On Making Relational Division Comprehensible

Lester I. McCann

mccann@uwp.edu

Computer Science Department

University of Wisconsin — Parkside

Kenosha, WI

Frontiers in Education

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Outline

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    - Using Set Containment
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- Division Pitfalls
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Background

- Relational database management systems are based on Codd's relational data model
  - Rooted in set theory
- Codd's original data languages:
  - Relational Calculus (non–procedural)
    - Based on First–Order Predicate Calculus
  - Relational Algebra (procedural)
    - Five fundamental operators: σ, π, $\times$, $-$, $\cup$
    - Three additional operators: $\cap$, $\bowtie$, $\div$

Division

- Division is considered the most challenging of the eight operators
  - Defined using three operators (π, $-$, and $\times$) and six operations
  - Based on finding values that are not answers
  - Not easily expressed in SQL
  - A challenge to explain to students
- Often an afterthought in database texts

- But necessary to answer a specific type of query!

What Division Does

- Division identifies the attribute values from a relation that are found to be paired with all of the values from another relation.
- Viewed another way:
  - As multiplication is to division in arithmetic, Cartesian Product ($\times$) is to Division in relational algebra.

Cartesian Product and Division

- Consider the unary relations $\boxed{m}$ and $\boxed{n}$, and their Cartesian Product $\boxed{o}$:

$\boxed{m}$

C
4
8

$\boxed{n}$

D
3
1
7

$\boxed{o}$

C	D
4	3
4	1
4	7
8	3
8	1
8	7

Cartesian Product and Division

- Division is the opposite of Cartesian Product:

$\boxed{o}$

C	D
4	3
4	1
4	7
8	3
8	1
8	7

$o \div n =$ $\boxed{m}$

C
4
8

$o \div m =$ $\boxed{n}$

D
3
1
7

Cartesian Product and Division

- Division is the opposite of Cartesian Product:

$\boxed{o}$ C	D
4	3
4	1
4	7
8	3
8	1
8	7

$o \div n =$

$\boxed{m}$ C
4
8

$o \div m =$

$\boxed{n}$ D
3
1
7

- That's easy! Who needs a formal definition? : -)

A More Practical Example

- Consider this subset of Date's famous Suppliers–Parts–Projects schema:

$\boxed{p}$

pno	pname	color	weight	city
P1	Nut	Red	12.0	London
...	...	...	...	...
P6	Cog	Red	19.0	London

$\boxed{spj}$

sno	pno	jno	qty
S1	P1	J1	200
...	...	...	...
S5	P6	J4	500

A More Practical Example (cont.)

Query: *Find the sno values of the suppliers that supply all parts of weight equal to 17.*

p	pno	pname	color	weight	city

spj	sno	pno	jno	qty

A More Practical Example (cont.)

Query: *Find the sno values of the suppliers that supply all parts of weight equal to 17.*

p	pno	pname	color	weight	city

spj	sno	pno	jno	qty

- Students can tell us that we need to create this schema:

α	sno	pno

β	pno

A More Practical Example (cont.)

- Constructing $\boxed{\alpha}$ and $\boxed{\beta}$ is straight–forward:

  $\alpha \leftarrow \pi_{sno,pno}(SPJ)$ and $\beta \leftarrow \pi_{pno}(\sigma_{weight=17}(P))$

$\boxed{\alpha}$

sno	pno
S1	P1
S2	P3
S2	P5
S3	P3
S3	P4
S4	P6
S5	P1
S5	P2
S5	P3
S5	P4
S5	P5
S5	P6

$\boxed{\beta}$

pno
P2
P3

Division in Relational Algebra

Idea: Find the values that *do not* belong in the answer, and remove them from the list of possible answers.

- In our P–SPJ example, the list of possible answers is just the available *sno* values in α:

$\boxed{\pi_{sno}(\alpha)}$

sno
S1
S2
S3
S4
S5

Division in Relational Algebra (cont.)

- All possible *sno–pno* pairings can be generated easily:

$\boxed{\pi_{sno}(\alpha)}$

sno
S1
S2
S3
S4
S5

$\times$

$\boxed{\beta}$

pno
P2
P3

$=$

$\boxed{\gamma}$

sno	pno
S1	P2
S1	P3
S2	P2
S2	P3
S3	P2
S3	P3
S4	P2
S4	P3
S5	P2
S5	P3

Division in Relational Algebra (cont.)

- If we remove from $\boxed{\gamma}$ all of the pairings also found in $\boxed{\alpha}$, the result will the values of *sno* that we **do not** want.
- See next slide!

Division in Relational Algebra (cont.)

$\boxed{\gamma}$

sno	pno
S1	P2
S1	P3
S2	P2
S2	P3
S3	P2
S3	P3
S4	P2
S4	P3
S5	P2
S5	P3

$-$

$\boxed{\alpha}$

sno	pno
S1	P1
S2	P3
S2	P5
S3	P3
S3	P4
S4	P6
S5	P1
S5	P2
S5	P3
S5	P4
S5	P5
S5	P6

$=$

$\boxed{\delta}$

sno	pno
S1	P2
S1	P3
S2	P2
S3	P2
S4	P2
S4	P3

Victim tuples are shown in magenta.

