# Embedding, Clustering and Coloring for Dynamic Maps

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*Joint work with*

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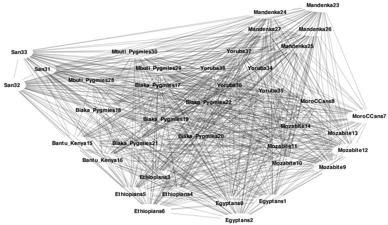
# GMAP

- Given high-dimensional data



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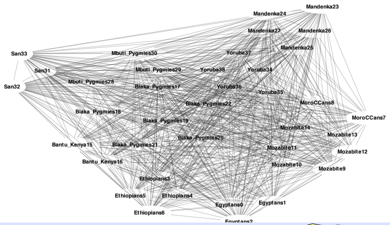
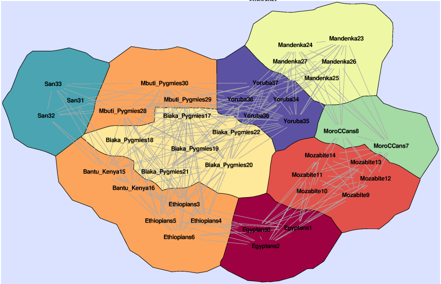
- Given high-dimensional data
- GMAP
  - clusters form countries
  - interconnected clusters share boundaries



# GMAP

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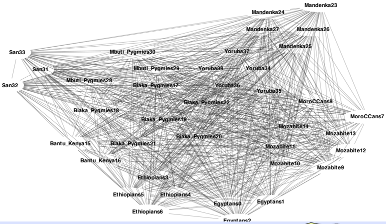
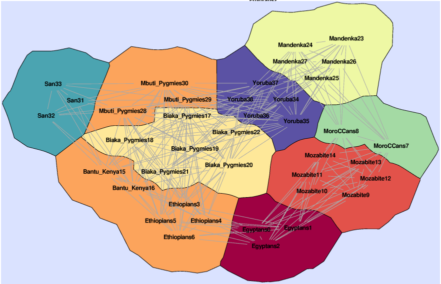
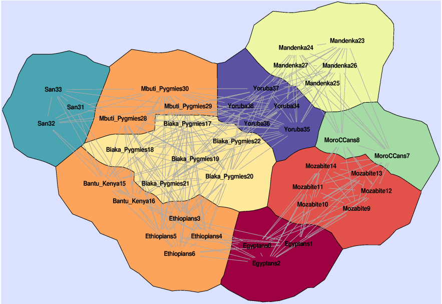
  - interconnected clusters share boundaries
  - visualize connectivity information (via edges)
  - visualize clustering information (via countries)

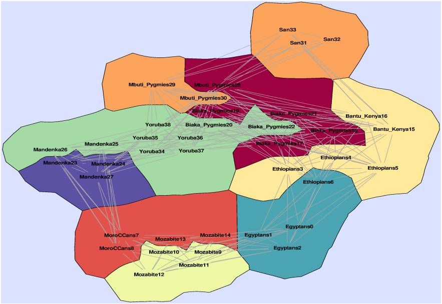


# Simulataneous map visualization

- Compare two or more maps obtained using different similarity metrics.



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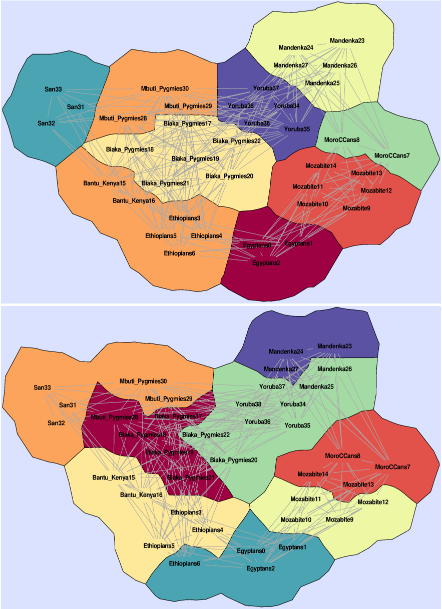
- Dynamic Map Layout



