

Polylogarithmic Approximation for Generalized Minimum Manhattan Networks

Aparna Das¹ Krzysztof Fleszar² Stephen Kobourov¹
Joachim Spoerhase² Sankar Veeramoni¹
Alexander Wolff²

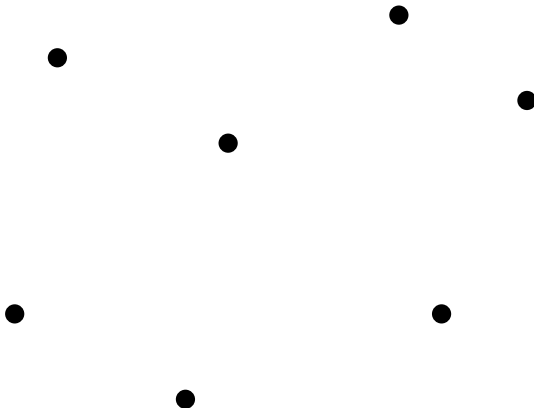
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University of Arizona

²Institut für Informatik
Universität Würzburg

14.06.2012
Mittagsseminar am Lehrstuhl 1

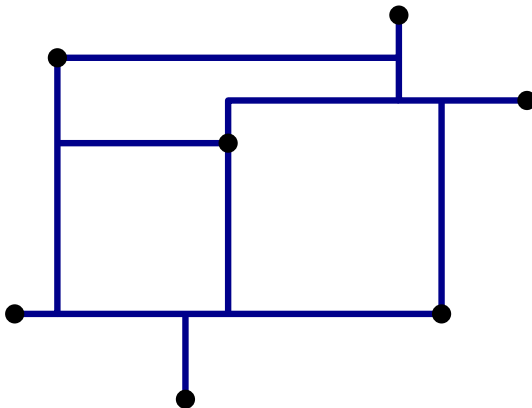
Minimum Manhattan Network Problem (MMN)

Problem Definition



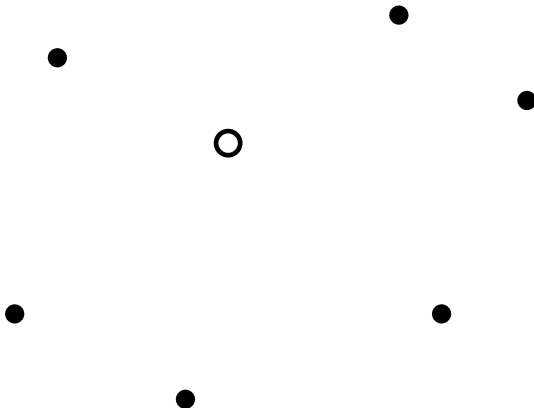
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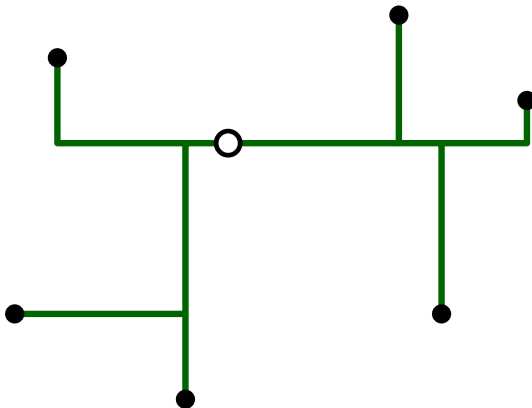
Rectilinear Steiner Arborescence Problem (RSA)

Problem Definition



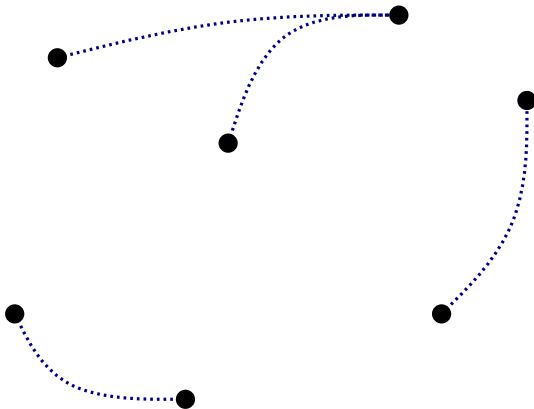
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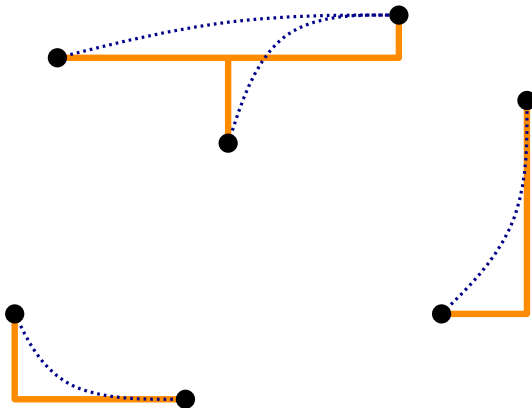
Generalized MMN Problem (GMMN)

Problem Definition



Generalized MMN Problem (GMMN)

Problem Definition



History

Introduction

- 1974: RSA
- 2000: RSA has PTAS
- 2000: RSA is NP-hard

- 1999: MMN
- 2004: MMN has 2-approximation
- 2009: MMN is NP-hard

- 2004: GMMN
- 2012: ...

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Previous Results

Introduction

| | 2D | Dimension $d > 2$ |
|------|---------------------|--------------------|
| RSA | $(1 + \varepsilon)$ | $O(n^\varepsilon)$ |
| MMN | 2 | $O(n^\varepsilon)$ |
| GMMN | ? | ? |

Our Contribution

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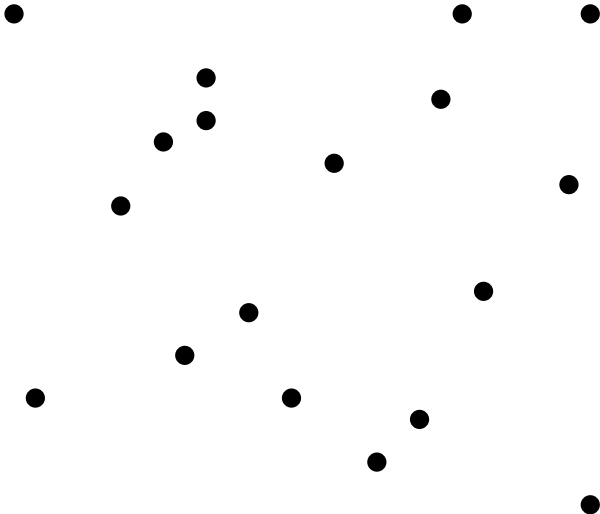
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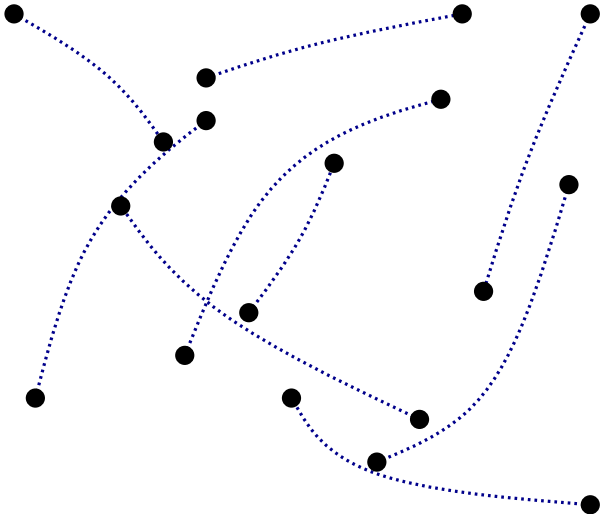
Problem Instance

Overview



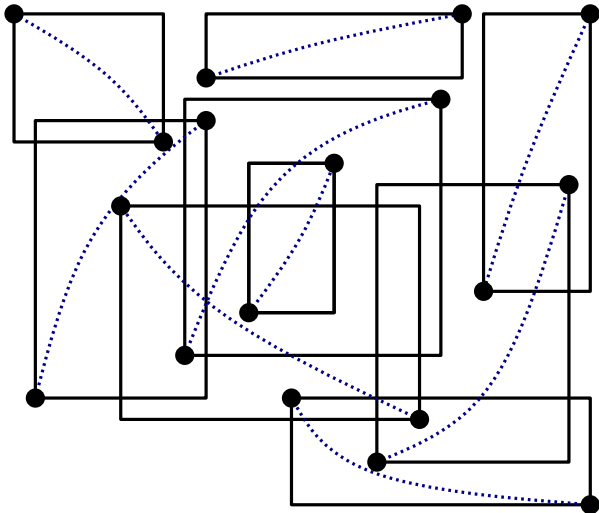
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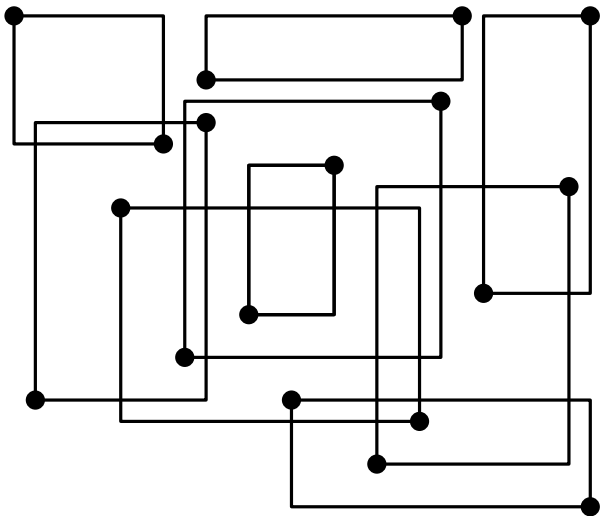
Problem Instance

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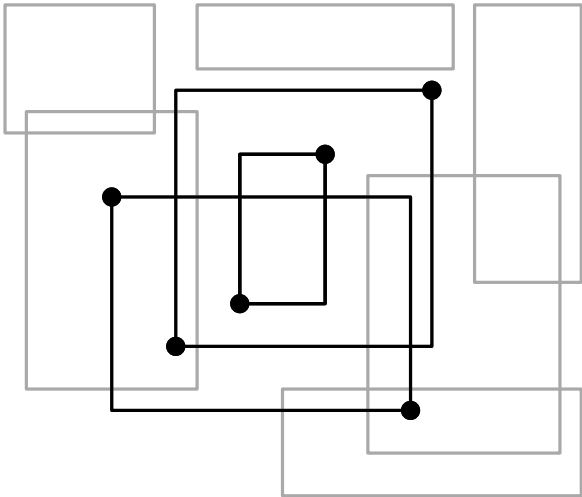
Partition ...

Overview



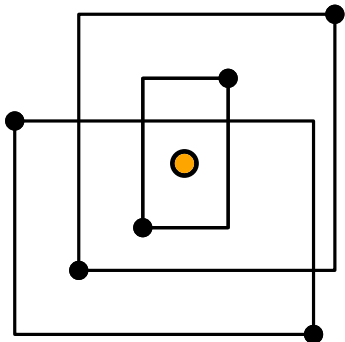
Partition ...

Overview



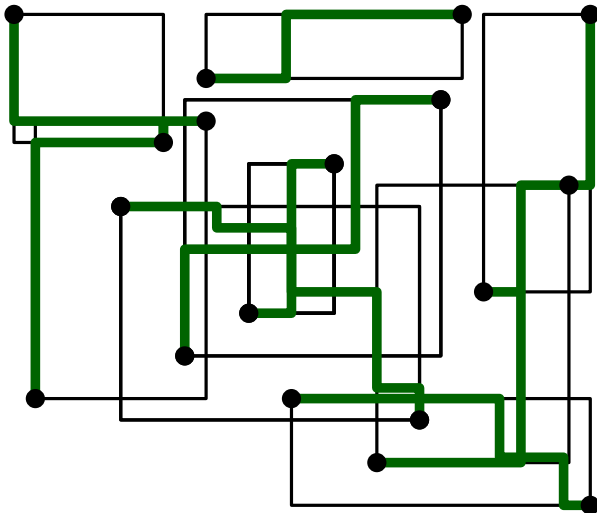
Partition ...

