Befriending Sequences

Friendly Sequences

A *friendly sequence* is one in which successive terms differ by one. Since a friendly sequence may be a repeat on which a longer sequence is based, the first and last terms must be friendly so that repeats are friendly.

Friendly sequences often make good candidates for threading and treadling sequences. And since they have alternating parity, they are applicable to weaves that have this requirement, such as overshot.

Close proximity amounts to friendship. Figure 1 shows a friendly sequence, which we'll label ☺. Notice, as required, its first and last terms are friendly.

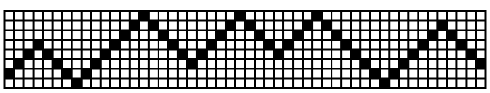

Figure 1. ☺ A Friendly Sequence

Figure 2 shows a fairly unfriendly sequence, which we'll label 😐, and Figure 3 shows a downright hostile sequence, ☹.

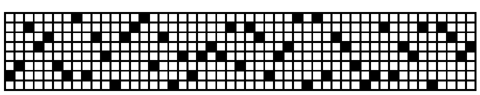

Figure 2. 😐 A Fairly Unfriendly Sequence

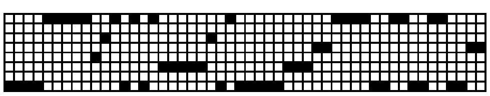

Figure 3. ☹ A Hostile Sequence

☺ exudes good vibes; it's a cheerful sequence. The tension and confusion in 😐 are evident, while ☹ reeks of discord.

Our goal here is to convert unfriendly sequences to friendly ones — to befriend unfriendly sequences. These are the rules:

- Only friendly terms may be added.
- Terms may not be deleted.
- Existing friends may not be separated.

Under these rules, befriending a friendly sequence does not change it.

The most straightforward and conservative approach is to add the fewest terms necessary to achieve a friendly result. This involves inserting a friend between pairs of equal, self-focussed terms and adding a run of friendly terms between unfriendly terms that are some distance apart.

When there is a pair of equal, self-focussed terms, there is a question of whether to insert a term that is one larger or one smaller. This can be done many ways. One natural way is to make the choice at random. Another way is to alternate between the two choices. In the examples that follow, choices are made at random.

A more enthusiastic approach is to allow some leeway in inserting friends between unfriendly terms — letting the friendly path wander a little, adding more friends than are strictly necessary. Wandering implies some degree of randomness. Of course, we expect friend-binding paths to be finite so that the befriending process terminates. For this reason, the choice of direction is biased toward the target friend in a manner that makes the probability of termination very high.

Figure 4 shows the results of befriending 😐 and ☹ in a conservative way. Figure 5 shows the results for more enthusiastic befriending. Note that enthusiastic befriending produces a more lively result than conservative befriending.

The horizontal tick marks at the left edges of these plots show the upper and lower bounds for the original sequences. Befriending in the way we've done it can add values larger or smaller than those in the original sequences.

What is a Friend?

The key question in befriending sequences is what constitutes a friend. We made what appeared to be a simple statement about this at the beginning of this article, but sequences for drafting may come from modular reduction in order to bring a sequence within the bounds of

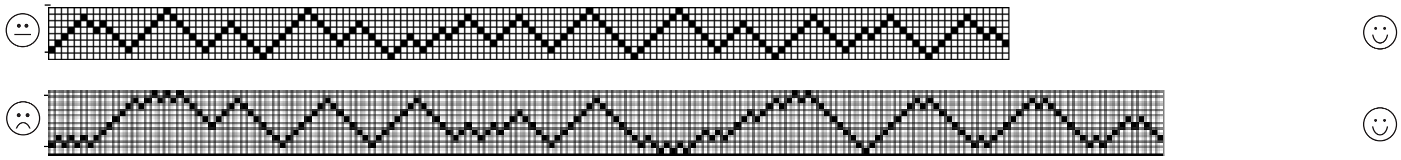

Figure 4. Conservatively Befriended Sequences

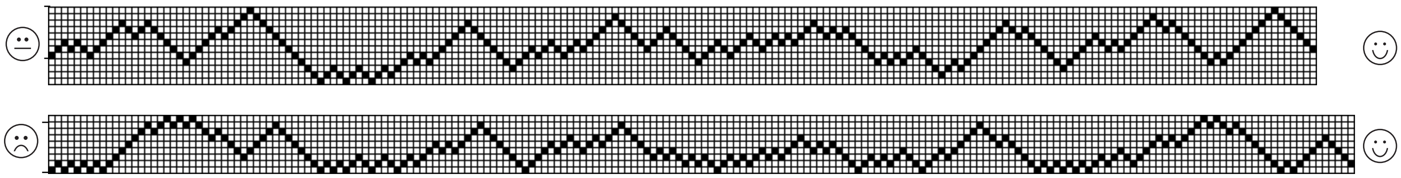

Figure 5. Enthusiastically Befriended Sequences

the number of shafts or treadles to be used [1]. In such cases, the modulus and 1 are friends.