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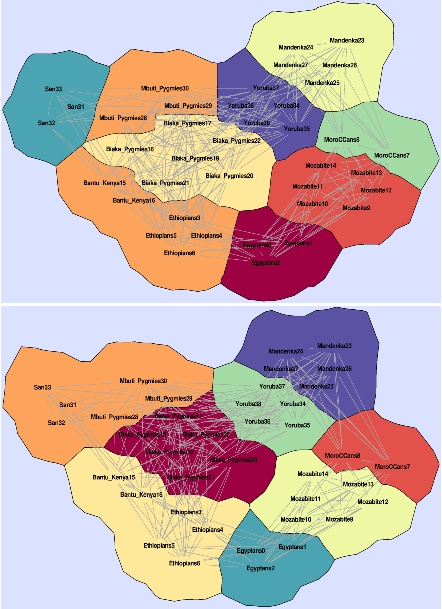

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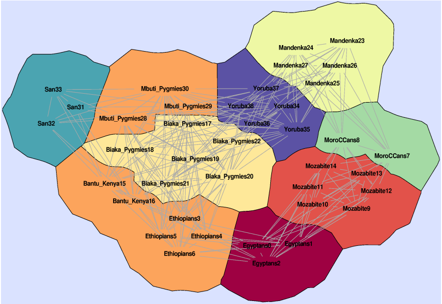
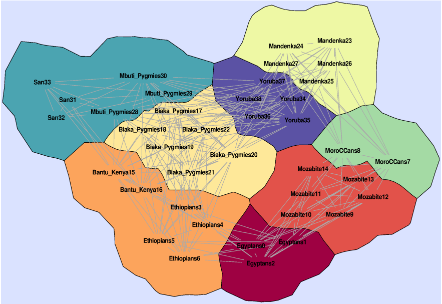
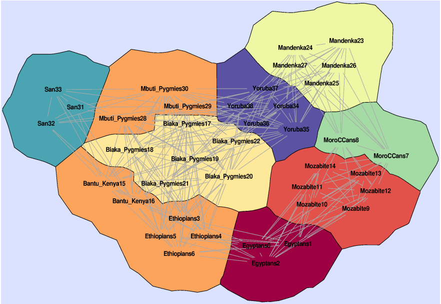
- Dynamic Map Layout
- Dynamic clustering
- Dynamic Map Coloring

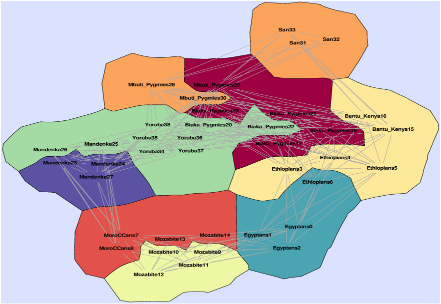


# Dynamic Map Layout

- two graphs with very similar topology can have very different drawings



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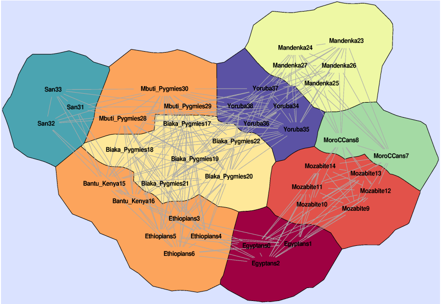
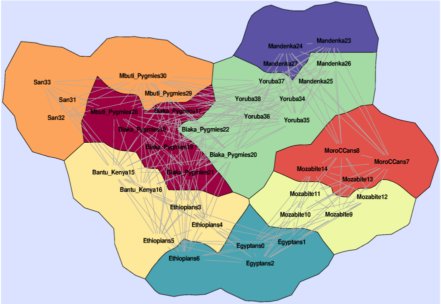
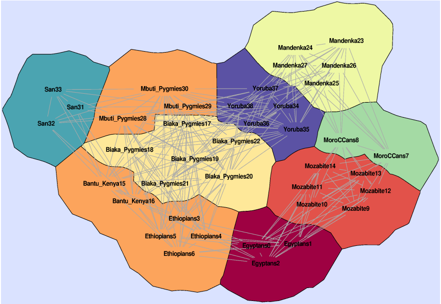
- compute node positions to preserve mental map