Overview

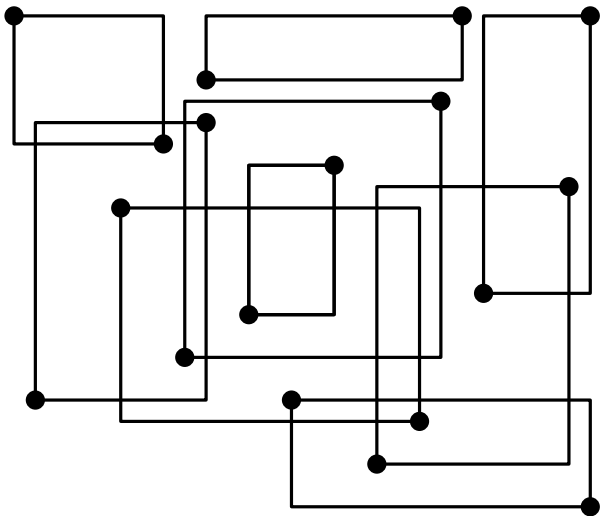


... And Reduce to RSA

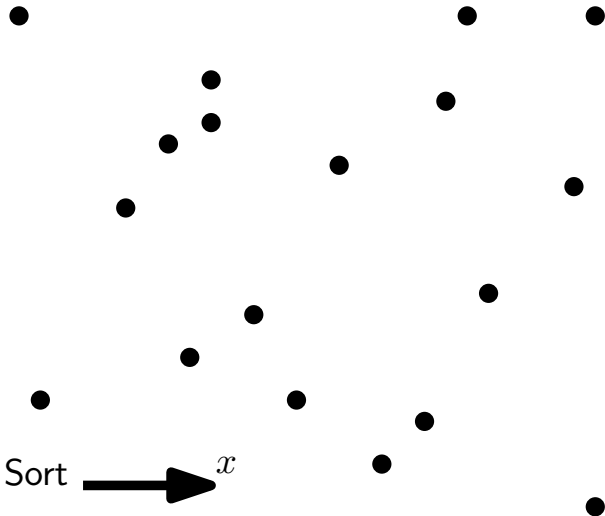
Overview



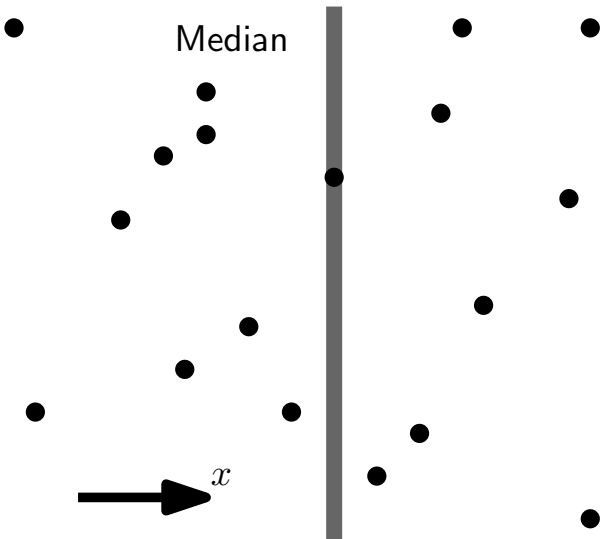
Partition Construction



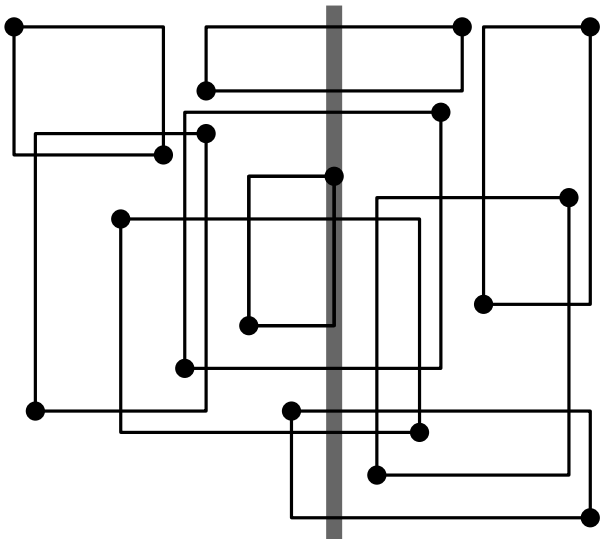
Partition Construction



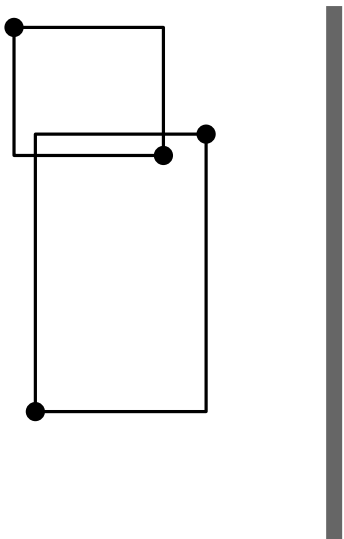
Partition Construction



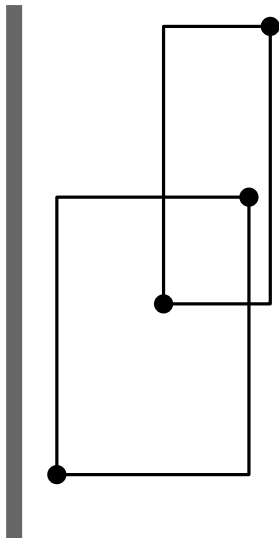
Partition Construction



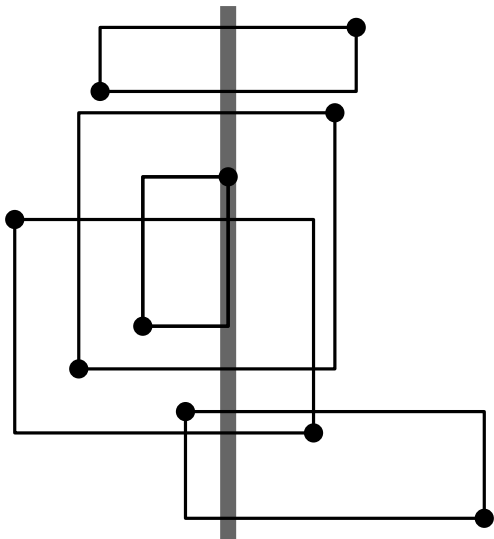
Left Sub-Instance Construction



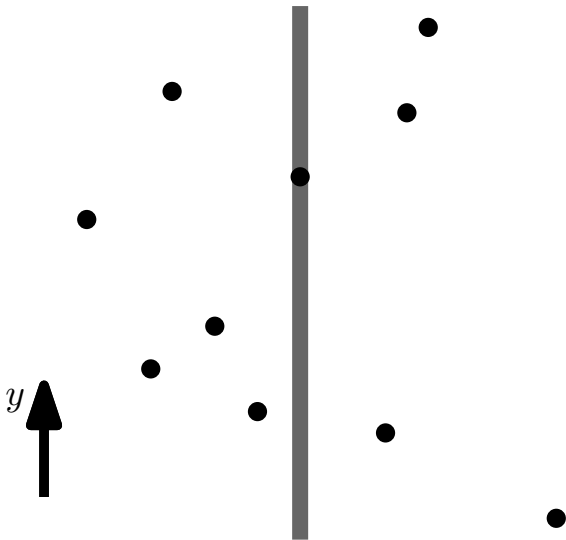
Right Sub-Instance Construction



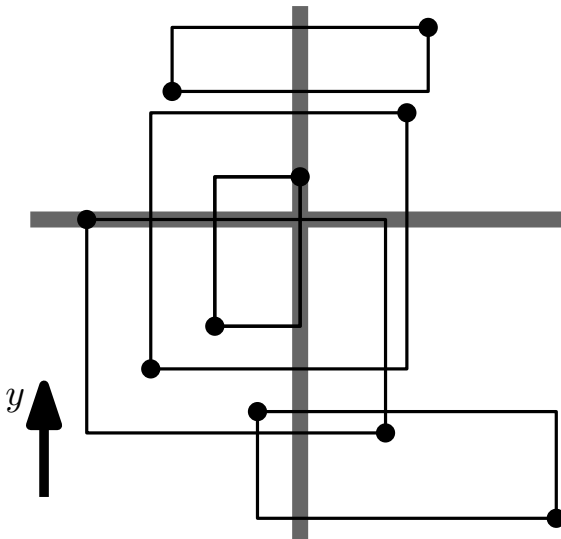
x-Separated Instance Construction



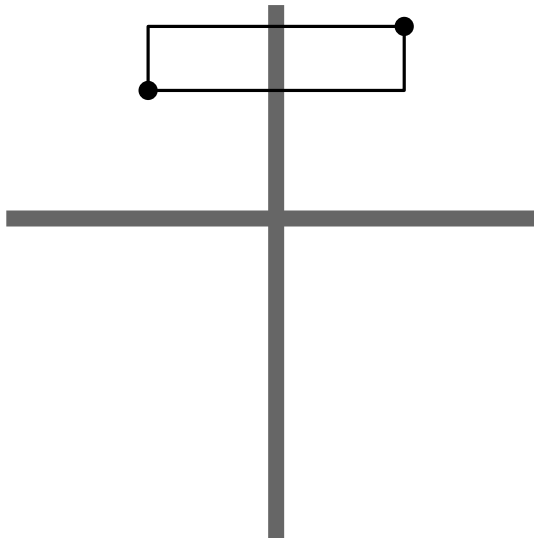
x-Separated Instance Construction



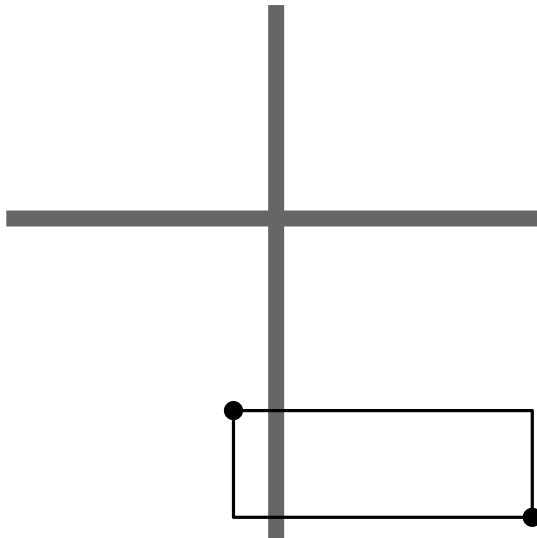
x-Separated Instance Construction



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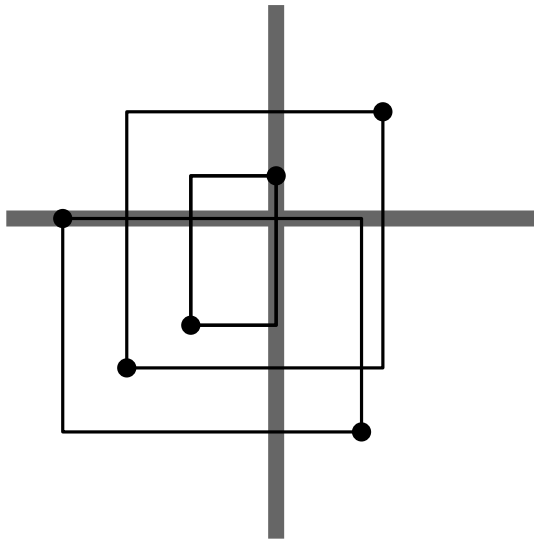


x-Separated Instance Construction



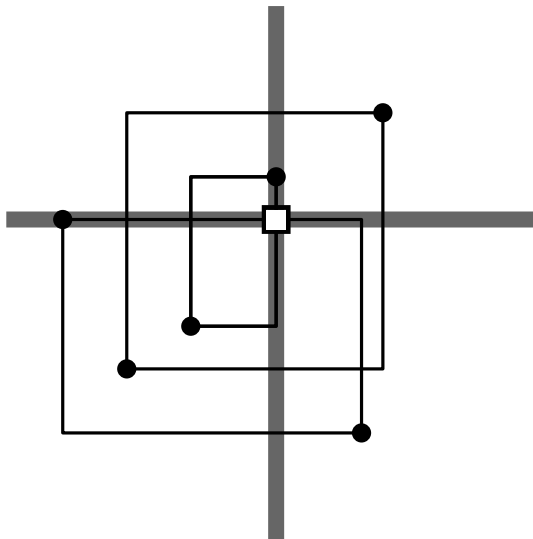
x-Separated Instance

Construction



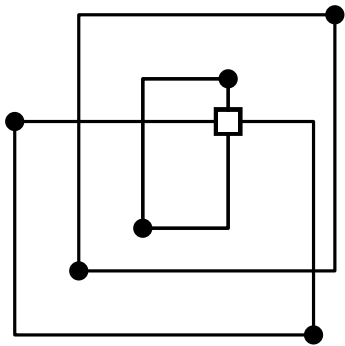
xy-Separated Instance

Construction



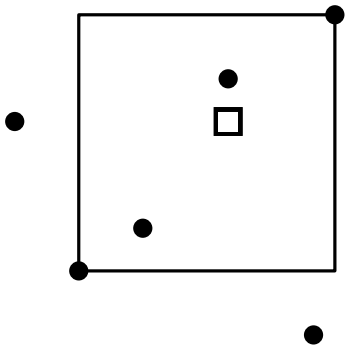
xy-Separated Instance

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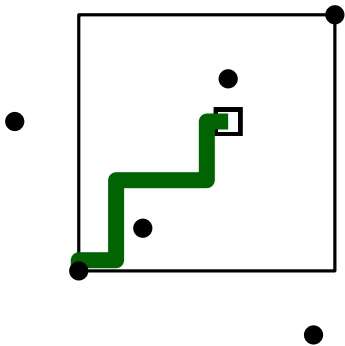
Observation

Construction



Observation

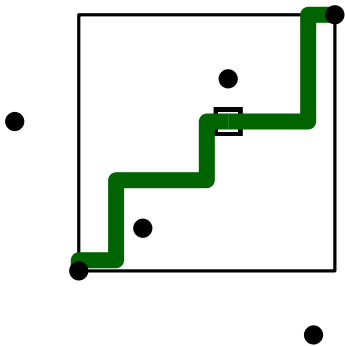
Construction



Observation

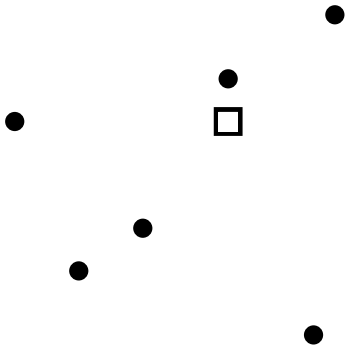
Construction

Manhattan-Path



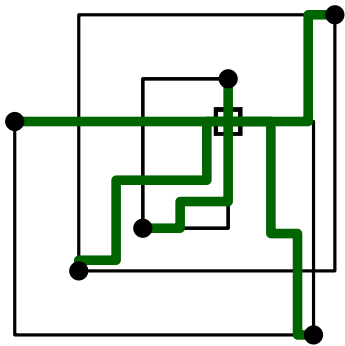
Observation

Construction



Reduce to RSA

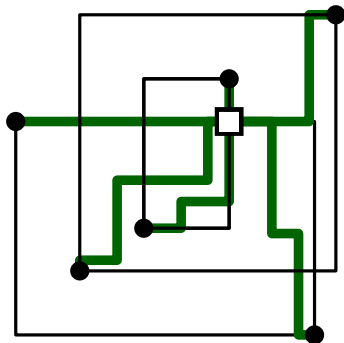
Construction



Cost of RSA

Cost Analysis

$$\text{cost} \leq (1 + \varepsilon) \cdot \text{OPT}_{\text{RSA}}$$

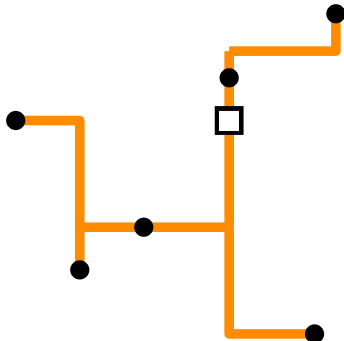


$\text{OPT}_{\text{RSA}} = ?$

Cost of RSA

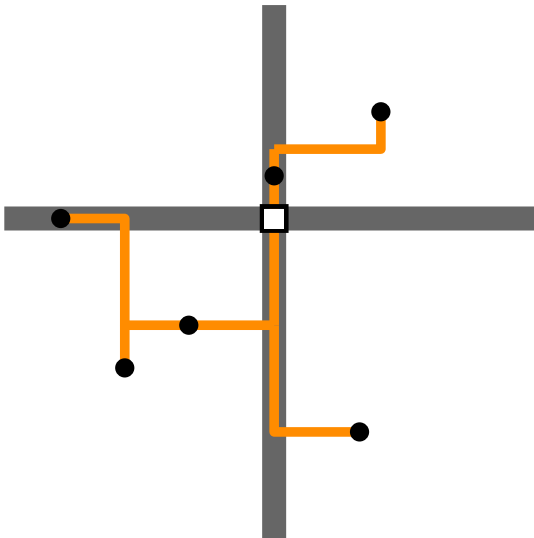
Cost Analysis

An optimum GMMN solution



Cost of RSA

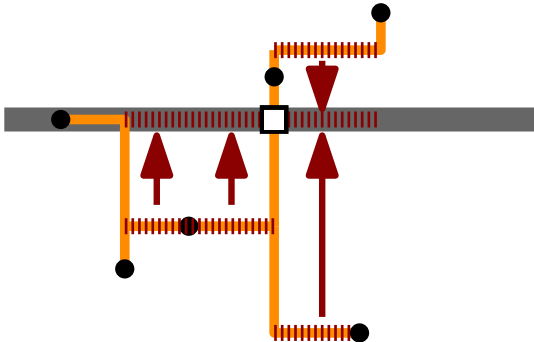
Cost Analysis



Cost of RSA

Cost Analysis

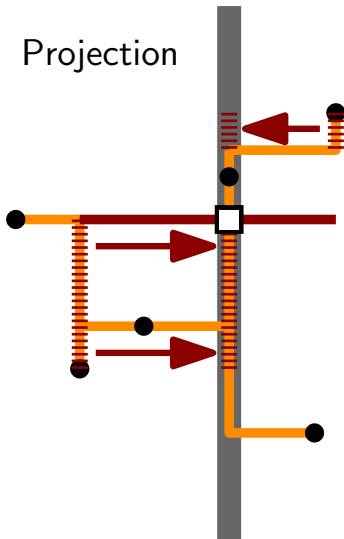
Projection



Cost of RSA

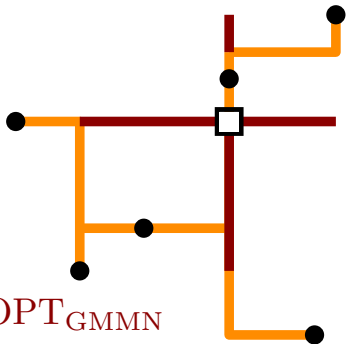
Cost Analysis

Projection



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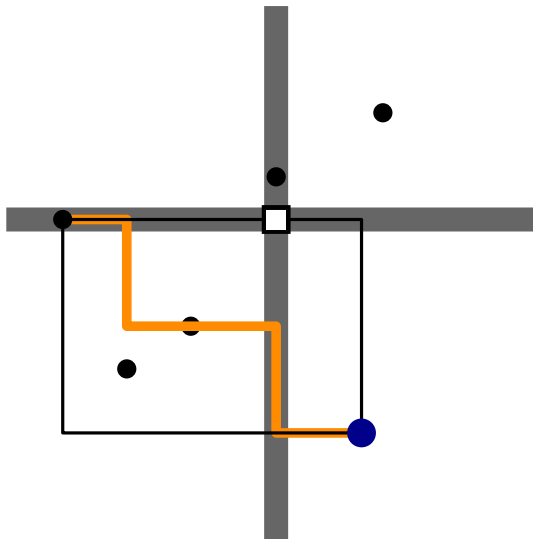
Cost Analysis



$$\text{cost} \leq 2 \cdot \text{OPT}_{\text{GMMN}}$$

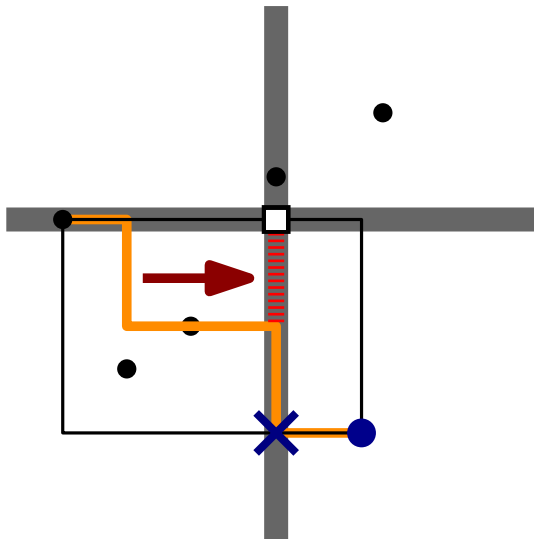
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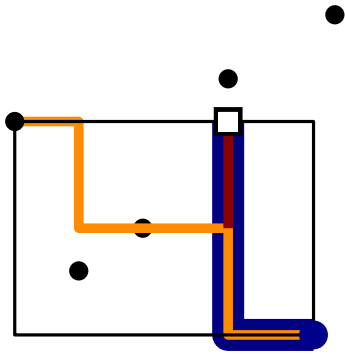
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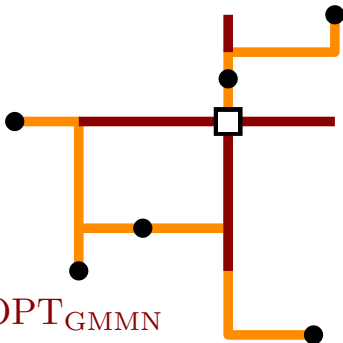
Cost Analysis