Modular reduction effectively wraps the sequence around a modular wheel whose modulus, *m*, is the number of shafts. Values not in the range $1 \le i \le m$ are replaced by their residues. See Figure 6.

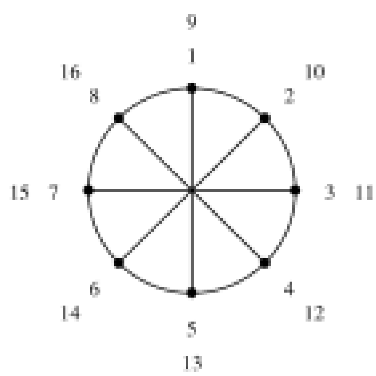

Figure 6. Arithmetic Shaft Modulo 8

The converse operation to modular reduction, which we call modular expansion, can be used to convert a sequence on *m* shafts to a sequence on *n* shafts, $n \ge m$, in which there is no wrap-around. The result is a sequence whose residues, shaft modulo *m*, produce the original sequence.

Figures 7 and 8 show an example of modular expansion.

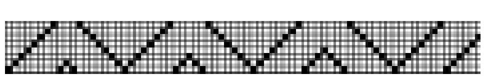

Figure 7. A Point Draw

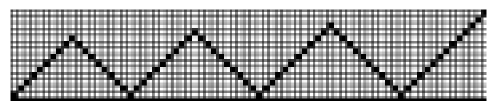

Figure 8. The Modular Expanded Point Draw

Note that modular expansion exposes the underlying pattern in this point draw.

The process of modular expansion is simple and relies on the fact that 1 and *m* are adjacent on the modular wheel.

Starting with $i = 1$, if term $t_i = m$ and $t_{i+1} = 1$, add *m* to t_{i+1} and all the remaining terms (shifting them upward by *m*). Similarly, if $t_i = 1$ and $t_{i-1} = m$, subtract *m* from t_{i-1} and all the remaining terms (shifting them downward by *m*). Note that adding or subtracting a multiple of *m* does not affect the residues.

When the process is complete, add enough multiples of *m* to bring the smallest value in the range 1 to *m*. (The smallest value can be less than 1 but it cannot be greater than *m*, since t_1 is not greater than *m* and is not changed by the process.)

If the expanded sequence is not friendly, it can be made friendly and then reduced according to the original modulus. Figure 9 shows a sequence that is not friendly when it is expanded, as shown in Figure 10. Figure 11 shows the result of conservatively befriending this sequence and Figure 12 shows the result of modular reduction of the sequence by its original modulus.

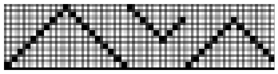

Figure 9. A Sequence

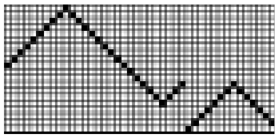

Figure 10. The Modular-Expanded Sequence

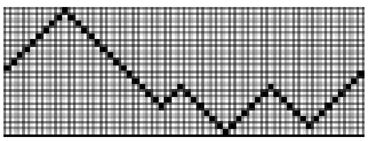

Figure 11. The Befriended Modular-Expanded Sequence

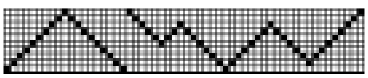

Figure 12. The Modular-Reduced Befriended Sequence

Design Applications of Friendly Sequences

Virtually any sequence can be befriended to produce a threading or treadling sequence that gives an aesthetically pleasing weave.

There are many sequences that can benefit from befriending. For example, the modular reduction of sequences of mathematical origin often produces a periodic but unfriendly sequence. Figure 13 shows the Fibonacci sequence shaft-modulo 8, which has period 12, and Figure 14 shows the result of conservatively befriending it, which has period 42. Tick marks show the boundaries of repeats.

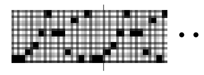

Figure 13. The Fibonacci Sequence Modulo 8

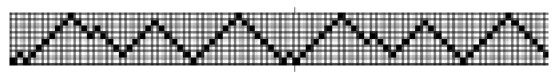

Figure 14. The Befriended Fibonacci Sequence Modulo 8

Although these are very different sequences, the second is derived from the first in a well-defined way.

The Appendix shows some examples of weaves derived by befriending equences with mathematical origins.