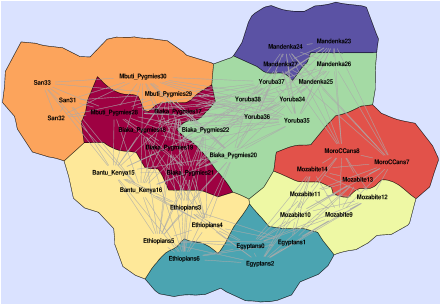


# Dynamic Map Layout

- two graphs with very similar topology can have very different drawings
- compute node positions to preserve mental map
- Approach :
  - initialize the node positions
  - apply multidimensional scaling (MDS) layout
  - Procrustes transform the coordinates of the nodes



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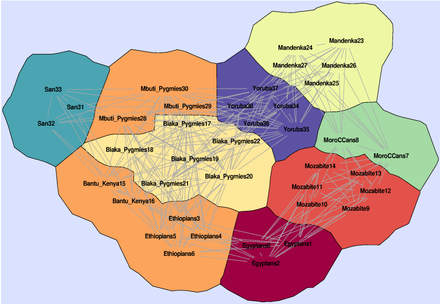
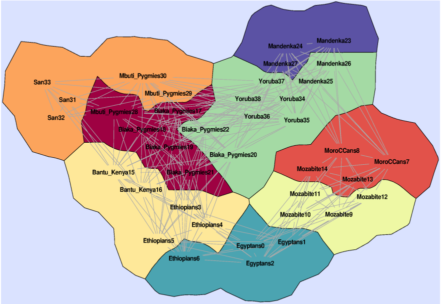
  - Node positions from first layout, $y_i, \; i = 1, 2, \ldots, |V|$
  - Node positions from second layout $x_i, \; i = 1, 2, \ldots, |V|$
  - find a translation vector $b$, scaling value $\rho$ and rotation matrix $T$ that minimize

$$\sum_{i=1}^{|V|} \|y_i - (\rho T x_i + b)\|^2. \tag{1}$$

# Dynamic clustering

- cluster stability $\Rightarrow$ preserve the clustering structure
- Modularity $\Rightarrow$ cluster quality measure
    - Modularity based clustering by Blondel et al.



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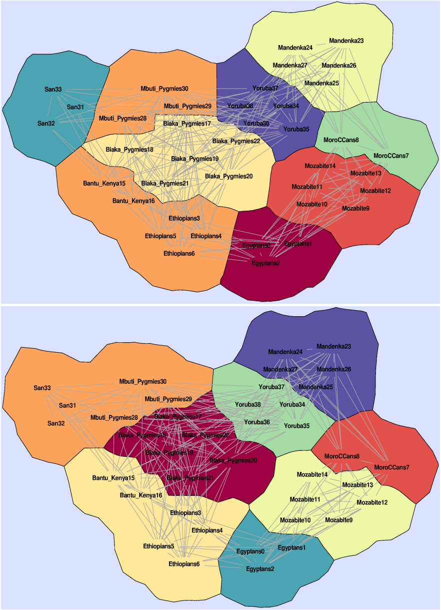

