T-Sequences, Part 3: Runs

Runs — integers in numerical sequence — occur very frequently in t-sequences.

A *simple run* consists of integers in order from a starting value to an ending value. If the starting value is less than the ending value, the run is up, else it is down. Figure 1 shows an up run followed by a down run.

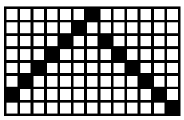

Figure 1. Simple Runs

There are two different kinds of runs that are composed of simple runs: *connected runs* as shown in Figure 2 and *disconnected runs*, as shown in Figure 3 (and in Figure 1).

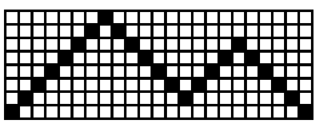

Figure 2. Connected Run

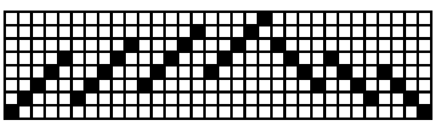

Figure 3. Disconnected Runs

Simple Runs

A simple run is denoted by

$i \rightarrow j$

For example,

$(2 \rightarrow 8) = [2, 3, 4, 5, 6, 7, 8]$

and

$(5 \rightarrow 1) = [5, 4, 3, 2, 1]$

Connected Runs

In a connected run, runs go up and down (or down and up) between a *beginning point*, *inflection points*, and an *ending point* with no gaps. Beginning points, inflection points, and ending points collectively are called *anchor points*.

Connected runs can be constructed using simple runs and concatenation (with duplicate removal [1]). For example, the connected run shown in Figure 2 can be constructed by

$$(1 \rightarrow 8) \mid (8 \rightarrow 2) \mid (2 \rightarrow 6) \mid (6 \rightarrow 1)$$

where parentheses are used to make the grouping of operations unambiguous.

Constructing a connected run using concatenation is unnecessarily cumbersome, however, since the run is completely described by its anchor points: the sequence [1, 8, 2, 6, 1]. This is emphasized in Figure 4, where the anchor points are set off by color. The sequence of anchor points is shown in Figure 5.

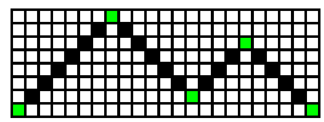

Figure 4. Highlighted Anchor Points

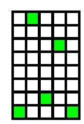

Figure 5. Anchor-Point Sequence

The operation for constructing a connected run from an anchor-point sequence S is denoted by $\rightarrow S$. Note that the operator symbol is in prefix position before its operand as opposed to the same symbol used to denote simple runs, which is in infix position between its operands.

For example,

$\rightarrow [1, 8, 2, 6, 1]$

produces the same connected run as the concatenation of simple runs shown earlier.

Disconnected Runs

In a disconnected run, there are breaks in the numerical sequence of values.

Disconnected runs can, of course, be constructed by concatenating simple runs. For example, the disconnected run shown in Figure 3 can be constructed by

$$(1 \rightarrow 5) \mid (2 \rightarrow 6) \mid (3 \rightarrow 7) \mid (4 \rightarrow 8) \mid (6 \rightarrow 3) \mid (5 \rightarrow 2) \mid (4 \rightarrow 1)$$

This also is unnecessarily cumbersome, since the runs are completely characterized by pairs of beginning and ending points: the sequences

[1, 2, 3, 4, 6, 5, 4]

and

[5, 6, 7, 8, 3, 2, 1]

Figure 6 shows the disconnected runs with the end points highlighted. Figures 7 and 8 show the sequences of beginning and end points.

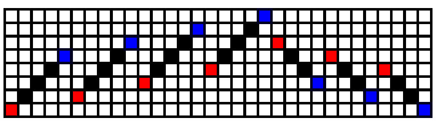

Figure 6. Highlighted End Points

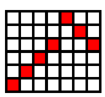

Figure 7. Beginning-Point Sequence

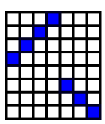

Figure 8. End-Point Sequence

The operation for constructing disconnected runs from sequences of end points is denoted by S / T, where S is the sequence of beginning points and T is the sequence of ending points.

For example, the disconnected runs shown in Figure 6 can be constructed by

[1, 2, 3, 4, 6, 5, 4] / [5, 6, 7, 8, 3, 2, 1]

It is worth noting that the sequences of beginning and ending points are concatenations of simple runs, so the same result can be obtained by

$$((1 \rightarrow 4) \mid (6 \rightarrow 4)) \; / \; ((5 \rightarrow 8) \mid (3 \rightarrow 1))$$

Although this form is more complicated than the one using explicit sequences, it reveals underlying structure in this disconnected run. Another form is perhaps more revealing:

$$((1 \rightarrow 4) \; / \; (5 \rightarrow 8)) \mid ((6 \rightarrow 4) \; / \; (3 \rightarrow 1))$$

This is the concatenation of a sequence of upward disconnected runs with a sequence of downward connected runs. This is, of course, evident in the grid plot.

Summary

$i \rightarrow j$	simple run
$\rightarrow S$	connected run
S / T	disconnected runs

Reference

1. Ralph E. Griswold, “T-Sequences, Part 2: Extension”, 2004:
http://www.cs.arizona.edu/patterns/weaving/webdocs/gre_ts02.pdf

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