There are, of course, endless other possibilities for using friendly sequences. That is the challenge of creative weave design.

Reference

1. *Drafting with Sequences*, Ralph E. Griswold, 2001:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_seqd.pdf

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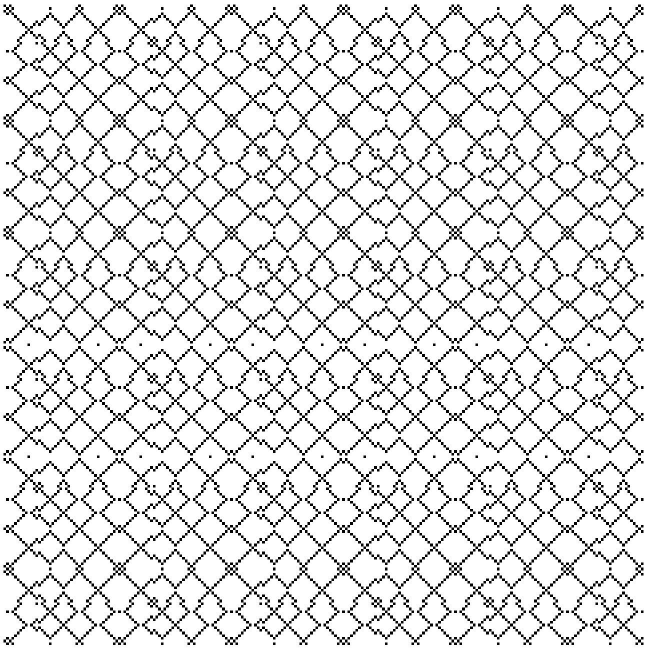
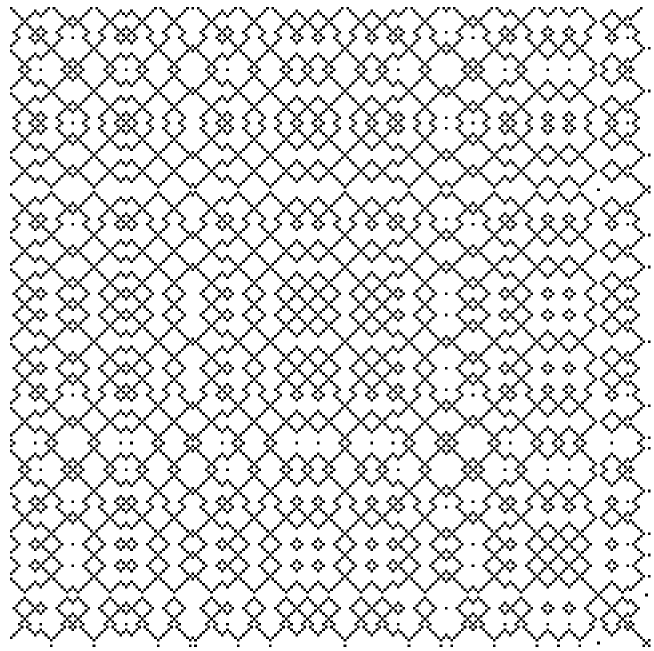
Appendix

These drawdown images have 240 ends and 240 picks. All for eight shafts and eight treadles, treadled as drawn in. Befriendings are conservative except as noted

