

T-Sequences, Part 1: Introduction

In conventional weaving drafts, the threading and treading are given as sequences of integers that specify the shafts and treadles used. Threading and treading sequences collectively are called *t-sequences*.

T-sequences have several characteristics:

- They consist of positive integers.
- They are limited in length — the number of ends used for threading and the number of picks for treading.
- The maximum value is small — the number of shafts or treadles used.

From here on, threading and treading will not be distinguished as far as t-sequences are concerned.

Patterns of great variety abound in t-sequences from drafts. See the grid plots on the last page of this article. These patterns in turn, combined with tie-ups, produce the patterns in weave structures.

This series of articles studies t-sequences in terms of the patterns that occur in them and provides a formalism for describing these patterns. More important, this formalism can be used as the basis for constructing new t-sequences.

Cataloging all the kinds of patterns that occur in t-sequences is a daunting task, and the amount of notation needed to describe them is large and forbidding. Yet, through it, t-sequence patterns can be understood and created.

The formalism that is introduced in these articles allows patterns to be described precisely and compactly and provides conceptual focus. There is no mathematics, *per se*, just as there is no mathematics in the notation used for weaving drafts. Like draft notation, the t-sequence notation must be understood to be useful.

Terminology and Notational Conventions

The term *sequence* implies linear order. The *terms* in a sequence come one after another. There is a first term, a second term, and so on.

T-sequence terms may be explicit, as in 1, 4, 6, and so on, or they may be given as *variables* that take on different values in different contexts. Variables are indicated by lowercase italic letters, such as *i*, *j*, and *k*. Subscripts may be used to distinguish different term variables, such as i_1 , i_2 , j_3 , and so on.

Sequences may be given explicitly by enclosing their terms in square brackets, as in

$$[1, 2, 3, 4, 3, 2, 3, 4, 5, 6, 7, 8, 7, 6, 5, 4]$$
$$[i_1, i_2, i_3, i_4, i_5, i_6, i_7, i_8]$$

It is also possible to have an empty sequence with no terms. Although an empty sequence is not useful in weaving, it may arise in operations used to create other t-sequences. The empty sequence is denoted explicitly by $[\]$ and is represented by the symbol Θ .

Ellipses are used to indicate one or more terms in a sequence that are not given explicitly, as in

$$[1, 2, 3, 4, 5, \dots 15, 16, 15, \dots 1]$$

Variables are used to name sequences so that they can be referred to without specifying their terms. Sequence variables are indicated by uppercase italic letters, such as *S*, *T*, and *U*. Sequence variables also may have subscripts to distinguish different sequences in a common context. Examples are S_1 , S_2 , and T_5 .

A specific sequence can be given a name. This is called *assignment* and is indicated by a colon followed by an equal sign, as in

$$S := [1, 2, 3, 4, 3, 2, 3, 4, 5, 6, 7, 8]$$

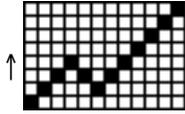
Then *S* can be used to refer to this sequence without giving all the terms.

Two t-sequences are identical, denoted by $S = T$, if they are the same, term by term.

Graphic Representation

Patterns in t-sequences usually are easier to detect in graphical representations than by examining sequences of integers.

In these articles, the values in grid plots increase upward:



In the grid plots used in these articles, the axes usually are not marked, since such markings tend to distract the human visual system and interfere with pattern recognition.

The bottom row corresponds to the value 1 and the left column corresponds to the first value in the sequence.

T-Sets and T-Numbers

Sometimes it is useful to specify the particular shafts/treadles used in a t-sequence. This is called the *t-set* of the t-sequence. Braces are used to denote t-sets, as in $\{1, 2\}$.

In many cases, all the shafts and treadles are used, as illustrated in the example above. For example, the t-set for the plot above is

$$\{1, 2, 3, 4, 5, 6, 7, 8\}.$$

Finally, to avoid having to say shaft/treadle numbers repeatedly, *t-numbers* is used to cover both.