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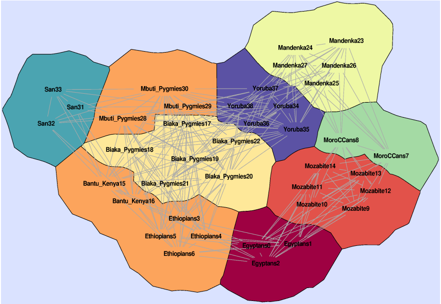
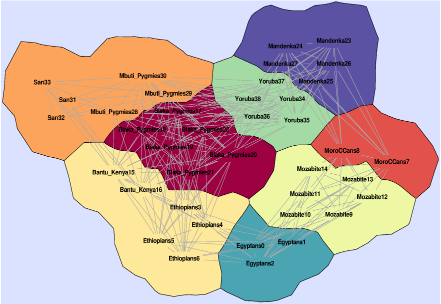
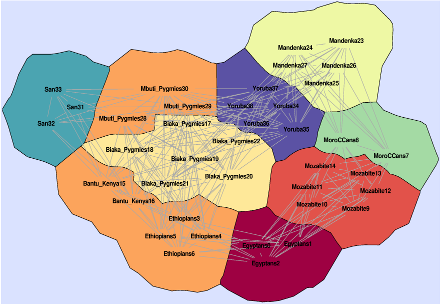
- Dynamic variation of Blondel's clustering algorithm
- Clustering of one map is used as pre-clustering.

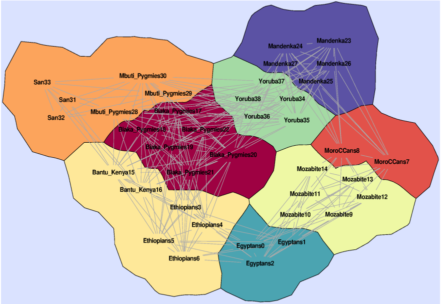


# Dynamic Map Coloring

- same color for similar countries



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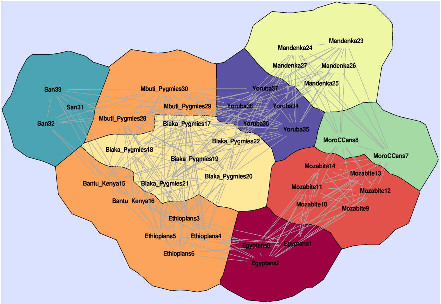


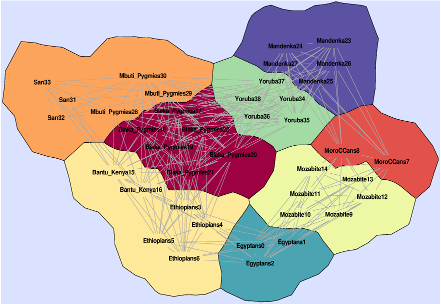

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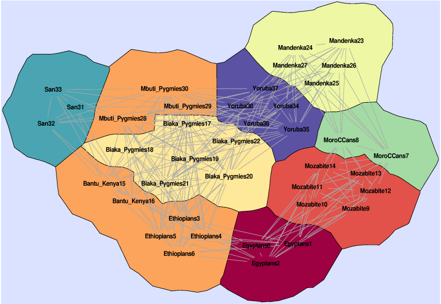
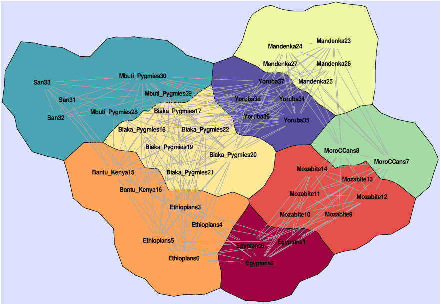
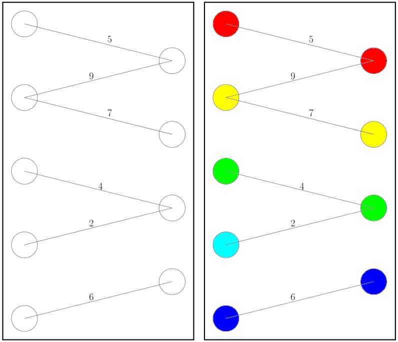
- similar countries ⇒ more nodes in common.
- can be modeled with maximum weighted matching (MWM) of a bipartite graph.