Cost of RSA

Cost Analysis

RSA solution



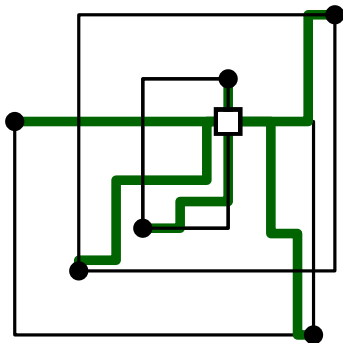
$$\text{cost} \leq 2 \cdot \text{OPT}_{\text{GMMN}}$$

$$\Rightarrow \text{OPT}_{\text{RSA}} \leq 2 \cdot \text{OPT}_{\text{GMMN}}$$

Cost of RSA

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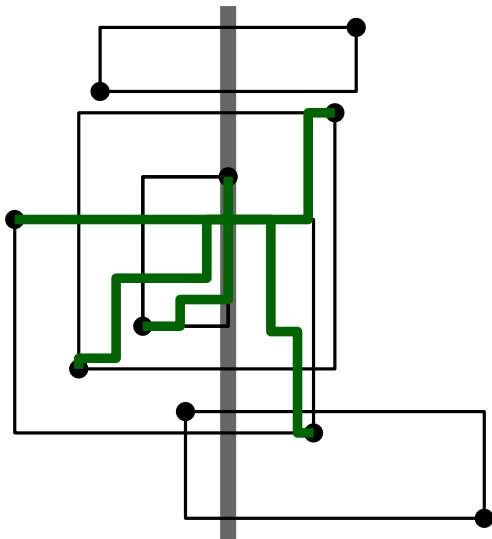


$$\Rightarrow \text{OPT}_{\text{RSA}} \leq 2 \cdot \text{OPT}_{\text{GMMN}}$$

$$\Rightarrow \text{cost} \leq (1 + \varepsilon) \cdot 2 \cdot \text{OPT}_{\text{GMMN}}$$

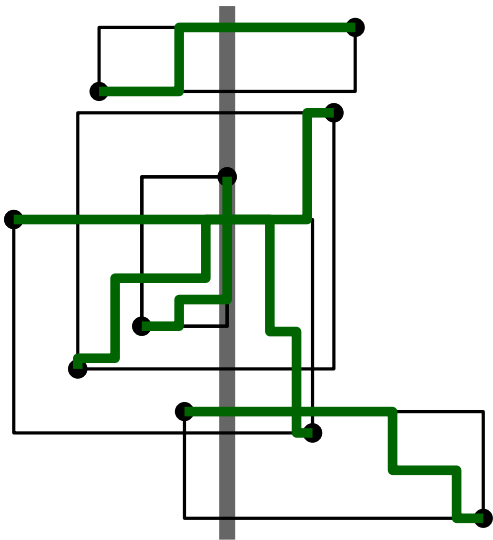
Cost of x -Separated Instance

Cost Analysis



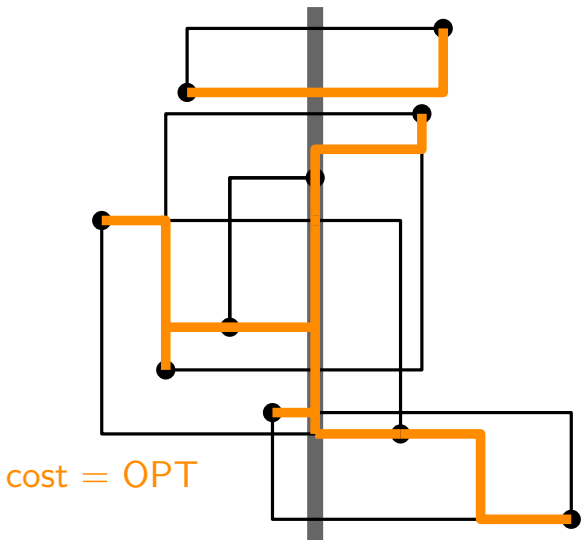
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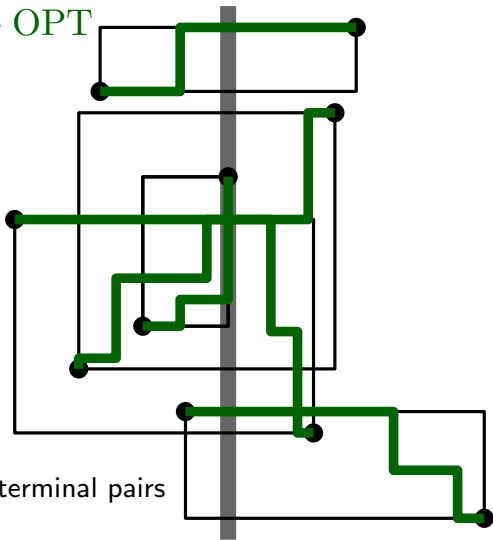
Cost Analysis



Cost of x -Separated Instance

Cost Analysis

$$\text{cost} \leq \rho_x(n) \cdot \text{OPT}$$



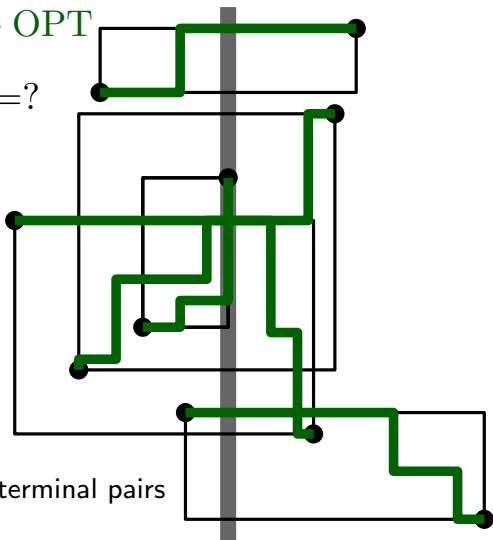
$n =$ number of terminal pairs

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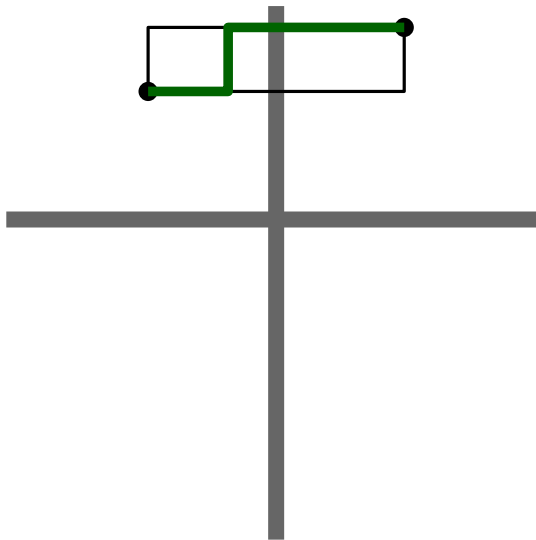
$$\rho_x(n) = ?$$



n = number of terminal pairs

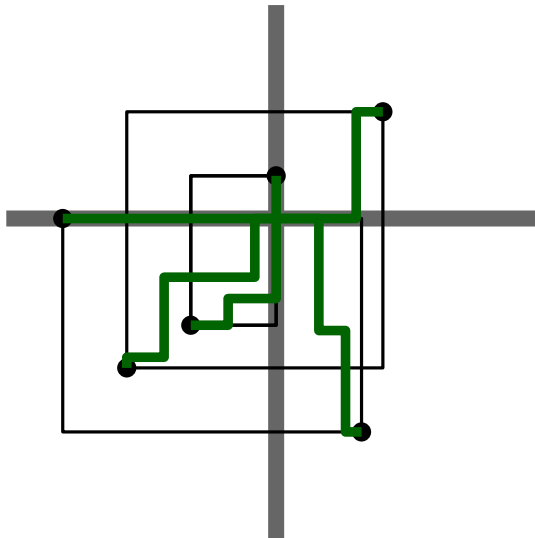
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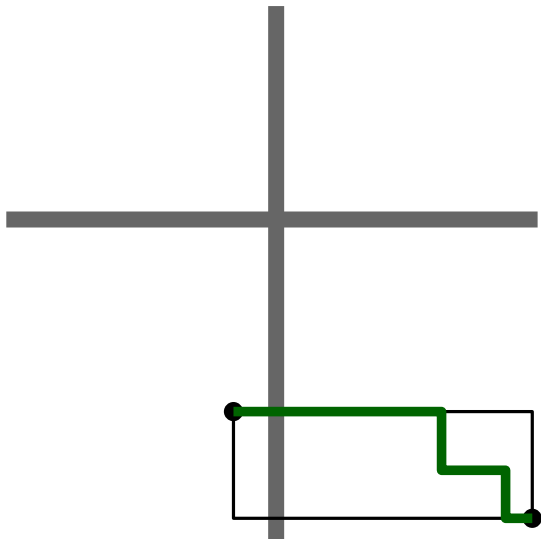
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Cost Analysis



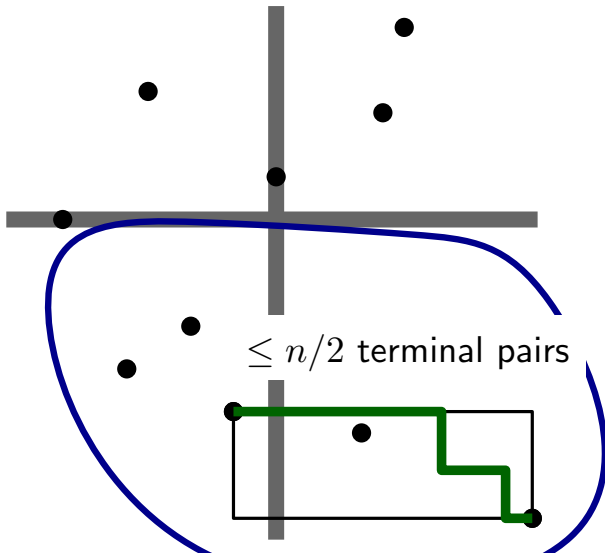
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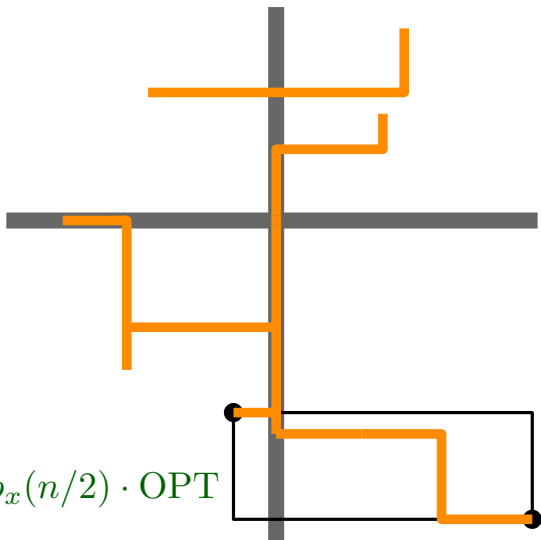
Cost of x -Separated Instance

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Cost of x -Separated Instance

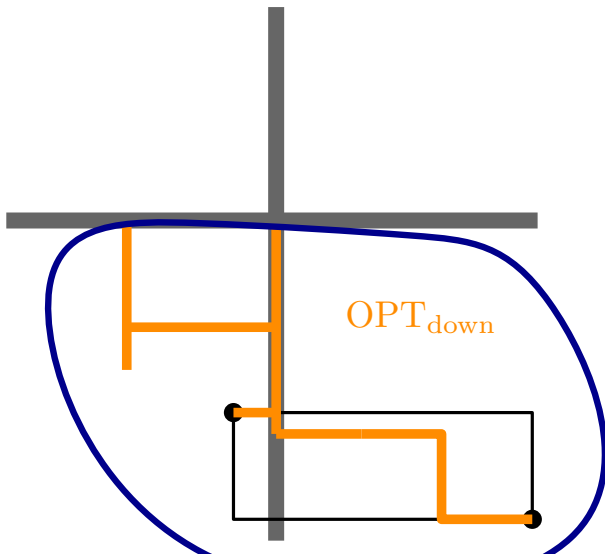
Cost Analysis



$$\text{cost} \leq \rho_x(n/2) \cdot \text{OPT}$$

Cost of x -Separated Instance

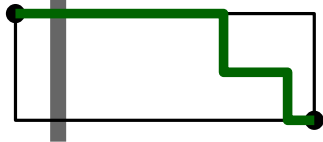
Cost Analysis



Cost of x -Separated Instance

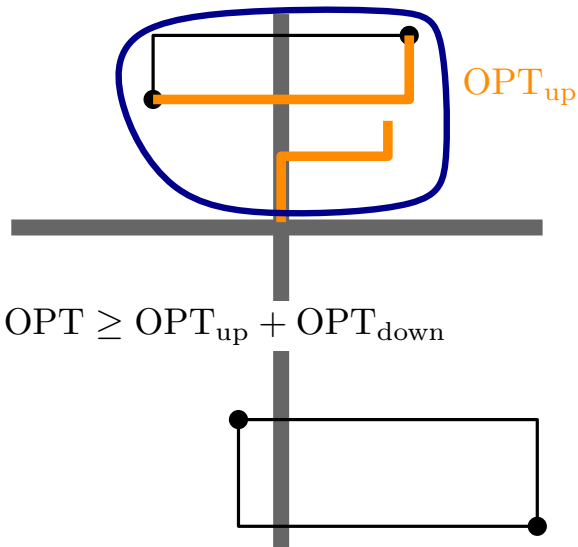
Cost Analysis

$$\text{cost} \leq \rho_x(n/2) \cdot \text{OPT}_{\text{down}}$$



Cost of x -Separated Instance

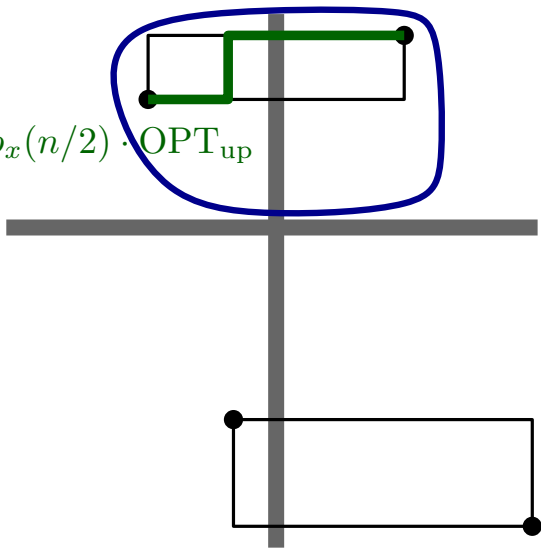
Cost Analysis



Cost of x -Separated Instance

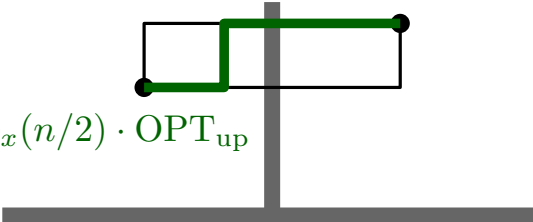
Cost Analysis

$$\text{cost} \leq \rho_x(n/2) \cdot \text{OPT}_{\text{up}}$$



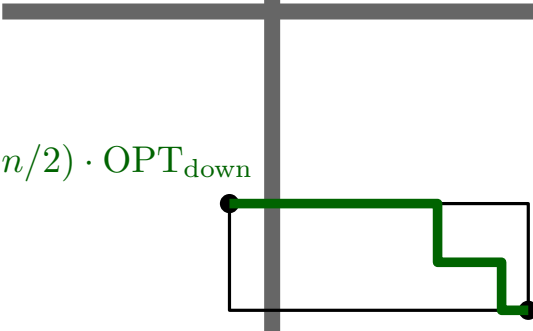
Cost of x -Separated Instance

Cost Analysis



A diagram showing a vertical grey line intersected by a horizontal grey line. A black path starts at a black dot on the left, goes right, then down, then right, then down, then right, ending at a black dot on the right. A green path follows the same route but is highlighted in green.

$$\text{cost} \leq \rho_x(n/2) \cdot \text{OPT}_{\text{up}}$$

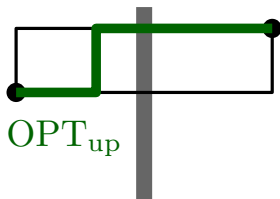


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Cost of x -Separated Instance