Sequence Metrics

There are three important properties associated with a sequence: its length, its minimum value (usually 1), and its maximum value, called its *bound*. These are given by functions whose names are lowercase Greek letters:

- $\lambda(S)$ length
- $\mu(S)$ minimum value
- $\gamma(S)$ maximum value (bound)

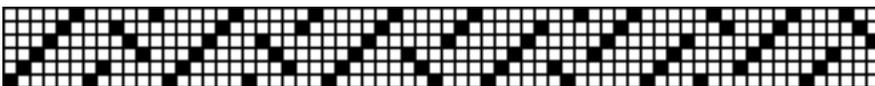
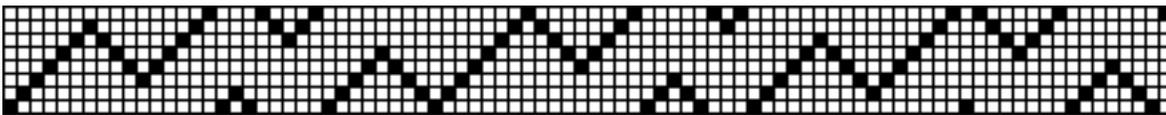
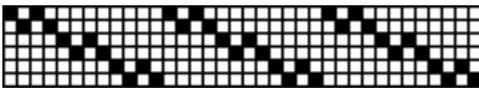
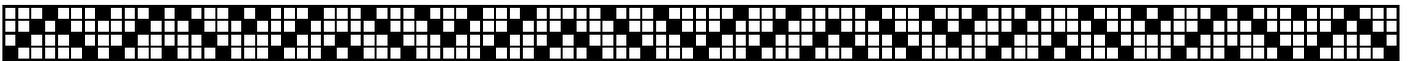
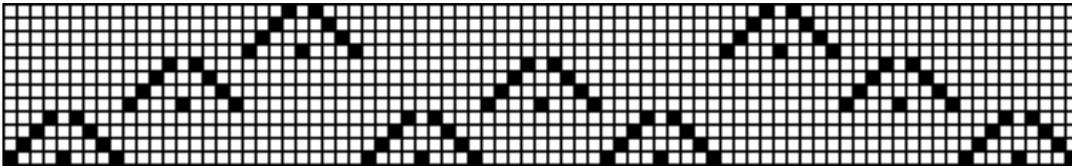
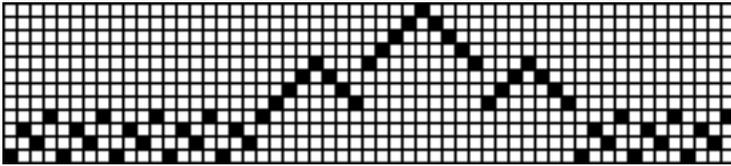
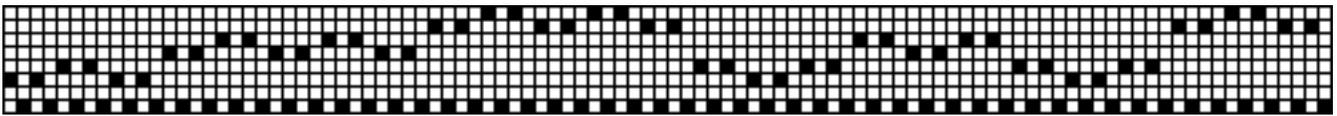
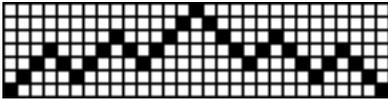
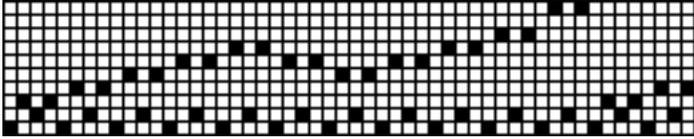
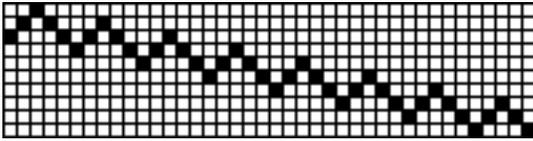
For the sequence S in the preceding section, $\lambda(S) = 12$, $\mu(S) = 1$, and $\gamma(S) = 8$.

Articles to Follow

Subsequent articles will cover the different kinds of patterns that occur in t-sequences, grouping them by common characteristics.

Ralph E. Griswold
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T-Sequences Represented Graphically