# maximum weighted matching

- Nodes - clusters(countries) in the two maps
- Edge weight - number of objects that are common between clusters



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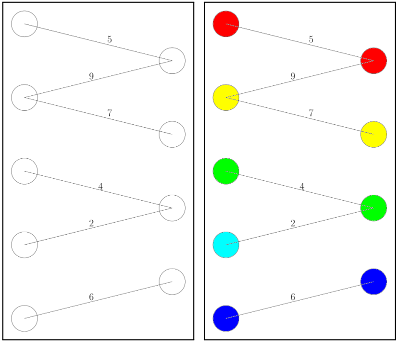

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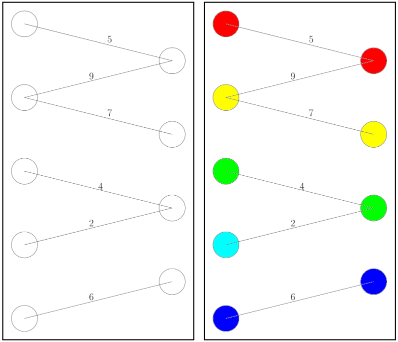
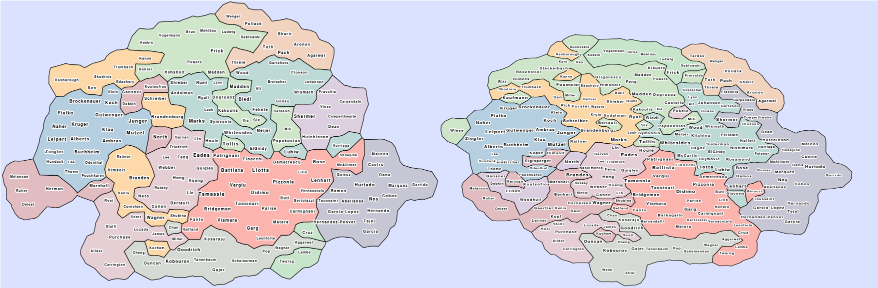
  - $m = O(n)$
  - $O(n^2 \log n)$



# Dynamic map visualization

- dynamic relational data
    - Research collaboration
    - last fm
    - social network



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- $w$ be a weight in the range from 0 to 100
- $M_1$ and $M_2$ be the two input matrices.
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- Use slider to find the "optimum" weight.

# DEMO Setup

- Two input similarity matrices with distance measured using PCA analysis.
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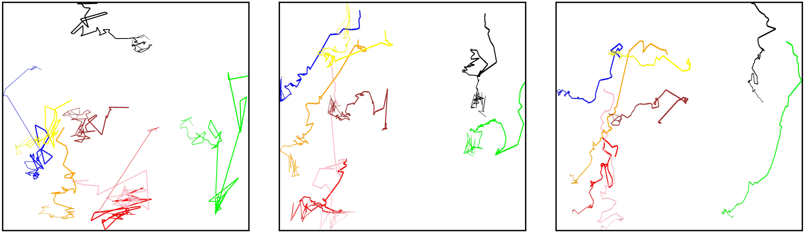
- DEMO

# Layout Stability

- Trajectories of randomly selected nodes
- Compared the average distance travelled by one node in one iteration.
  - independent layout - 21.41 pixels.
  - layout initialized with positions from the previous frame - 13.19 pixels
  - Procrustes transformation - 8.43 pixels.



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$$rand(C, C^{'}) = 1 - \frac{2(|S_{11}| + |S_{00}|)}{n(n-1)} \tag{2}$$

- 0 if the two clusterings are identical
- 1 if one clustering is a singleton clusters and the other one with all nodes in the same cluster.

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- Without our heuristic, value = 0.0631
- With the heuristic, value = 0.0252.
- improvement of a little more than 60% with the heuristic.

# Conclusion

- a practical approach for visualizing
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  - dynamic relational data
  - multiple relationships on the same data
- dynamic cluster stability - Modified modularity clustering algorithm
- color stability using MWM
- layout stability using affine transformations.

# Future Work

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- deploy the system in a generic dynamic data setting.


Thank You!