Cost Analysis

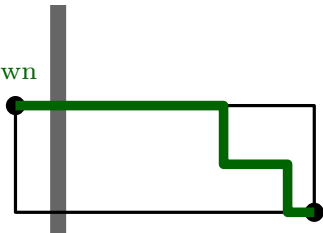


$$\text{cost} \leq \rho_x(n/2) \cdot \text{OPT}_{\text{up}}$$

$$\leq \rho_x(n/2) \cdot (\text{OPT}_{\text{up}} + \text{OPT}_{\text{down}})$$

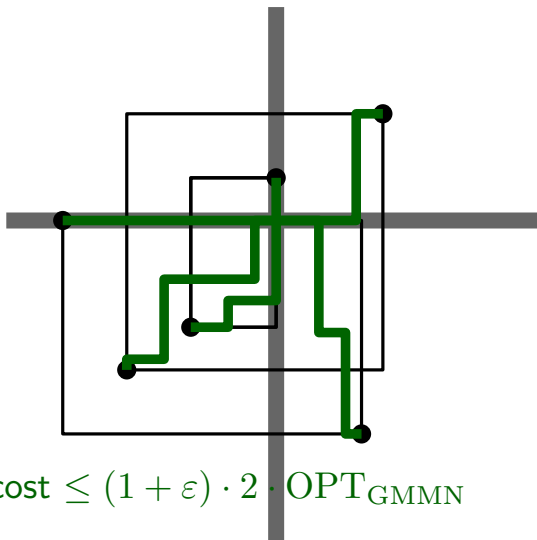
$$\leq \rho_x(n/2) \cdot \text{OPT}$$

$$\text{cost} \leq \rho_x(n/2) \cdot \text{OPT}_{\text{down}}$$



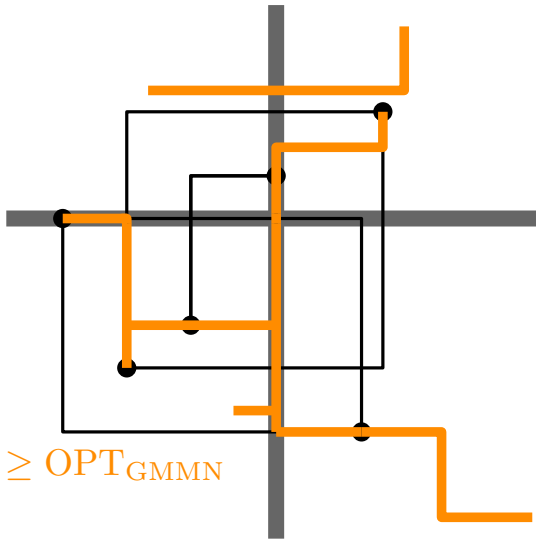
Cost of x -Separated Instance

Cost Analysis



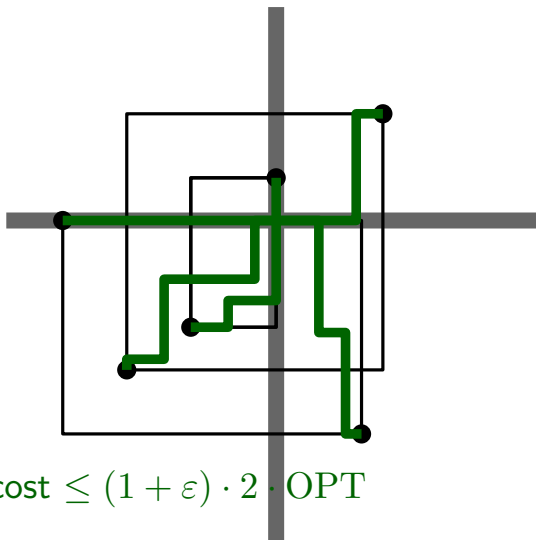
Cost of x-Separated Instance

Cost Analysis



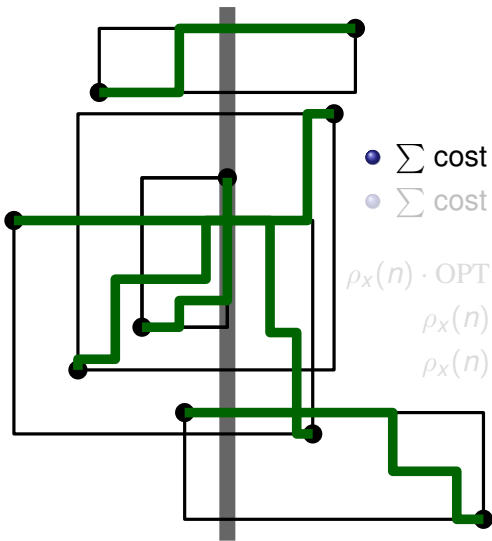
Cost of x -Separated Instance

Cost Analysis



Cost of x -Separated Instance (Summed Up)

Cost Analysis



- $\sum \text{cost} \leq \rho_x(n/2) \cdot \text{OPT} + (1 + \varepsilon) \cdot 2 \cdot \text{OPT}$
- $\sum \text{cost} \leq \rho_x(n) \cdot \text{OPT}$

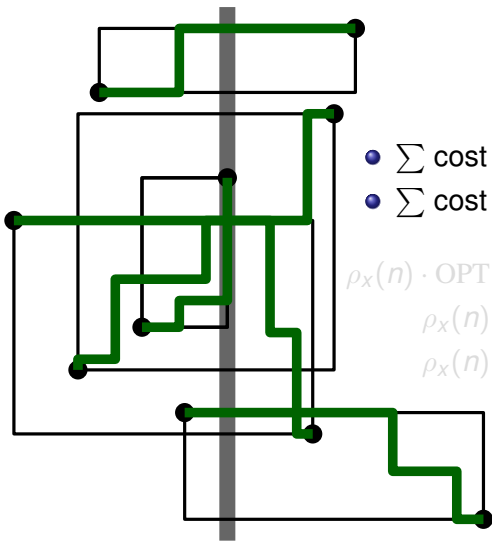
$$\rho_x(n) \cdot \text{OPT} \leq \rho_x(n/2) \cdot \text{OPT} + (1 + \varepsilon) \cdot 2 \cdot \text{OPT}$$

$$\rho_x(n) \leq \rho_x(n/2) + (1 + \varepsilon) \cdot 2$$

$$\rho_x(n) \leq \log n \cdot (1 + \varepsilon) \cdot 2$$

Cost of x -Separated Instance (Summed Up)

Cost Analysis



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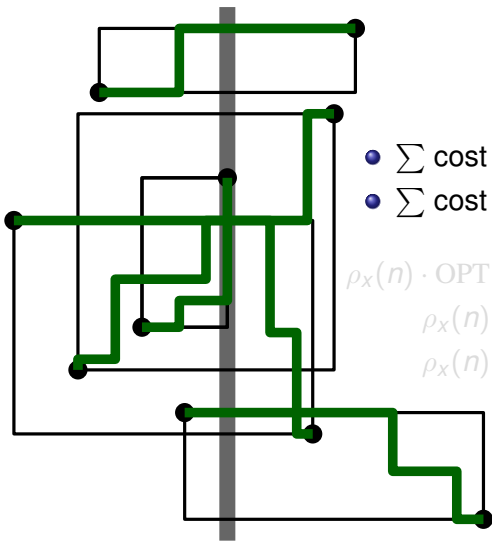
$$\rho_x(n) \cdot \text{OPT} \leq \rho_x(n/2) \cdot \text{OPT} + (1 + \varepsilon) \cdot 2 \cdot \text{OPT}$$

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Cost of x -Separated Instance (Summed Up)

Cost Analysis



- $\sum \text{cost} \leq \rho_x(n/2) \cdot \text{OPT} + (1 + \varepsilon) \cdot 2 \cdot \text{OPT}$
- $\sum \text{cost} = \rho_x(n) \cdot \text{OPT}$

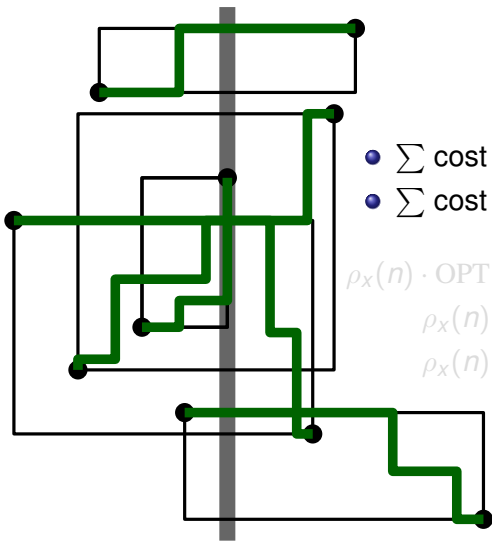
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Cost Analysis



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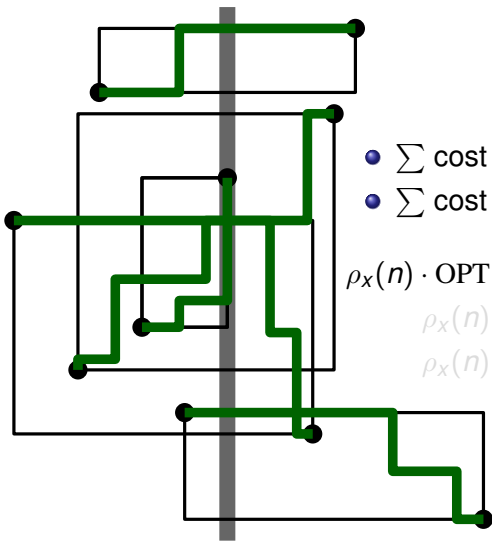
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Cost of x -Separated Instance (Summed Up)

Cost Analysis



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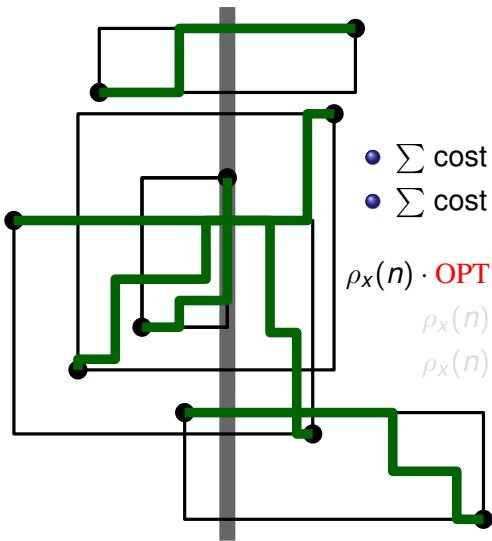
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Cost of x -Separated Instance (Summed Up)

Cost Analysis



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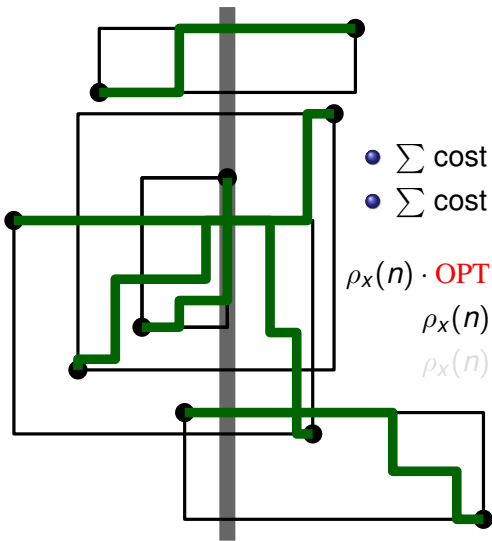
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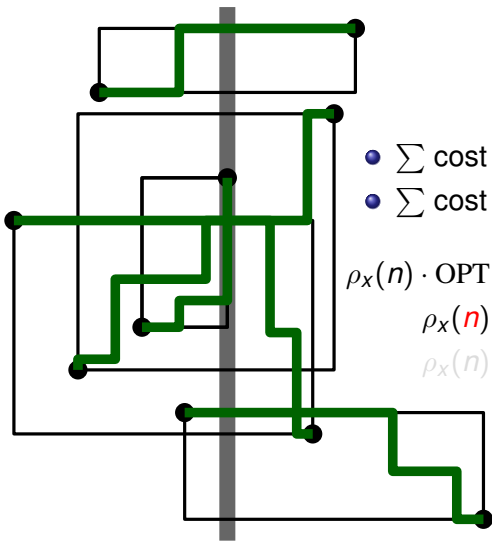
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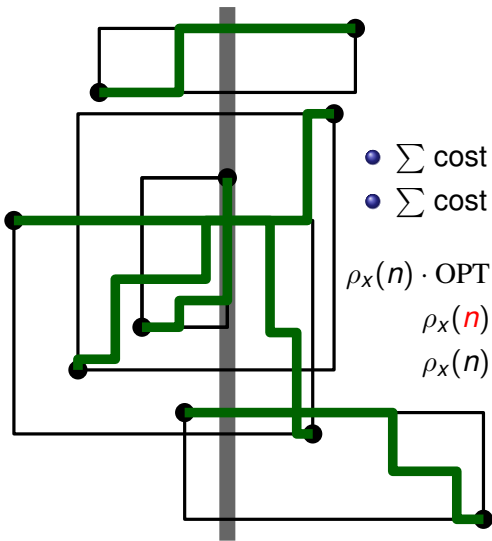
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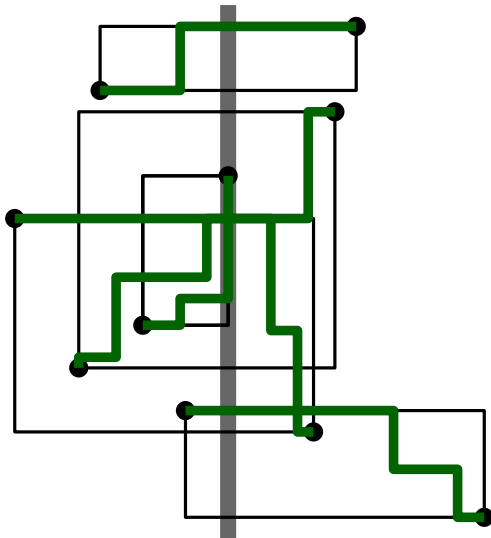
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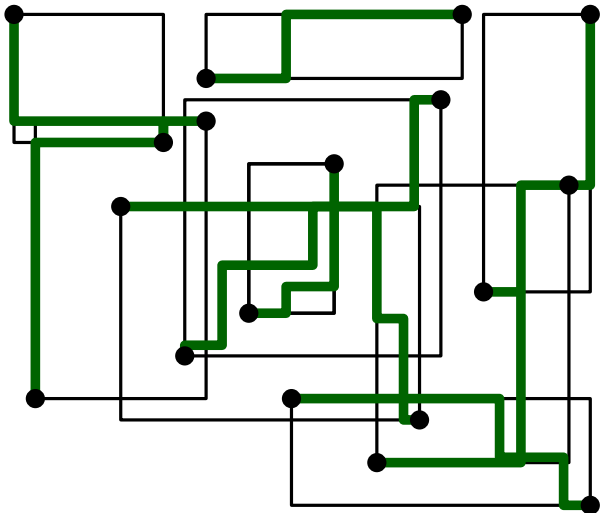
General Instance

Cost Analysis



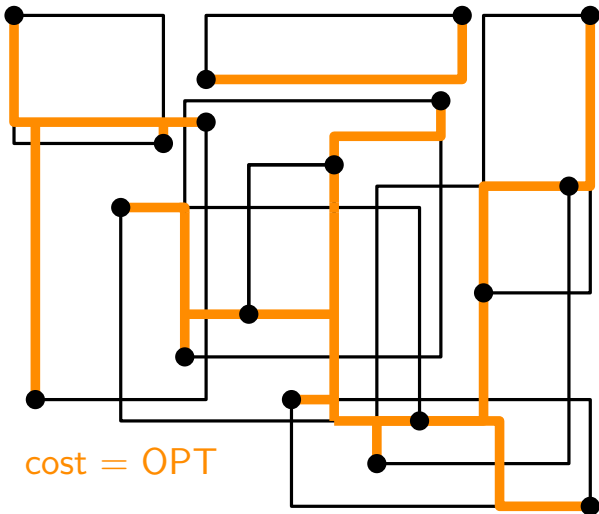
General Instance

Cost Analysis



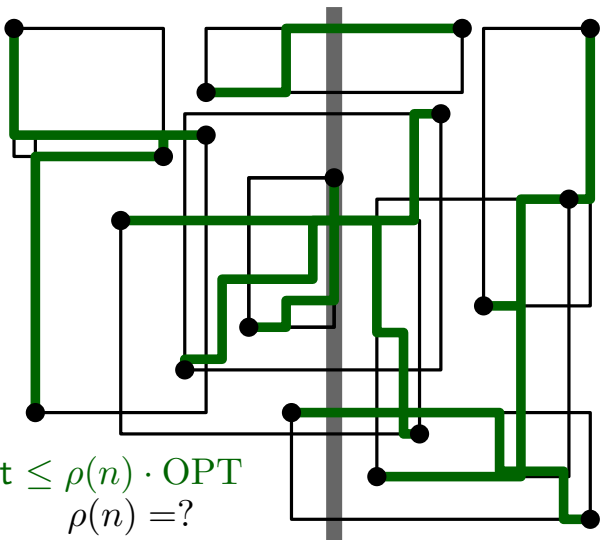
General Instance

Cost Analysis



General Instance

Cost Analysis



$$\text{cost} \leq \rho(n) \cdot \text{OPT}$$
$$\rho(n) = ?$$

Total cost

Cost Analysis

- $\sum \text{cost} \leq \rho(n/2) \cdot \text{OPT} + \rho_x(n)\text{OPT}$
- $\sum \text{cost} = \rho(n) \cdot \text{OPT}$

$$\rho(n) \leq \log n \cdot \rho_x(n)$$

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Main Result in 2D

Theorem

2D-GMMN admits an $O(\log^2 n)$ -approximation.

