## Satin Counters

Few aspects of weaving require more mathematics than simple arithmetic. When a subject does require more, authors of books on weaving sometimes provide descriptions that are anything but clear, even to a person with some knowledge of mathematics. And sometimes the descriptions are incomplete or even incorrect.

Part of the problem is that the authors think they are writing for an audience most of whom not only know little mathematics beyond the most basic but also often are hostile to or fearsome of mathematics. (Hostility is a good cover for fear, the latter being socially less acceptable.) Of course, authors themselves may have the same problems with mathematics. Part of the problem comes from trying to express in words things for which ordinary language is inadequate.

### How (or How Not) to Determine Satin Counters

Sidebar on what satin counters are by Ruth/Marg

Satin counters provide an example. Here are five quotations on the subject from sources dating from 1888 to 1994.

E. A. Posselt, author of many books on weaving and textiles in the late eighteenth and early nineteenth centuries, in 1882 writes [1]:

> *Divide the number of harness for the satin into two parts, which must neither be equal nor the one a multiple of the other; again it must not be possible to divide both parts by a third number.*

Harness is used here as a collective noun.

Charles Z. Petzold, writing in 1900, gives this (incomplete) rule [2]:

> *The mathematical formula is found by dividing the number of harnesses of the desired sateen into two parts. The numbers thus found should not be equal, neither should they be a multiple of each other.*

Ann Sutton, writing in 1982, describes determining satin counters in this way [3]:

> *Divide the number of ends (or shafts) on which the satin … is to be woven into two unequal parts, so that one shall not be a measure of the other, nor shall it be divisible by a common number.*

"Measure of the other" is British English and means (I think — my problem) "divisible by the other".

S. A. Zielinski, in his massive *Master Weaver Library*, says [4]:

> *Find two numbers which give a sum equal to the number of frames. None of these numbers can be 1; the two numbers cannot divide one another, or by any other number at the same time.*

Madelyn van der Hoogt, explains what a satin counter cannot be [5]:

> *The satin counter cannot be 1, or the interlacement forms a twill. It cannot be one fewer than the number in the unit … , or the interlacement forms a twill in the opposite direction. The counter cannot share a divisor with the number in the unit, or some warp threads interlace more than once and others not at all.*

Grammatical errors, tortured prose, questionable meaning, missing parts, and definition by elimination aside, what does this all mean?

### Mathematics to the Rescue (?)

#### Integer Division and Prime Numbers

The result of the division of two integers (whole numbers) such as 3 / 2 is usually taken to be 1-1/2 *[will fix typography]* or 1.5; that is 1 plus a fractional part. In true integer division, the fractional part is omitted and the result is just 1. If there is no fractional part, the second integer is said to evenly divide the first. For example, in 4 / 2, 2 evenly divides 4, and 2 is said to be a divisor of 4.

A prime number is one that has no divisors other than 1 and itself. An example is 7. The smallest prime is 2, the only even prime. The first few primes are 2, 3, 5, 7, 11, 13, 17, … The is no limit to the number of primes. This is sometimes stated as "the number of primes is infinite".

Two integers are relatively prime if they have no common divisor. For example, 5 and 12 are relatively prime, but 4 and 12 are not, since 4 divides both 4 and 12.

A mathematician might state it this way:

For $n$ shafts, find relatively prime $i$ and $j$ such that $1 < i < n$, $i$

$+ j = n$, and $1 < j < n$.

The mathematician talks in terms of "variables"— $i$, $j$, and $n$ — and describes the conditions they must satisfy. The condition "$1 < i < n$" can be phrased in plain English as "$i$ is greater than 1 and less than $n$" and the phrase "$i + j = n$" as "$i$ and $j$ add to $n$" In fact, it might seem clearer as "$j = n - i$", but the mathematician prefers to talk in terms of constraints (requirements), for which "$i + j = n$" is appropriate.

All that having been said, most readers would understand the mathematical statement above except that they probably would not know what "relatively prime" means, which is the conceptual core of the definition.

There are many variations on the so-called mathematical description. (Actually, mathematicians are capable of and sometimes take delight in using arcane symbols and convoluted prose to make what really is simple into something that is incomprehensible to the layperson and requires effort even for other mathematicians to understand [6].)

It is possible to provide a definition of satin counter in simple mathematical terms along with an explanation of the core concept that is intelligible to most readers. Simple examples, especially of things that work, as opposed to those that do not, are a great help in understanding such things. Unfortunately, many mathematicians consider it beneath them to do this in their writing. A famous computer scientist openly stated that the use of examples is a sign of intellectual weakness.

## A Table of Satin Counters

Having spent more than a page on the problems with describing satin counters, I'll finish by giving a table.

In a sense, a table aren't that bad: there are not that many different counters for the number of shafts that are available for hand looms and there's no need for a weaver to compute them.

Here is a table for 2 to 24 shafts. Only the smaller of the two counter pairs is given; the other is easy to determine by simple subtraction. For example, for 13 shafts and the small counter of 4, the large counter is $13 - 4 = 9$. Incidentally, most descriptions of satin counters say that the smaller is preferable, or at least more often used, without saying why. Indeed, why?

| *shafts* | *small counters* | *number* |
|---|---|---|
| 2 | | 0 |
| 3 | | 0 |
| 4 | | 0 |
| 5 | 2 | 1 |
| 6 | | 0 |

| | | |
|---|---|---|
| 7 | 2 3 | 2 |
| 8 | 3 | 1 |
| 9 | 2 4 | 2 |
| 10 | 3 | 1 |
| 11 | 2 3 4 5 | 4 |
| 12 | 5 | 1 |
| 13 | 2 3 4 5 6 | 5 |
| 14 | 3 5 | 2 |
| 15 | 2 4 7 | 3 |
| 16 | 3 5 7 | 3 |
| 17 | 2 3 4 5 6 7 8 | 7 |
| 18 | 5 7 | 2 |
| 19 | 2 3 4 5 6 7 8 9 | 8 |
| 20 | 3 7 9 | 3 |
| 21 | 2 4 5 8 10 | 5 |
| 22 | 3 5 7 9 | 4 |
| 23 | 2 3 4 5 6 7 8 9 10 11 | 10 |
| 24 | 5 7 11 | 3 |

As is well known to weavers, true satin requires at least five shafts and cannot be woven with six shafts. By the way, if the number of shafts is a prime, $p > 2$, any number $2 \le i \le p-1$ is a valid counter: If $i + j = p$, i and j must be relatively prime — otherwise a common factor would divide $p$.