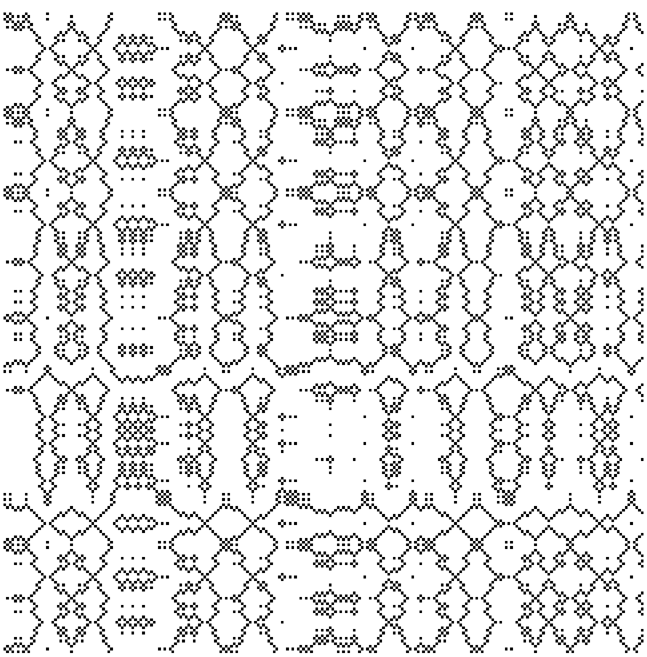
Direct Tie-Ups



Fibonacci Sequence

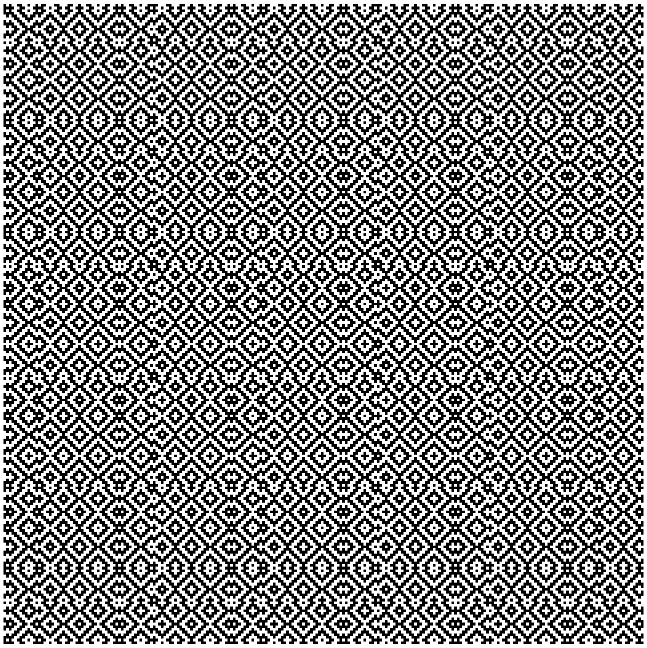


Primes

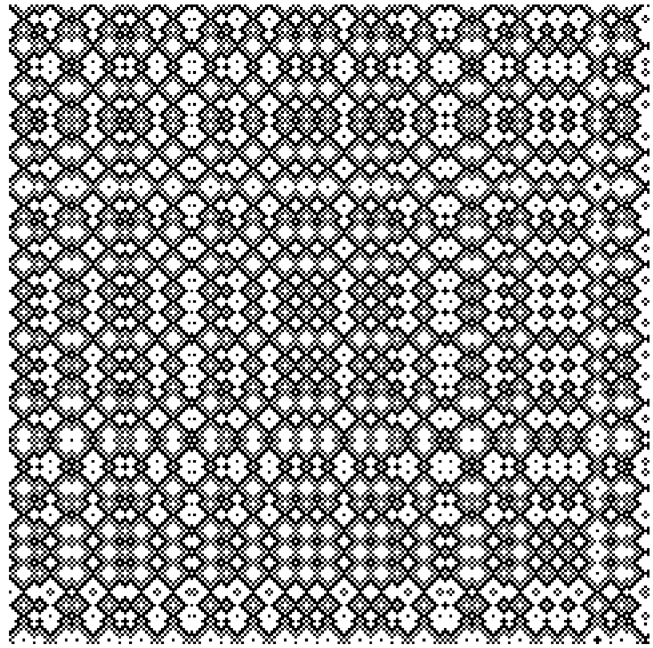


Fibonacci Sequence, Enthusiastic

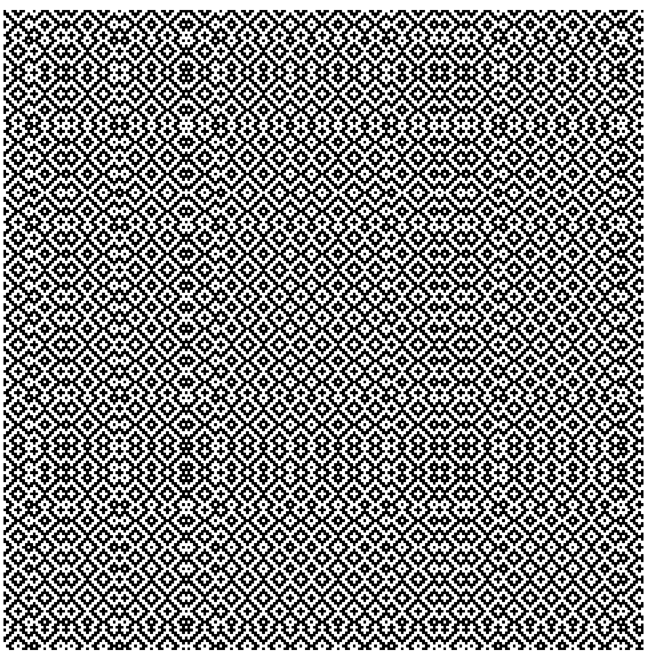
Twill Tie-Ups