Main Result in Higher Dimension

| Dimension | 1.Step (Partition) | 2.Step (RSA) | Altogether |
|-----------|--------------------|--------------|-------------------|
| 2 | $O(\log^2 n)$ | $O(1)$ | $O(\log^2 n)$ |
| $d > 2$ | $O(\log^d n)$ | $O(\log n)$ | $O(\log^{d+1} n)$ |

Theorem

GMMN in dimension d admits an $O(\log^{d+1} n)$ -approximation.

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Main Result in Higher Dimension

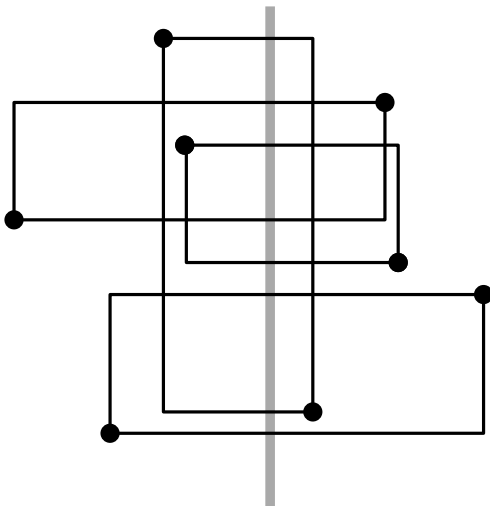
| Dimension | 1.Step (Partition) | 2.Step (RSA) | Altogether |
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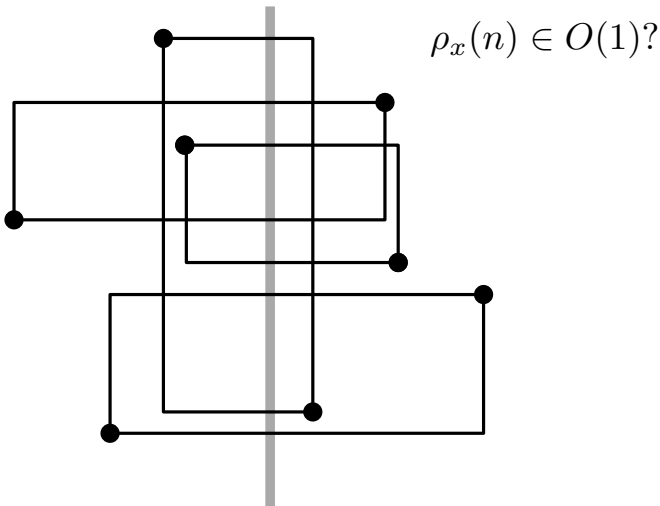
Optimization in 2D

Introduction



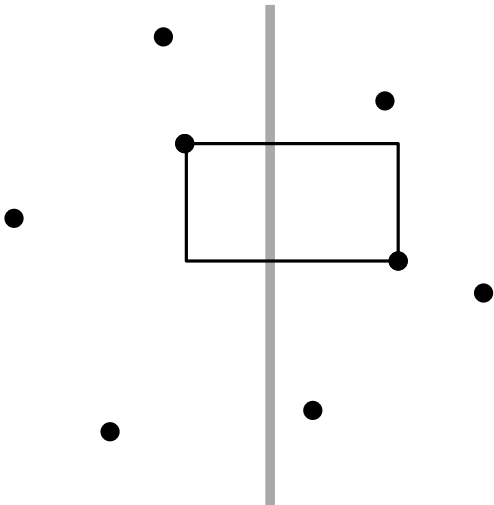
Optimization in 2D

Introduction



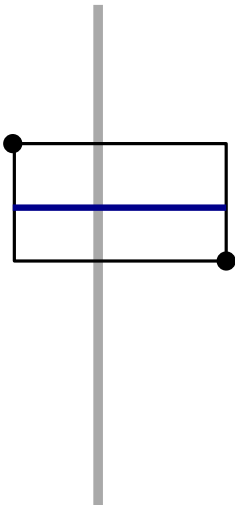
Idea

Introduction



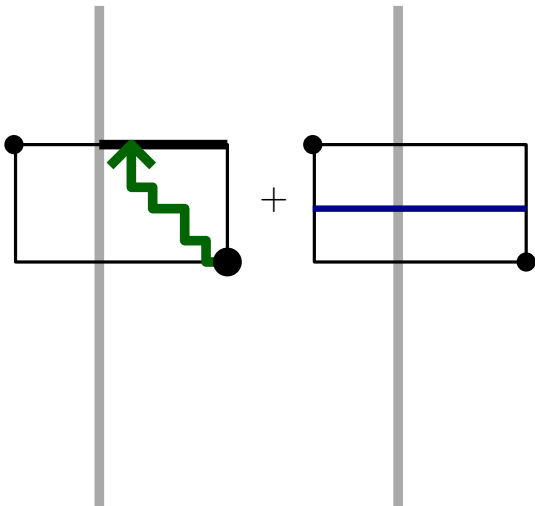
Idea

Introduction



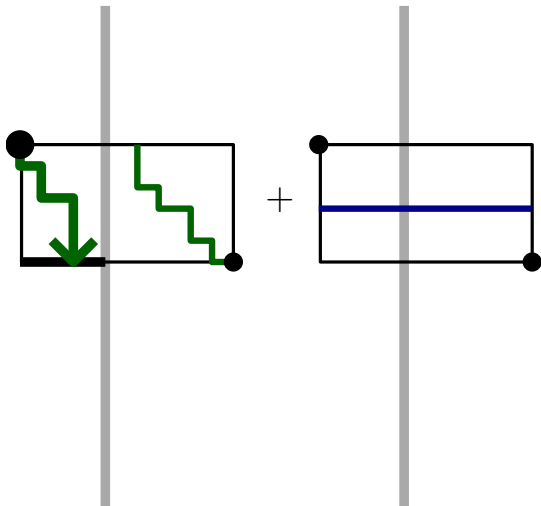
Idea

Introduction



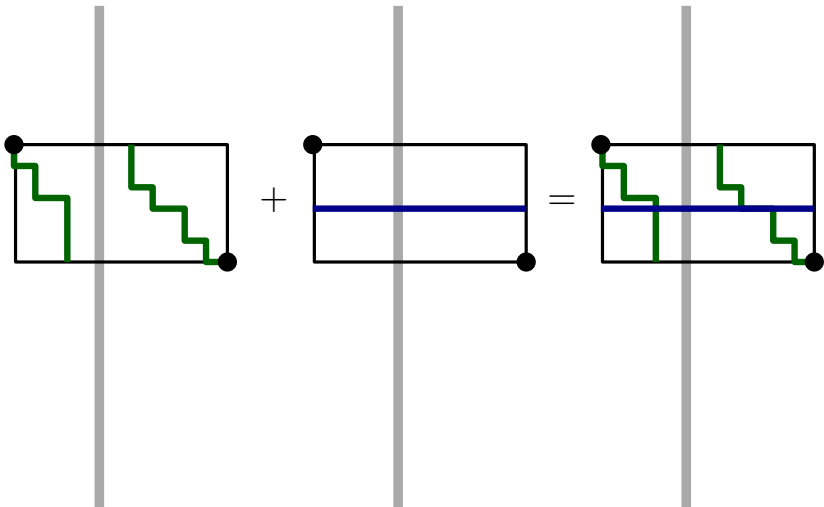
Idea

Introduction



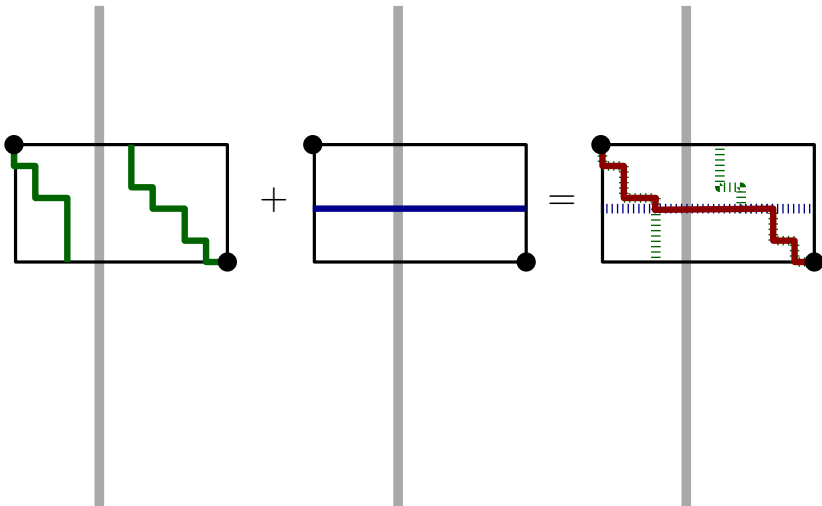
Idea

Introduction



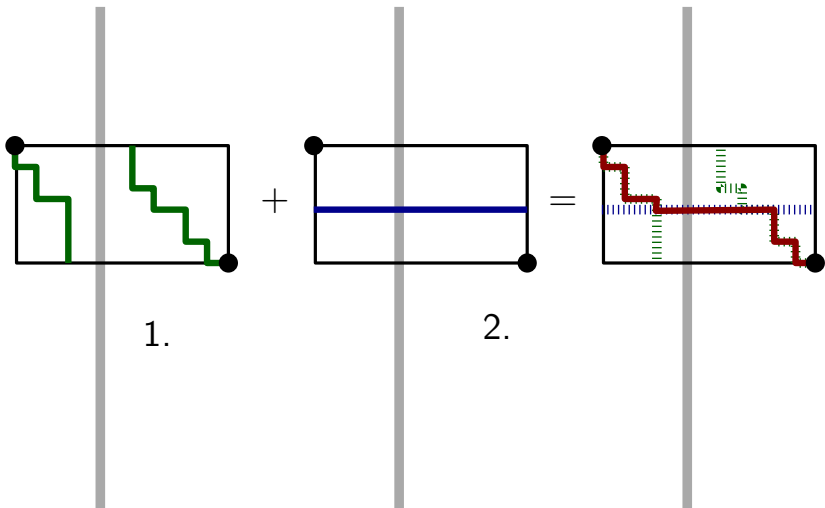
Idea

Introduction



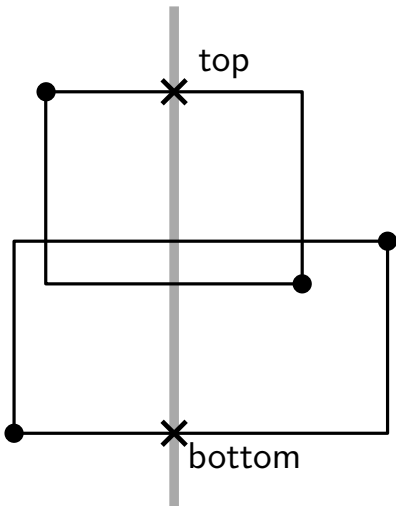
Idea

Introduction



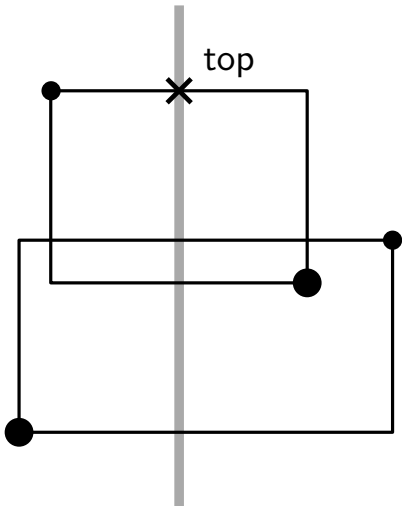
Construction

First Step



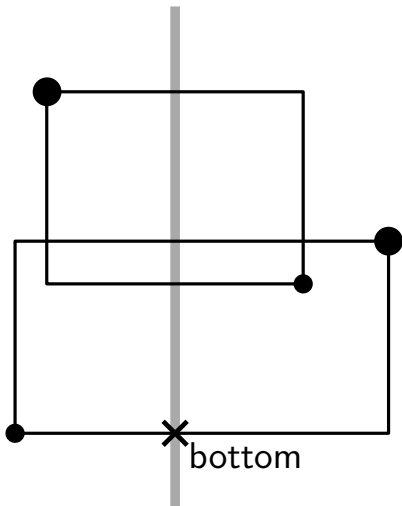
Construction

First Step



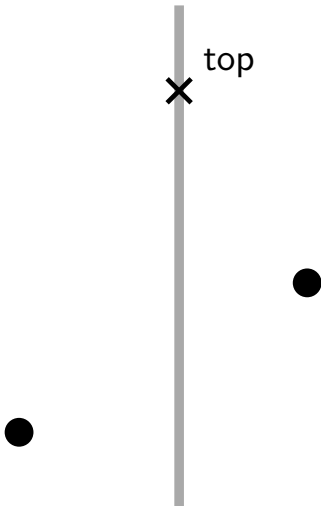
Construction

First Step



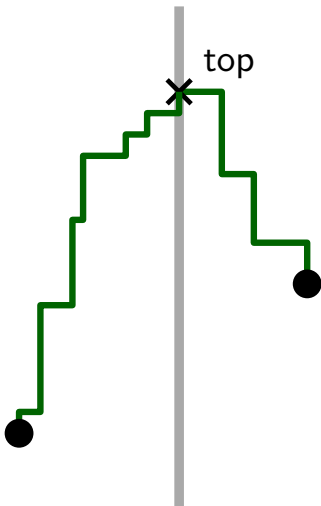
Construction

First Step



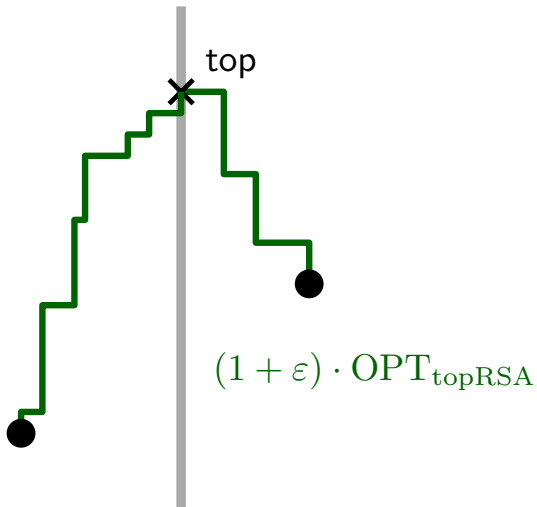
Construction

First Step



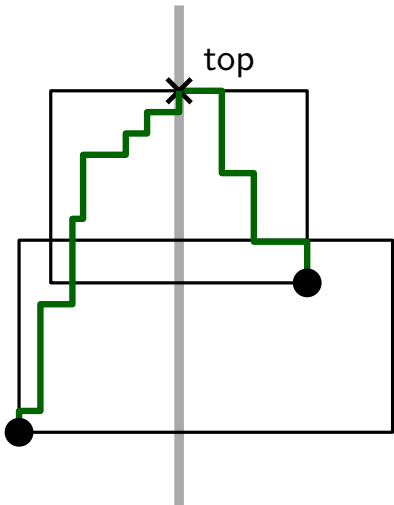
Construction

First Step



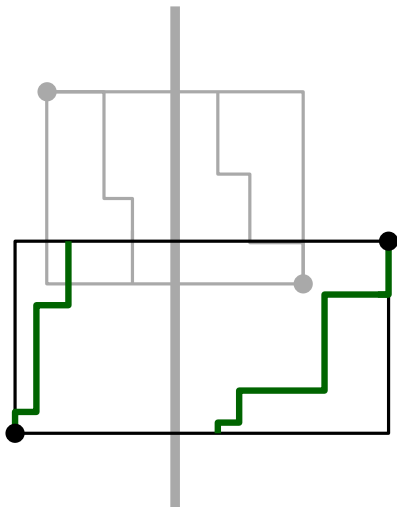
Construction

First Step



Construction

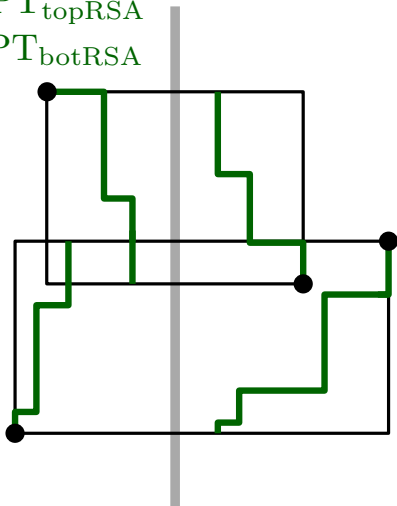
First Step



Cost Analysis

First Step

$$(1 + \varepsilon) \cdot \text{OPT}_{\text{topRSA}} \\ + (1 + \varepsilon) \cdot \text{OPT}_{\text{botRSA}}$$



Cost Analysis

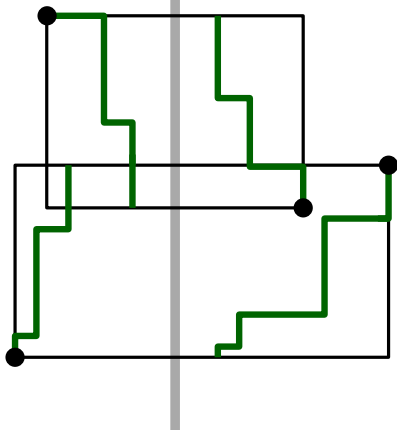
First Step

$$(1 + \varepsilon) \cdot \text{OPT}_{\text{topRSA}}$$

$$+ (1 + \varepsilon) \cdot \text{OPT}_{\text{botRSA}}$$

$$\text{OPT}_{\text{topRSA}}?$$

$$\text{OPT}_{\text{botRSA}}?$$



Cost Analysis

First Step

- $\text{OPT}_{\text{topRSA}}, \text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$

Lemma

Step 1 costs $(2 + 2\epsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})$.

