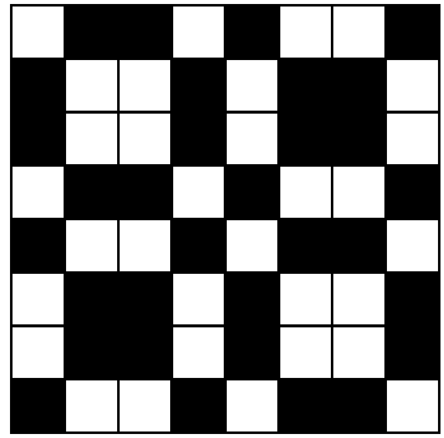
Replacing this cell by its block subdivides it:

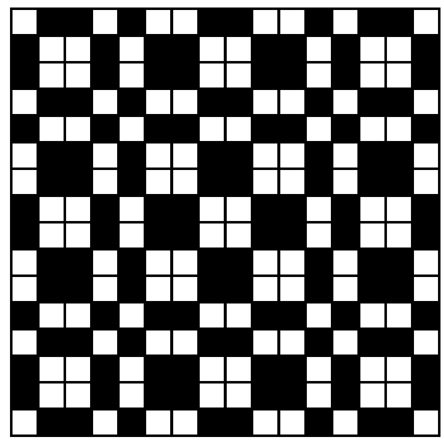


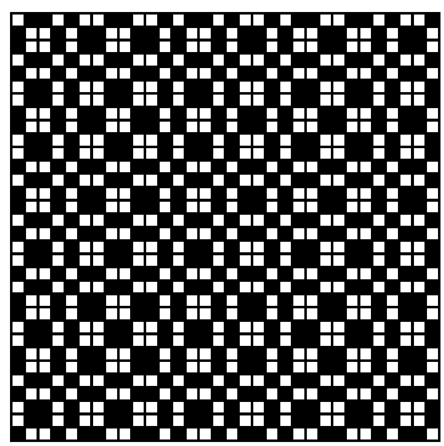
Repeating the process, each of the four cells in the pattern above is subdivided:



Successive iterations continue the subdivision:







Of course, if the process is viewed in this way, the cells eventually become too small to see. It is the idea of subdivision that is important. It corresponds to the notion of self similarity that characterizes fractals, in which the same pattern appears as magnification increases. This pattern is the Morse-Thue carpet [5], which makes an interesting non-repeating weave.

There are limitations to the kinds of patterns that block substitution can produce. For 2 × 2 blocks, there are only $2^8 = 256$ possible rules and, with initial patterns that have only one cell, black or white, there are only $2^9 = 1,024$ block substitution systems. However, substitution blocks can be larger, as can the initial pattern. And more colors can be used.

Subsequent articles will explore these and other possibilities.

References

1. *A New Kind of Science*, Stephen Wolfram, Wolfram Media, 2002.

2. *Drawdown Automata, Part 1: Basic Concepts*, Ralph E. Griswold: http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_dda1.pdf

3. *Designing with L-Systems, Part 1: String Rewriting Systems*, Ralph E. Griswold: http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_ls01.pdf

4. “Pattern Extension Methods”, Ralph E. Griswold, *Complex Weavers Journal*, May 2002, pp. 35-37.

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