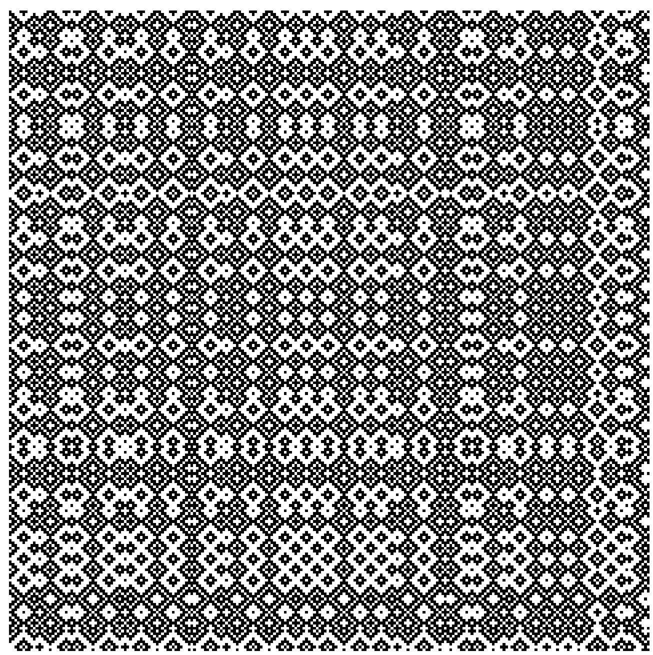
Fibonacci Sequence, 2/2



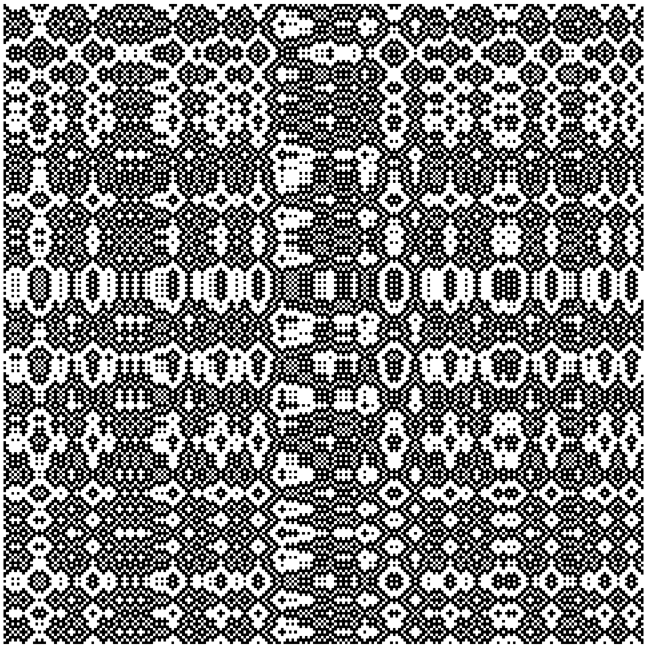
Primes, 2/4/1/1



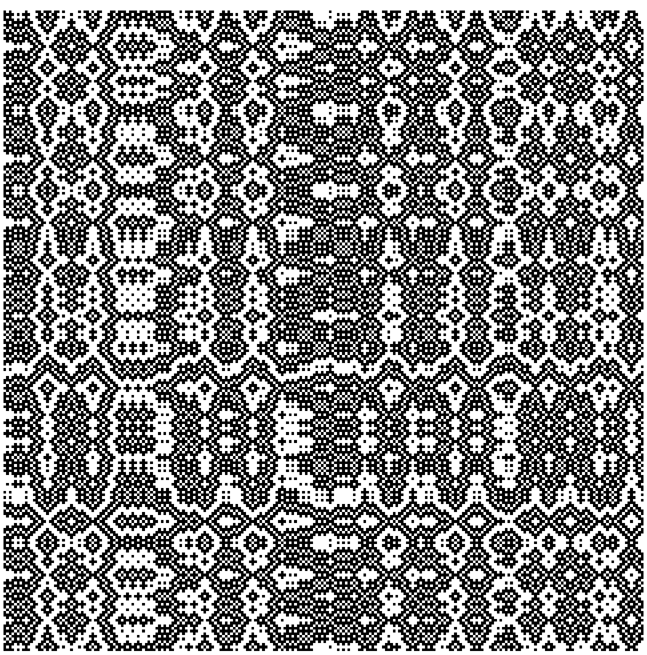
Primes, 2/2



Primes, 2/1/2/3



Primes, 2/1/2/3, Enthusiastic



Fibonacci Sequence, 2/1/2/3, Enthusiastic