Cost Analysis

First Step

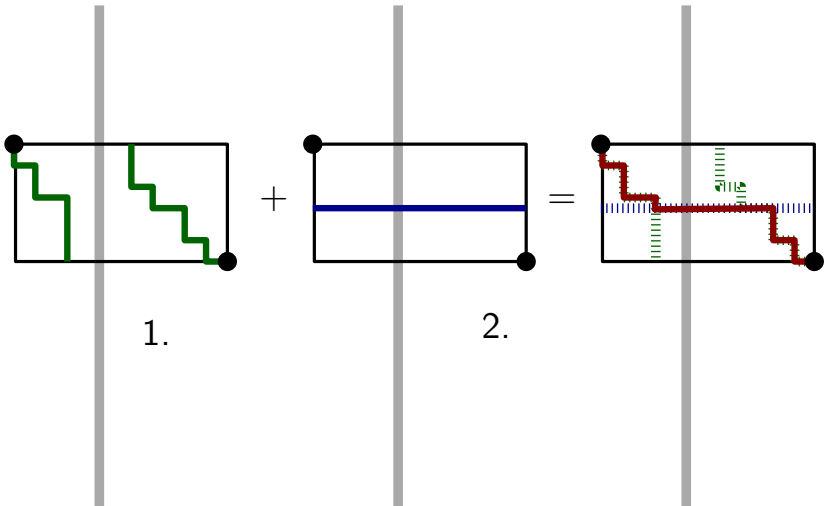
- $\text{OPT}_{\text{topRSA}}, \text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$

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Step 1 costs $(2 + 2\epsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})$.

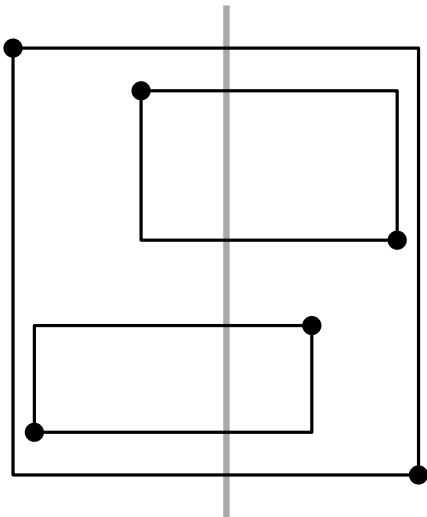
Second Step

Second Step



Observations

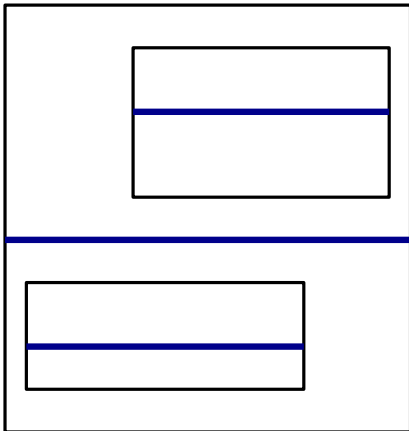
Second Step



Observations

Second Step

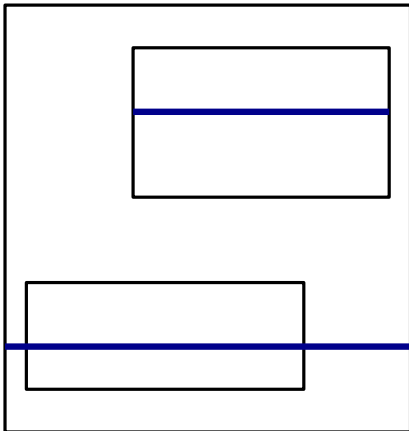
Stabbing



Observations

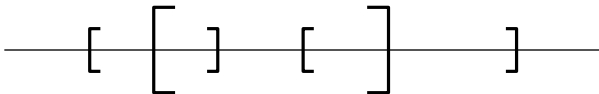
Second Step

Stabbing



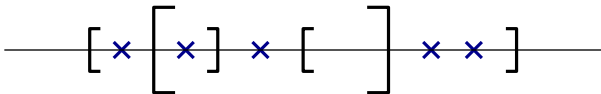
Excursion: Piercing

Second Step



Excursion: Piercing

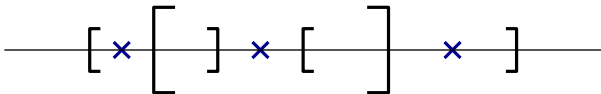
Second Step



Excursion: Piercing

Second Step

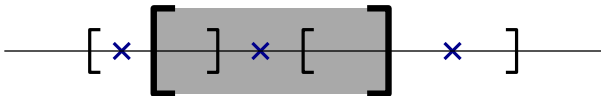
Minimal Piercing



Excursion: Piercing

Second Step

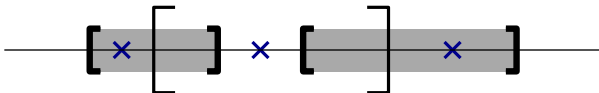
Minimal Piercing



Excursion: Piercing

Second Step

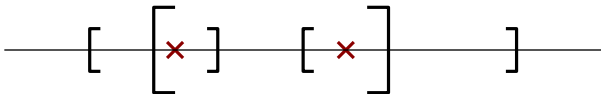
Minimal Piercing



Excursion: Piercing

Second Step

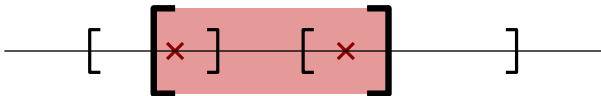
Optimum Piercing



Excursion: Piercing

Second Step

Optimum Piercing



Excursion: Piercing

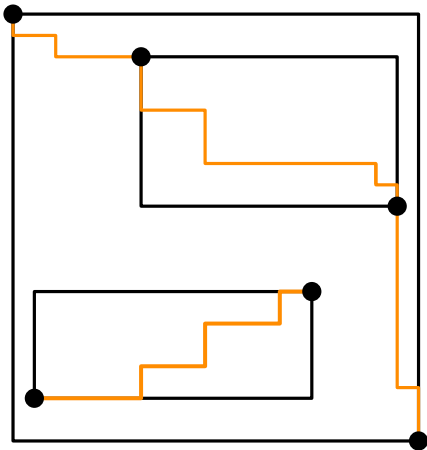
Second Step

Lemma

Minimal piercing $\leq 2 \cdot$ optimum piercing.

