Operations on Patterns, Part 2: Geometrical Transformations

In the first article in this series [1], we introduced the basic notions related to operations on patterns. This article deals with geometric operations.

There are two kinds of geometrical operations that can be performed on patterns: rotations and flips.

Rotations

A pattern can be rotated in increments of 90º: 90º, 180º, and 270º. Rotation by 360º leaves the pattern unchanged. We will assume clockwise rotation as a convention.

We will indicate the rotations of a pattern *P* by 90º, 180º, and 270º by ⊕*P*, ⊕*P*, and ⊕*P*, respectively.

Figure 1 shows the rotations of a square pattern and Figure 2 shows the rotations of an oblong pattern.

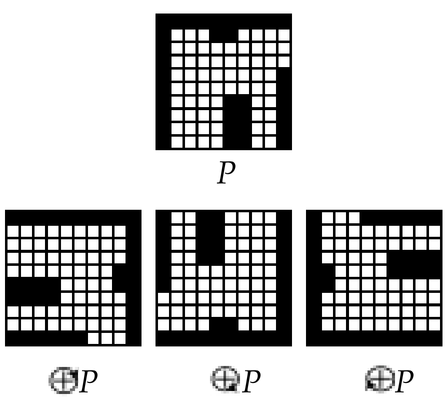

Figure 1. Rotations of a Square Pattern

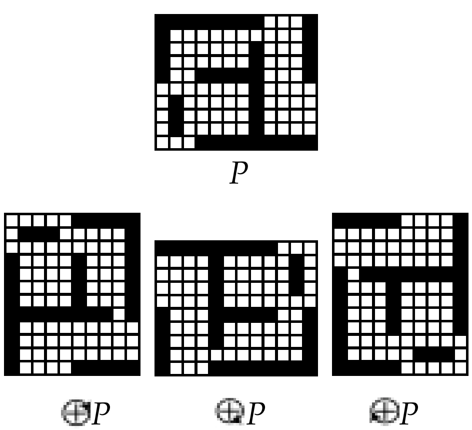

Figure 2. Rotations of an Oblong Pattern

Note that:

$\eta(P) = \omega(⊕P) = \omega(⊕P)$
$\omega(P) = \eta(⊕P) = \eta(⊕P)$
$\eta(P) = \eta(⊕P)$
$\omega(P) = \omega(⊕P)$.

Flips

There are four flips:

- horizontal, around a vertical axis
- vertical, around a horizontal axis
- right, around the left diagonal (from the upper-left corner to the lower-right)
- left, around the right diagonal (from the upper-right corner to the lower-left)

We will indicate these flips of a pattern *P* by ↔*P*, ↕*P*, ⤢*P*, and ⤡*P*, respectively.

Figure 3 shows these flips for a square pattern and Figure 4 shows them for an oblong pattern.

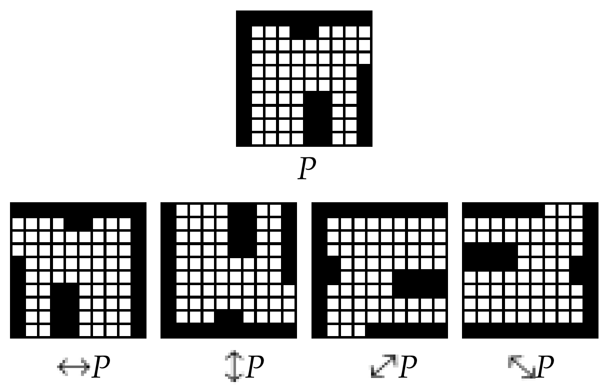

Figure 3. Flips of a Square Pattern

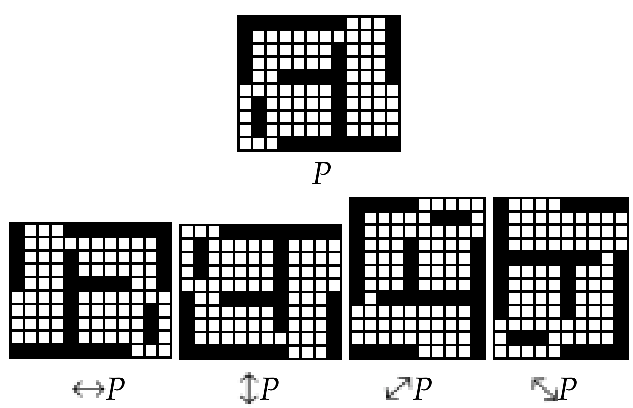

Figure 4. Flips of an Oblong Pattern

Note that:

$$\eta(P) = \eta(\leftrightarrow P) = \eta(\updownarrow P)$$

$$\omega(P) = \omega(\leftrightarrow P) = \omega(\updownarrow P)$$

$$\eta(P) = \omega(⤢ P) = \omega(⤡ P)$$

$$\omega(P) = \eta(⤢ P) = \eta(⤡ P)$$

Compound Operations

For rotations in increments of 90°, only one operation, ⊕*P*, is needed. Applying it twice results in rotation by 180°, and applying three times results in rotation by 270°:

⊕⊕*P* = ⊕*P*
⊕⊕⊕*P* = ⊕*P*

There also are relationships between rotations and flips. For example,

⊕⤢*P* = ↔*P*

In fact, all the geometrical operations can be obtained by using just ⊕*P* and ⤢*P* or by using just ⤢*P*, ↔*P*, and ↕*P*. For a discussion of these relationships, see Reference 2.

Although the relationships between the geometrical operations are interesting, for pattern construction, it's more convenient to have the whole set available.

Summary

⊕*P*	rotation by 90°
⊕*P*	rotation by 180°
⊕*P*	rotation by 270°
↔*P*	horizontal flip
↕*P*	vertical flip
⤢*P*	right flip
⤡*P*	left flip

Reference

1. *Drafting with Sequences*, Ralph E. Griswold, 2004:
http://www.cs.arizona.edu/patterns/ weaving/webdocs/gre_seqd.pdf.

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