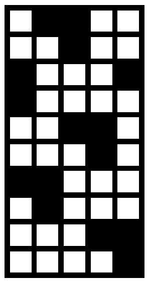
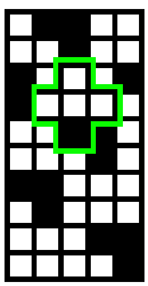
Constrained Patterns, Part 2: Neighborhood Analysis

In the first article on constraints, we introduced the concept of neighborhood constraints [1]. In this article, we'll look at the problem of determining the neighborhood constraint set of a pattern.

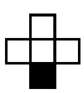
Consider the following pattern:



All that is necessary to determine the constraint set for this pattern is to examine every cell and record the template for its neighborhood.



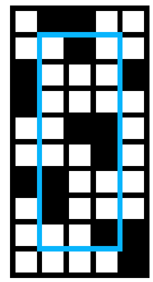
For example, the template for the cell outlined above is



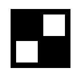
This process is straightforward except for cells at the edges, which have incomplete neighborhoods. There are several ways to handle such cells:

1. Don't include the edge cells in the analysis.
2. Assume that the pattern repeats so that the edges wrap around.
3. Don't assume the pattern repeats (for example, the Morse-Thue carpet does not [2]) but include partial neighborhoods of the edge cells.

Method 1 amounts to analyzing a subpattern, shown by the blue outline below:

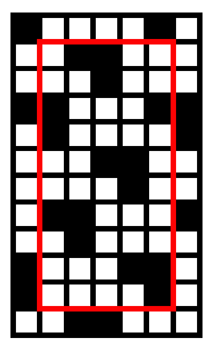


The problem with this approach is that the constraint set obtained may not be complete. For example, the unit motif for plain weave is a 2 × 2 pattern:



This pattern only has edge cells. If they are ignored, there is no constraint set at all, which obviously is incorrect.

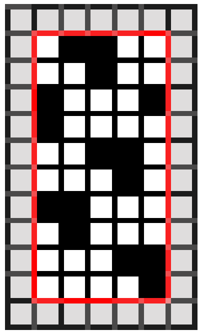
Method 2 can be handled by augmenting the pattern, adding cells around the edges that correspond to what would appear if the pattern were contained in a repeat:



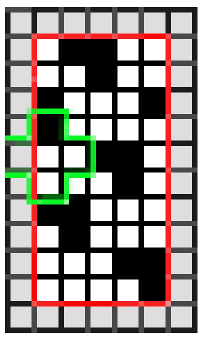
Now the analysis can proceed for the cells enclosed in the red rectangle above; effectively there are no edge cells.

This method is fine for repeating patterns, but it produces erroneous results for aperiodic patterns such as the Morse-Thue carpet.

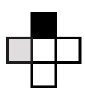
Method 3 tries to deal with this situation by adding edges with unknown cell colors:



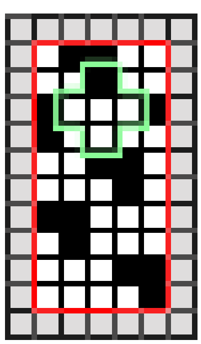
In this case, an edge cells such as the one outlined below has a neighborhood template with a cell of unspecified color:



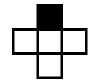
Here is that template:



We can add this partial constraint to the set. But note that there is other cells in the pattern with complete templates that have the same three cells as the partial constraint:



This neighborhood,



"covers" the incomplete one, so we do not need to keep the incomplete one.

If partial constraints remain after analyzing all cells, one possibility is to just "force them" by arbitrarily coloring the unspecified cells.

What to do about an aperiodic patterns an open question. One can analyze a portion of it using Method 3. But how can one tell if the constraint set obtained is complete? Would analyzing a larger portion add to the constraint set?

In the case of the Morse-Thue carpet, analyzing a modest portion yields a constraint set with 18 templates. Analyzing larger portions do not increase the size of the constraint set. It seems reasonable, examining the method by which the Morse-Thue carpet is constructed, that this constraint set applies to the entire, unlimited pattern.

But for other patterns, such as random ones, there is no basis for such an assumption. In fact, the constraint set for a randomly generated pattern may include all 32 possible constraints.

On the other hand, what is the point of trying to determine the neighborhood constraint set for a pattern without structure?

References

1. Griswold, Ralph E. "Constrained Patterns, Part 1: Basic Concepts", 2002: http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_con1.pdf

2. Griswold, Ralph E. "Pattern Extension Schemata, Part 1", 2002: http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_sch1.pdf

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