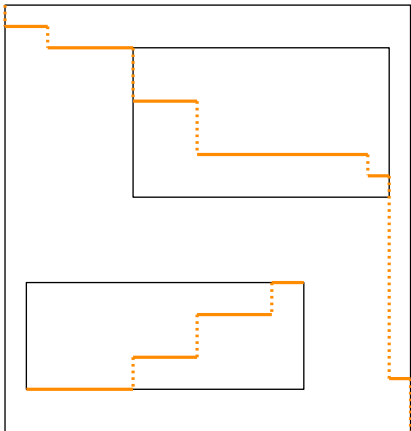
OPT is a Piercing

Second Step



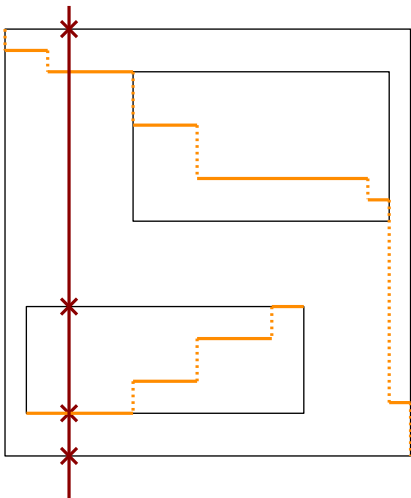
OPT is a Piercing

Second Step



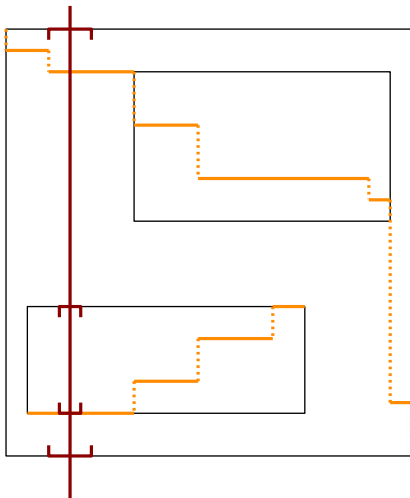
OPT is a Piercing

Second Step



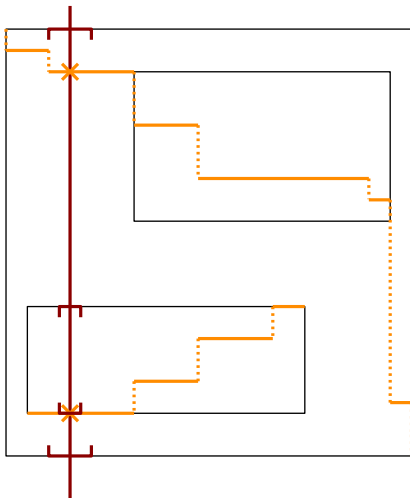
OPT is a Piercing

Second Step



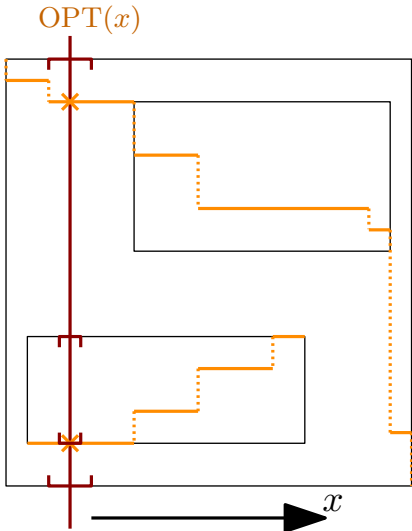
OPT is a Piercing

Second Step



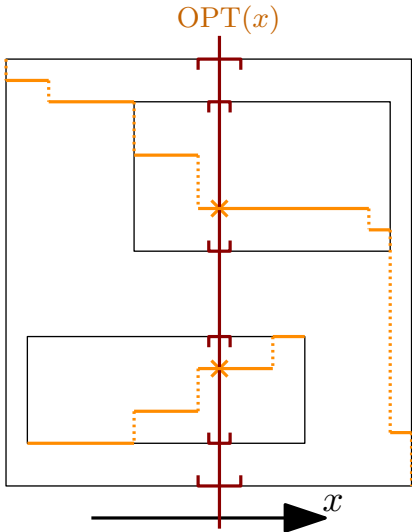
OPT is a Piercing

Second Step



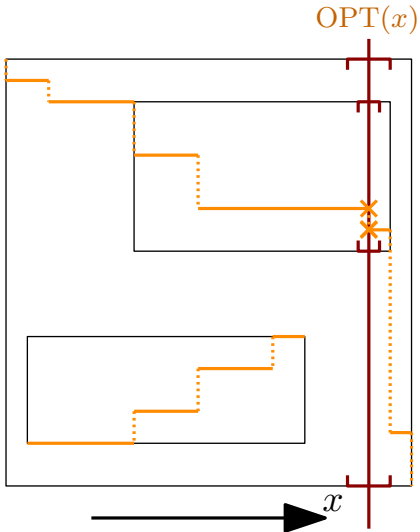
OPT is a Piercing

Second Step



OPT is a Piercing

Second Step



OPT is a Piercing

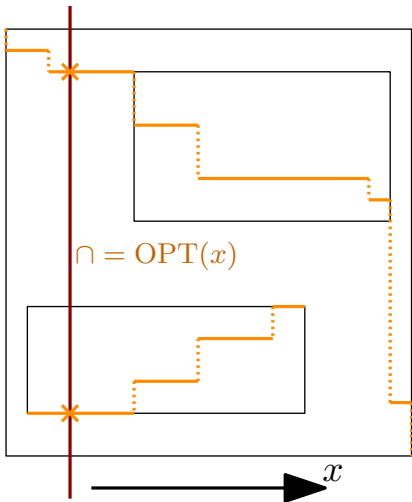
Second Step

Lemma

$\forall x : \textit{optimum piercing}(x) \leq \text{OPT}(x)$

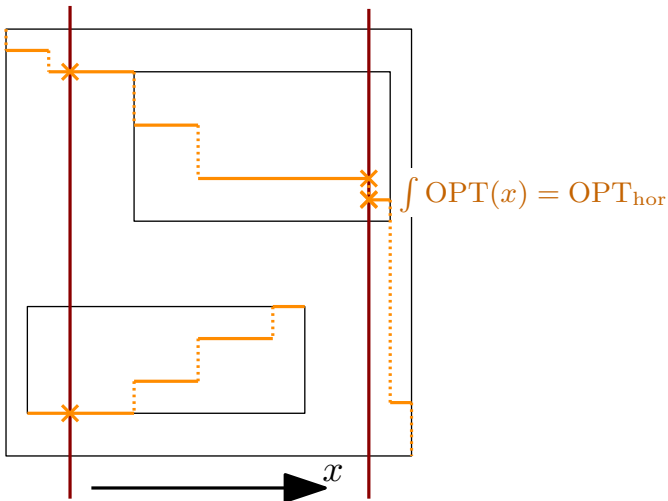
Piercing Summed Up

Second Step



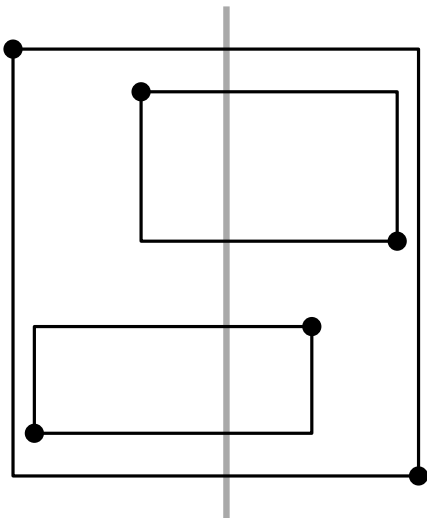
Piercing Summed Up

Second Step



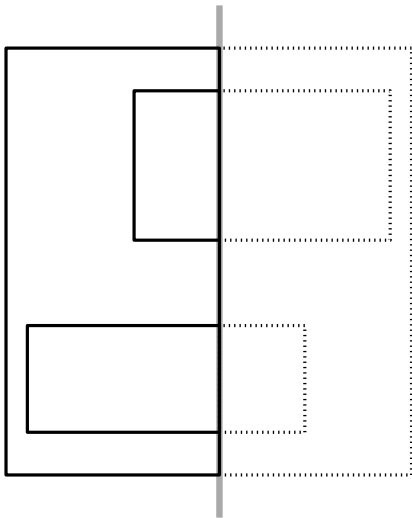
Stabbing the Left Side

Second Step



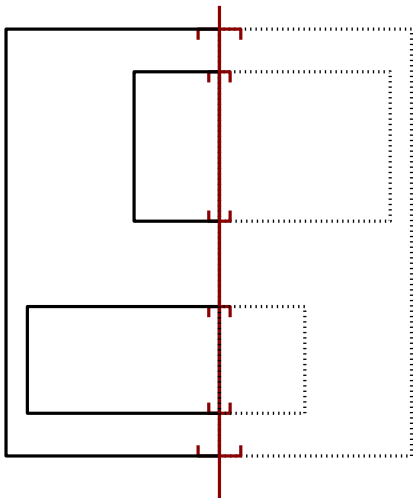
Stabbing the Left Side

Second Step



Stabbing the Left Side

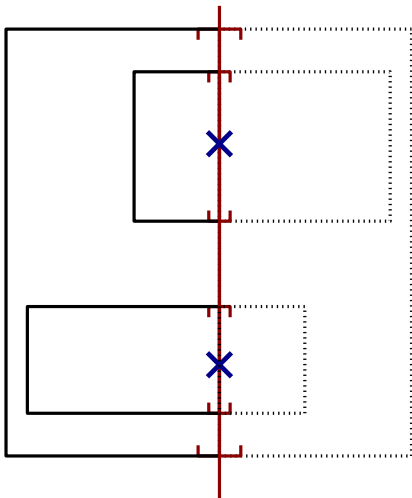
Second Step



Stabbing the Left Side

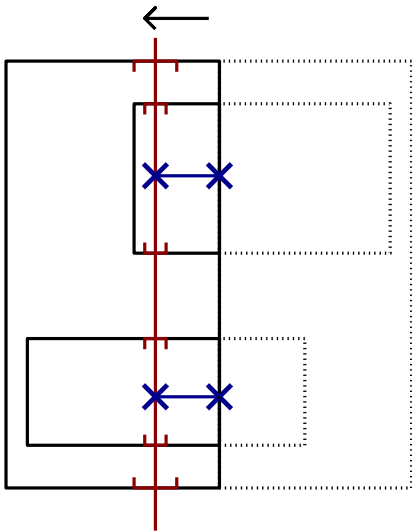
Second Step

Minimal Piercing



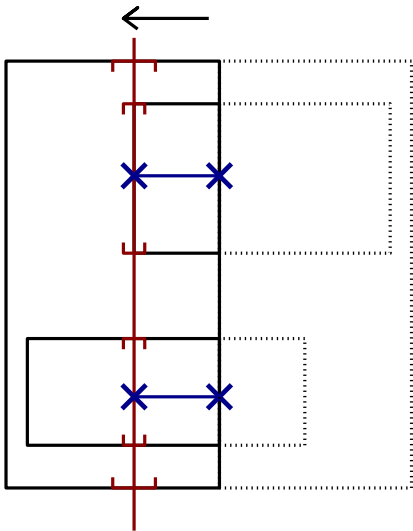
Stabbing the Left Side

Second Step



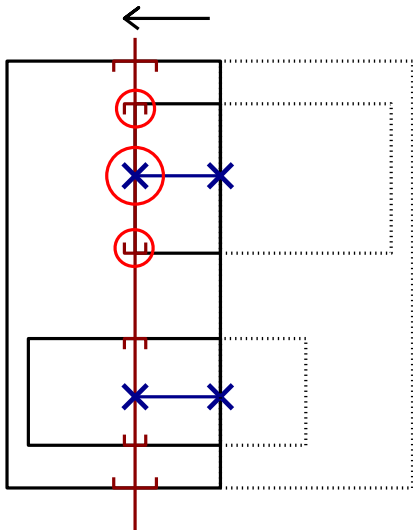
Stabbing the Left Side

Second Step



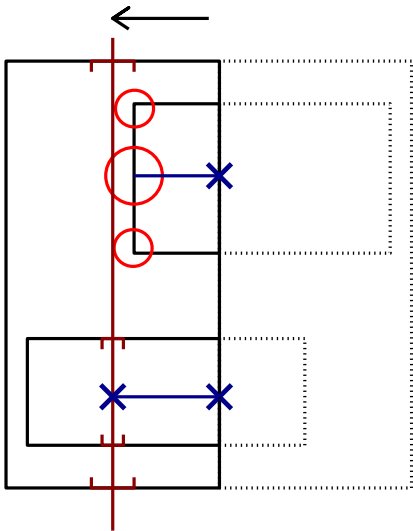
Stabbing the Left Side

Second Step



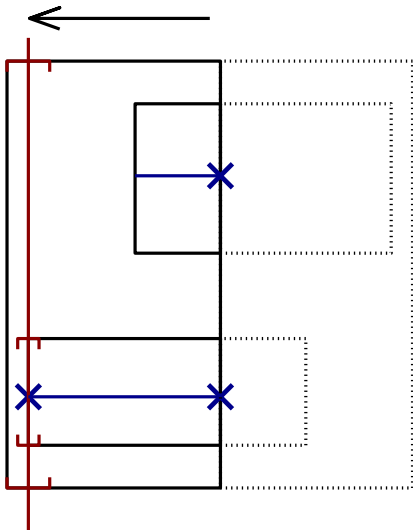
Stabbing the Left Side

Second Step



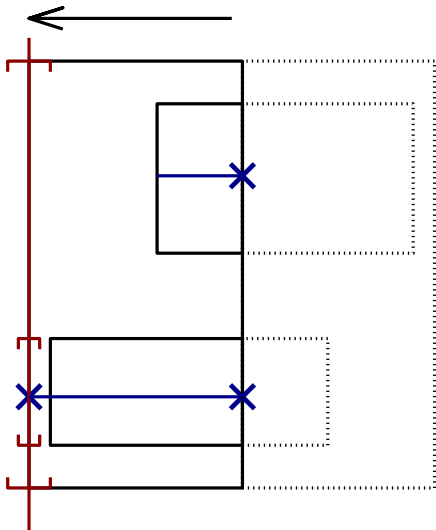
Stabbing the Left Side

Second Step



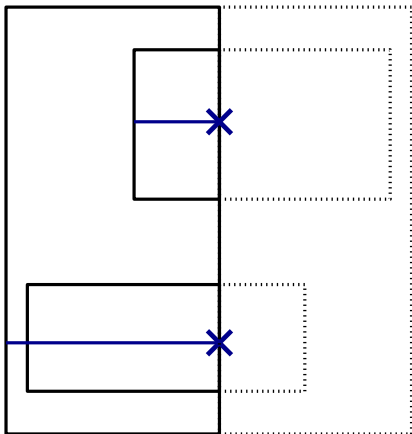
Stabbing the Left Side

Second Step



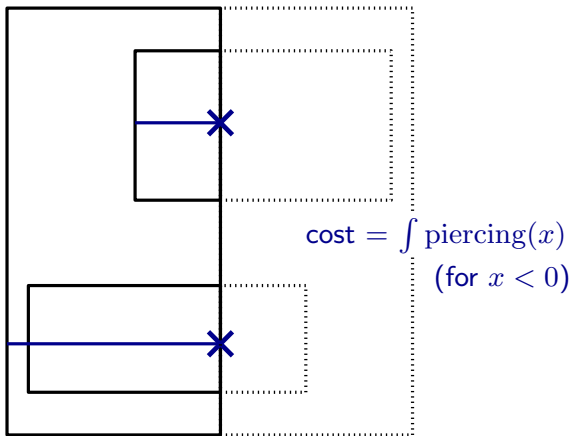
Stabbing the Left Side

Second Step



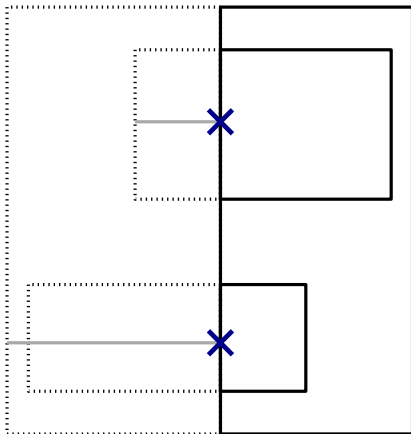
Stabbing the Left Side

Second Step



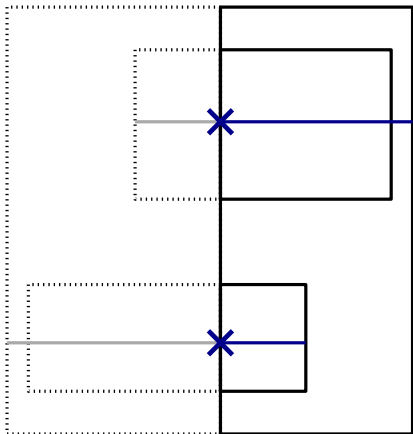
Stabbing Both Sides

Second Step



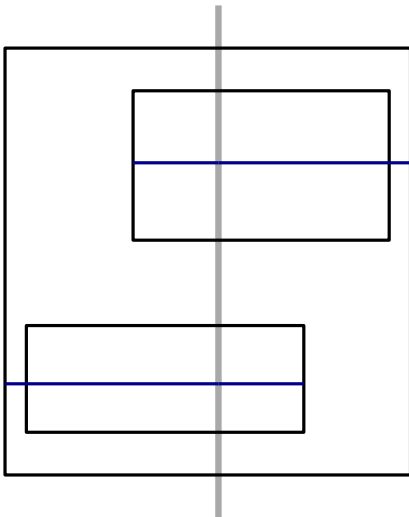
Stabbing Both Sides

Second Step



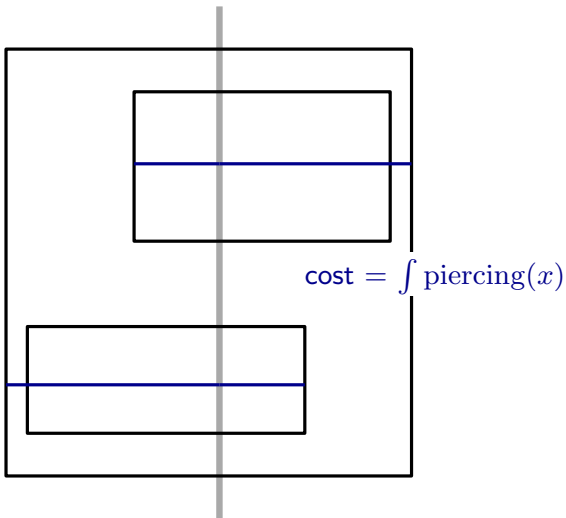
Stabbing Both Sides

Second Step



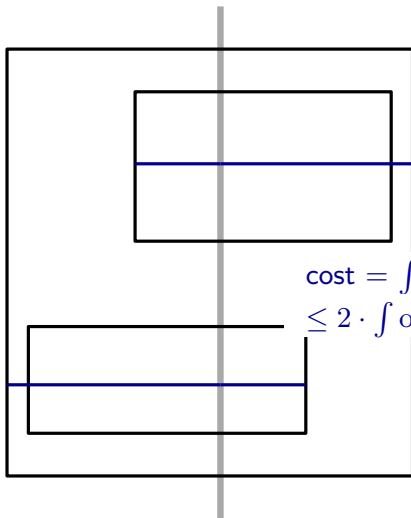
Stabbing Both Sides

Second Step



Stabbing Both Sides

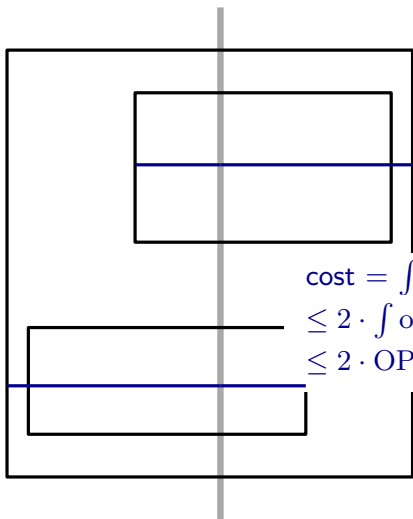
Second Step



$$\begin{aligned} \text{cost} &= \int \text{piercing}(x) \\ &\leq 2 \cdot \int \text{optimum piercing} \end{aligned}$$

Stabbing Both Sides

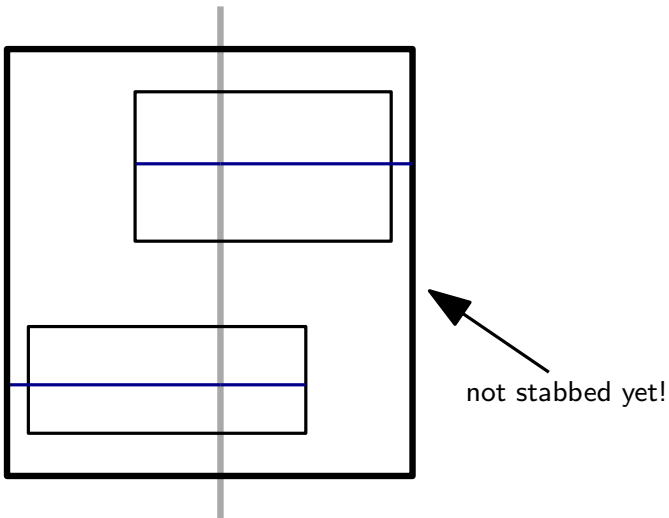
Second Step



$$\begin{aligned} \text{cost} &= \int \text{piercing}(x) \\ &\leq 2 \cdot \int \text{optimum piercing} \\ &\leq 2 \cdot \text{OPT}_{\text{hor}} \end{aligned}$$

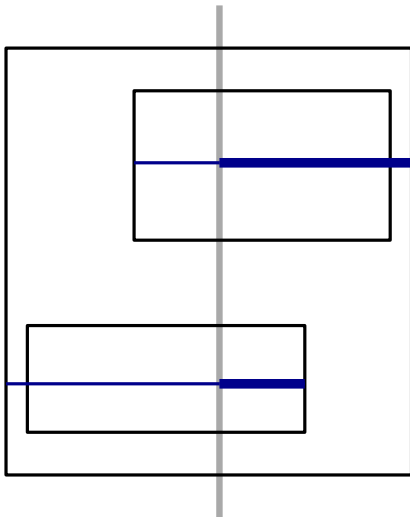
Mirrors Step

Second Step



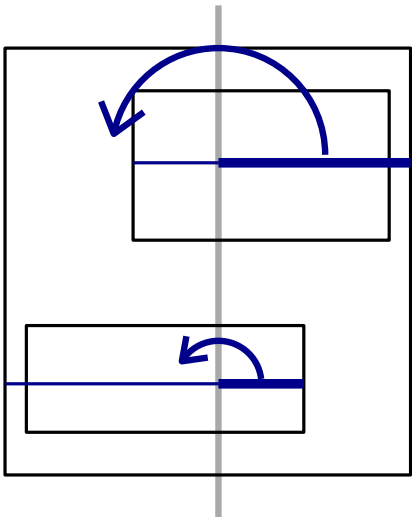
Mirrors Step

Second Step



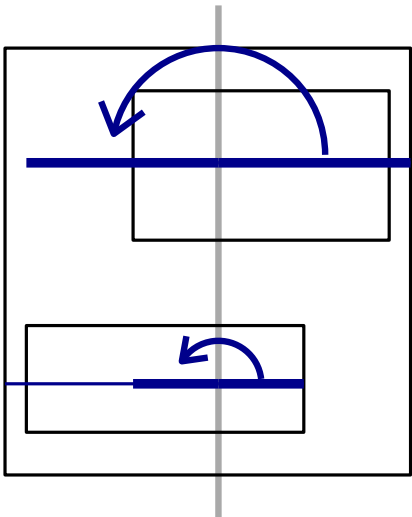
Mirrors Step

Second Step



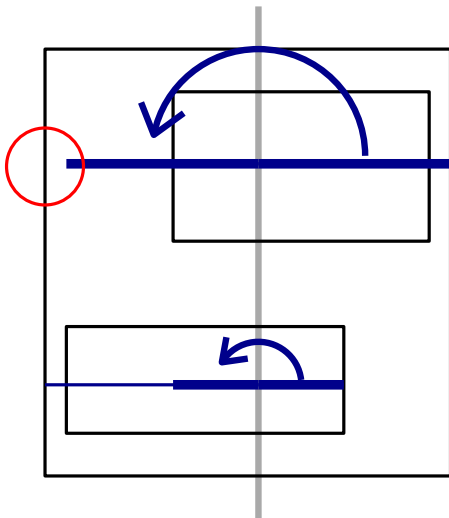
Mirrors Step

Second Step



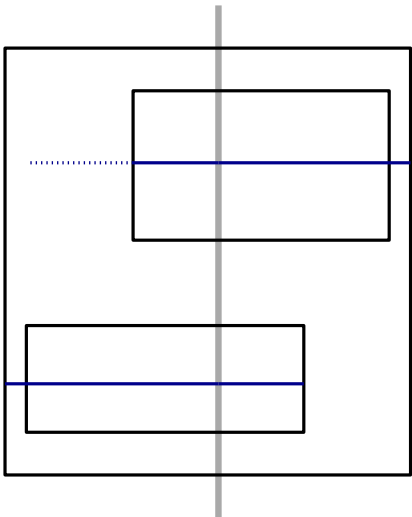
Mirrors Step

Second Step



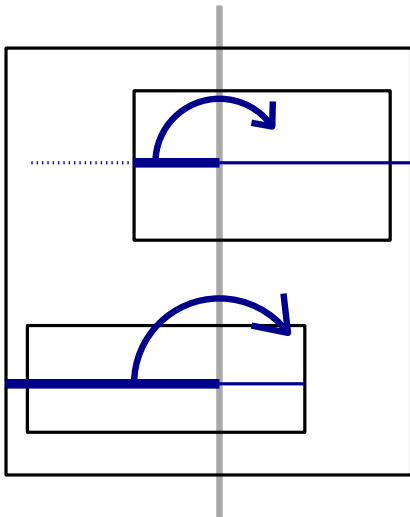
Mirrors Step

Second Step



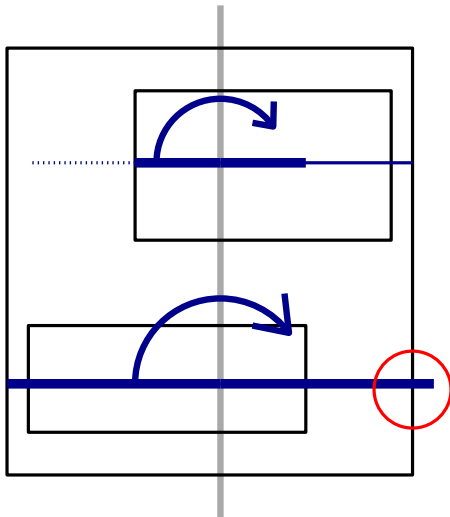
Mirrors Step

Second Step



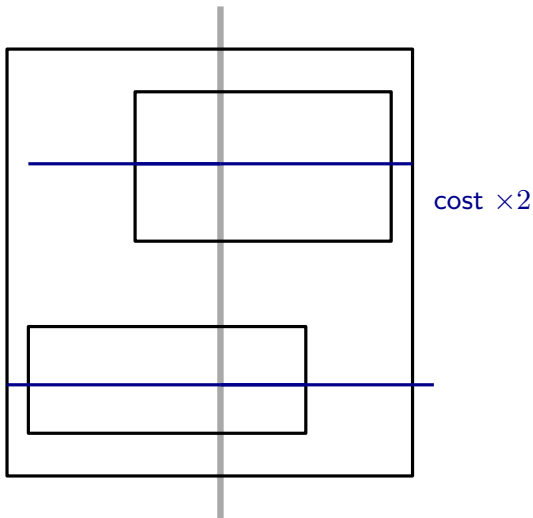
Mirrors Step

Second Step



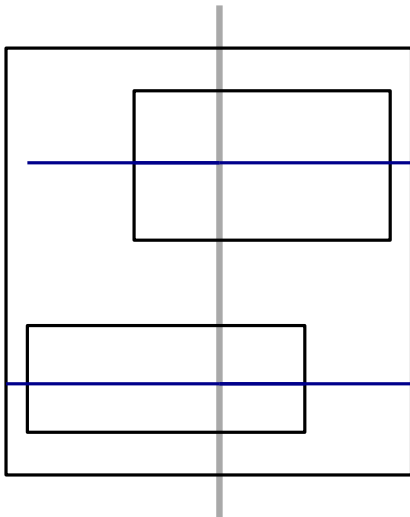
Mirrors Step

Second Step



Mirrors Step

Second Step

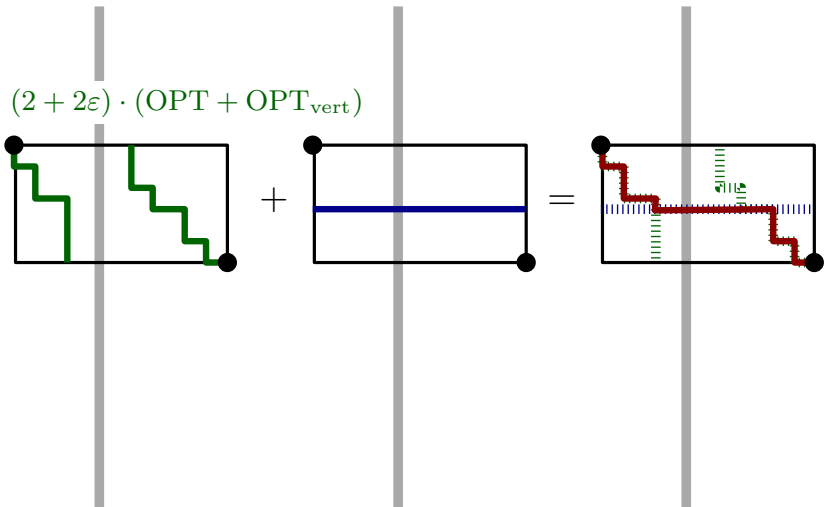


$$\text{cost} \leq 4 \cdot \text{OPT}_{\text{hor}}$$

Total Cost

Both Steps Together

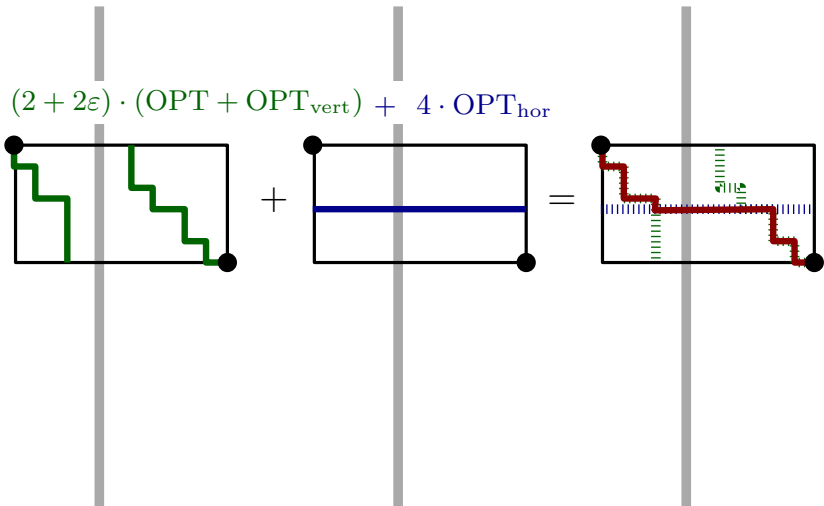
$$(2 + 2\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})$$



Total Cost

Both Steps Together

$$(2 + 2\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + 4 \cdot \text{OPT}_{\text{hor}}$$



Total Cost

Both Steps Together

$$(2 + 2\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + 4 \cdot \text{OPT}_{\text{hor}}$$

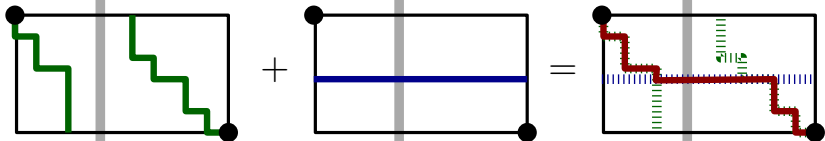


$$\leq (6 + \varepsilon') \cdot \text{OPT} \quad \text{for } \varepsilon' = \varepsilon/2$$

Total Cost

Both Steps Together

$$(2 + 2\varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + 4 \cdot \text{OPT}_{\text{hor}}$$



$$\leq (6 + \varepsilon') \cdot \text{OPT} \quad \text{for } \varepsilon' = \varepsilon/2$$

$$\Rightarrow \rho_x(n) = (6 + \varepsilon) \in O(1)$$

Optimized Result in 2D

Both Steps Together

Theorem (New)

2D-GMMN admits a $((6 + \epsilon) \cdot \log n)$ -approximation.

Theorem (Before)

2D-GMMN admits a $O(\log^2 n)$ -approximation.

Optimized Result in 2D

Both Steps Together

Theorem (New)

2D-GMMN admits a $O(\log n)$ -approximation.

Theorem (Before)

2D-GMMN admits a $O(\log^2 n)$ -approximation.

Optimized Result in 2D

Both Steps Together

Theorem (New)

2D-GMMN admits a $O(\log n)$ -approximation.

Theorem (Before)

2D-GMMN admits a $O(\log^2 n)$ -approximation.

Summary

Current Results

| | 2D | Dimension $d > 2$ |
|------|---------------------|-------------------|
| RSA | $(1 + \varepsilon)$ | $O(\log n)$ |
| MMN | 2 | $O(\log^{d+1} n)$ |
| GMMN | $O(\log n)$ | $O(\log^{d+1} n)$ |

Open Problems

Future Results?

- $O(1)$ -Approximation for 2D-GMMN?
- $O(1)$ -Approximation for 3D-RSA?
- APX-Hardness of 2D-GMMN?

Outline

- 1 Introduction
 - Problem Definition
 - Introduction
- 2 $\log^2 n$ Algorithm
 - Overview
 - Construction
 - Cost Analysis
 - Cost Analysis
- 3 $\log n$ Algorithm
 - Introduction
 - First Step
 - Second Step
 - Both Steps Together
- 4 Summary
 - Current Results
 - Future Results?
 - Detailed Analysis

Appendix

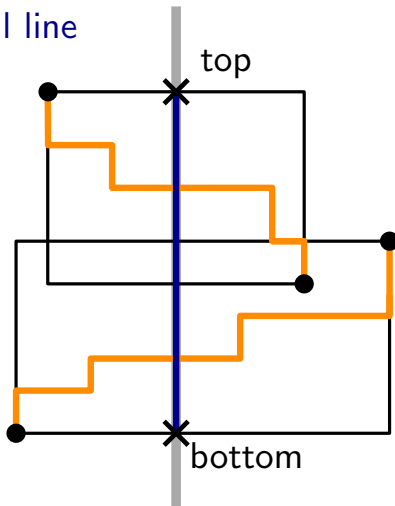
Detailed Analysis

(RSA Cost Analysis)
(Minimal Piercing Analysis)

Cost Analysis

Detailed Analysis

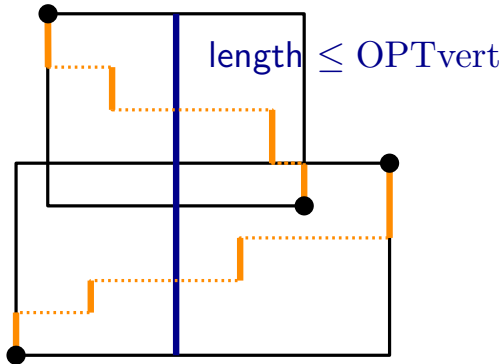
Add vertical line



Cost Analysis

Detailed Analysis

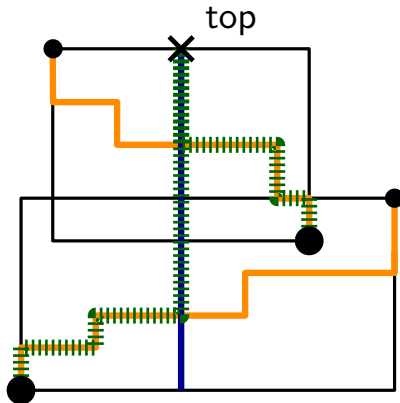
Add vertical line



Cost Analysis

Detailed Analysis

Optimal solution + vertical line = top RSA



Cost Analysis

Detailed Analysis

- $\text{OPT}_{\text{topRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- $\text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- Our solution cost:

$$\begin{aligned}
 & (1 + \varepsilon) \cdot \text{OPT}_{\text{topRSA}} + (1 + \varepsilon) \cdot \text{OPT}_{\text{botRSA}} \\
 & \leq (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) \\
 & \leq 2 \cdot (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})
 \end{aligned}$$

(Go To Short Analysis)

(Go To Open Problems)

Cost Analysis

Detailed Analysis

- $\text{OPT}_{\text{topRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- $\text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- Our solution cost:

$$\begin{aligned}
 & (1 + \varepsilon) \cdot \text{OPT}_{\text{topRSA}} + (1 + \varepsilon) \cdot \text{OPT}_{\text{botRSA}} \\
 & \leq (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) \\
 & \leq 2 \cdot (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})
 \end{aligned}$$

(Go To Short Analysis)

(Go To Open Problems)

Cost Analysis

Detailed Analysis

- $\text{OPT}_{\text{topRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- $\text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- Our solution cost:

$$\begin{aligned}
 & (1 + \varepsilon) \cdot \text{OPT}_{\text{topRSA}} + (1 + \varepsilon) \cdot \text{OPT}_{\text{botRSA}} \\
 & \leq (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) \\
 & \leq 2 \cdot (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})
 \end{aligned}$$

(Go To Short Analysis)

(Go To Open Problems)

Cost Analysis

Detailed Analysis

- $\text{OPT}_{\text{topRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- $\text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- Our solution cost:

$$\begin{aligned}
 & (1 + \varepsilon) \cdot \text{OPT}_{\text{topRSA}} + (1 + \varepsilon) \cdot \text{OPT}_{\text{botRSA}} \\
 & \leq (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) \\
 & \leq 2 \cdot (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})
 \end{aligned}$$

(Go To Short Analysis)

(Go To Open Problems)

Cost Analysis

Detailed Analysis

- $\text{OPT}_{\text{topRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- $\text{OPT}_{\text{botRSA}} \leq \text{OPT} + \text{OPT}_{\text{vert}}$
- Our solution cost:

$$\begin{aligned}
 & (1 + \varepsilon) \cdot \text{OPT}_{\text{topRSA}} + (1 + \varepsilon) \cdot \text{OPT}_{\text{botRSA}} \\
 & \leq (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) + (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}}) \\
 & \leq 2 \cdot (1 + \varepsilon) \cdot (\text{OPT} + \text{OPT}_{\text{vert}})
 \end{aligned}$$

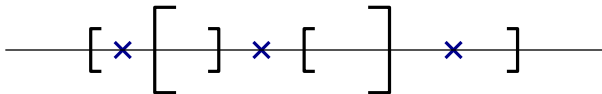
(Go To Short Analysis)

(Go To Open Problems)

Excursion: Piercing

Detailed Analysis

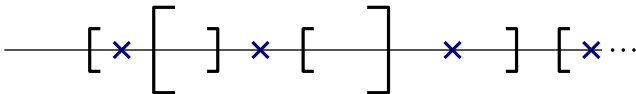
Minimal Piercing $\leq 2 \cdot$ Optimum Piercing



Excursion: Piercing

Detailed Analysis

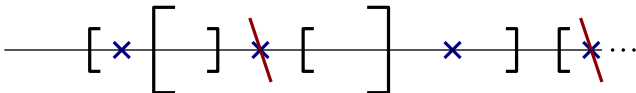
Minimal Piercing



Excursion: Piercing

Detailed Analysis

Minimal Piercing



Excursion: Piercing

Detailed Analysis

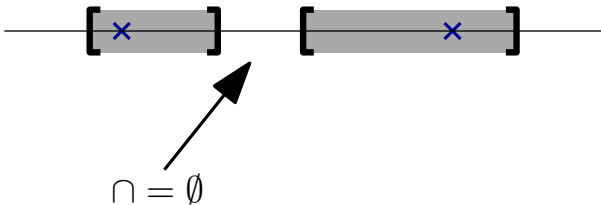
Minimal Piercing



Excursion: Piercing

Detailed Analysis

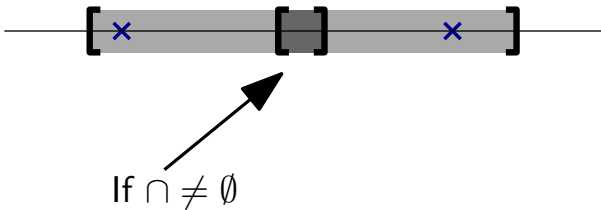
Minimal Piercing



Excursion: Piercing

Detailed Analysis

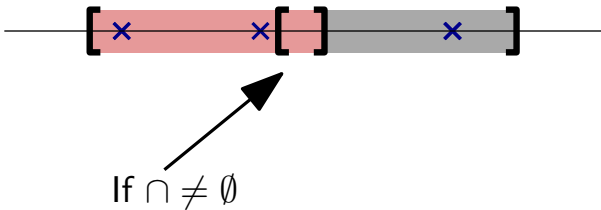
Minimal Piercing



Excursion: Piercing

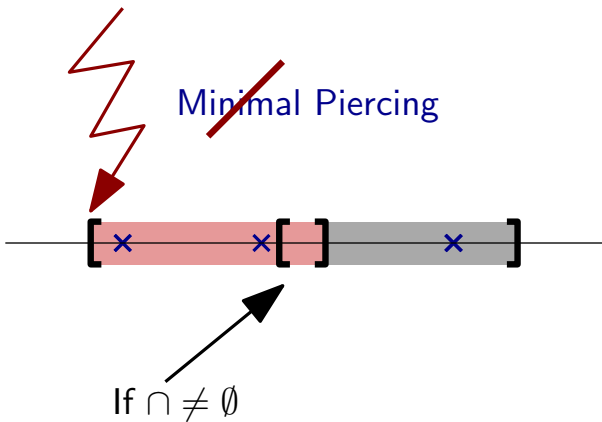
Detailed Analysis

Minimal Piercing



Excursion: Piercing

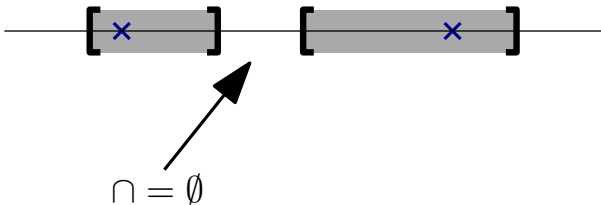
Detailed Analysis



Excursion: Piercing

Detailed Analysis

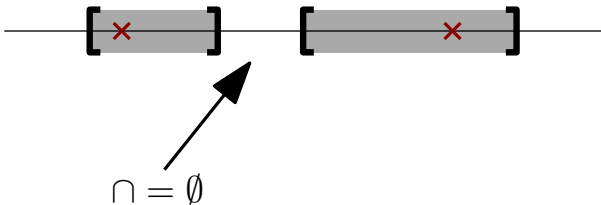
Minimal Piercing



Excursion: Piercing

Detailed Analysis

Optimum Piercing



Excursion: Piercing

Detailed Analysis

Lemma

Minimal piercing $\leq 2 \cdot$ optimum piercing.

- (Continue With Piercing)
- (Go To Open Problems)