Note that S5 is not represented in $\boxed{\delta}$.

Division in Relational Algebra (cont.)

- All that remains is to remove the 'non–answer' *sno* values from the set of possible answers:

$\boxed{\pi_{sno}(\alpha)}$

sno
S1
S2
S3
S4
S5

$-$

$\boxed{\pi_{sno}(\delta)}$

sno
S1
S2
S3
S4

$=$

$\boxed{\div}$

sno
S5

Relational Algebra Summary

- The complete division expression:

$$\alpha \div \beta = \pi_{A-B}(\alpha) - \pi_{A-B}((\pi_{A-B}(\alpha) \times \beta) - \alpha)$$

3 1 2

- Ignoring the projections, there are just three steps:
  1. Compute all possible attribute pairings
  2. Remove the existing pairings
  3. Remove the non–answers from the possible answers
- This is well within the grasp of DB students!

Moving On to SQL

- Most DB texts cover division when they cover Relational Algebra
  - But they often ignore/hide it in their SQL coverage!
  - Leaves students believing division isn’t important — not good!
- Why do they overlook division in SQL?
  - No built–in division operator
  - Standard SQL expressions of division are complex
- Division in SQL need not be confusing

Expressing Division in SQL

- I know of four ways to do division in SQL...

1. Direct conversion of the Relational Algebra expression
2. By applying a quantification tautology
3. By using set containment
4. By comparing set cardinalities

- ... but books frequently choose to use 2 — the hard one!

#1: From Relational Algebra

- Recall the Relational Algebra formulation:

$$\alpha \div \beta = \pi_{A-B}(\alpha) - \pi_{A-B}((\pi_{A-B}(\alpha) \times \beta) - \alpha)$$

- We need to know that in SQL …
  - … `EXCEPT` means difference (–)
  - … a join without the `WHERE` clause produces a Cartesian Product
  - … nested `SELECT`s sometimes need an alias

    … `(SELECT ...) as alias` …

#1: From Relational Algebra (cont.)

- The direct translation from Relational Algebra:

$$\alpha \div \beta = \pi_{A-B}(\alpha) - \pi_{A-B}((\pi_{A-B}(\alpha) \times \beta) - \alpha)$$

```
select distinct sno from spj
except
select sno
  from ( select sno, pno
           from (select sno from spj) as t1,
                (select pno from p where weight=17) as t2
         except
         select sno, pno from spj
       ) as t3;
```

where α would be `select sno, pno from spj`

and β is `select pno from p where weight=17`

#2: By Logical Tautology

- Consider our original English P–SPJ query:

  Find the sno values of the suppliers that supply all parts of weight equal to 17.

- Now consider this rewording that makes the quantifications more explicit:

  Find the sno values such that <u>for</u> <u>all</u> parts of weight 17 <u>there</u> <u>exist</u> suppliers that supply them all

- Problem: For this we need $\forall a(\exists b\, f(a,b))$, but SQL does not support universal quantification.

#2: By Logical Tautology (cont.)

- Solution: We can apply this tautology:

$$\forall a(\exists b\, f(a,b)) \leftrightarrow \overline{\exists} a(\overline{\exists} b\, f(a,b))$$

- Wording before conversion:

Find the sno values such that for all parts of weight 17 there exist suppliers that supply them all

- Wording after conversion:

Find sno values such that there do not exist any parts of weight 17 for which there do not exist any suppliers that supply them all

#2: By Logical Tautology (cont.)

- The resulting SQL version (with intentional misspellings of ‘local’ and ’global’):

```
select distinct sno from spj as globl
 where not exists
      ( select pno from p
          where weight = 17 and not exists
               ( select * from spj as locl
                   where locl.pno = p.pno
                     and locl.sno = globl.sno));
```

- Imagine presenting this to undergrads who have just a lecture or two of SQL under their belts.
- You **do** get the chance to talk about scoping of aliases...

#3: Set Containment

- Consider this: If a supplier supplies a superset of the parts of weight 17, the supplier supplies them all.
  - If only SQL had a superset (containment) operator...

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  - If only SQL had a superset (containment) operator...

- Logic to the rescue!

  If $A \supseteq B$, $B - A$ will be empty (or, $\overline{\exists}(B - A)$)

  where
  - A contains the parts of weight 17 that a supplier supplies
  - B contains all available parts of weight 17.

#3: Set Containment (cont.)

- The resulting SQL query scans through the sno values, computes A based on the current sno, and includes it in the quotient if the difference is empty:

```
select distinct sno from spj as globl
 where not exists (
    ( select pno from p where weight = 17 )
    except
    ( select p.pno
        from p, spj
       where p.pno = spj.pno
         and spj.sno = globl.sno )
 );
```

- The lack of a double negation makes this approach easier to understand.

#4: A Counting We Will Go

- The effect of the set containment approach is to indirectly count the members of each of the two sets, in hopes that the sums are equal.
- Thanks to SQL's `count()`, we can do the counting directly.
- The plan:
  - We find the suppliers that supply parts of weight 17 and how many of those parts each supplies.
  - A `having` clause compares each count to the total number of parts of weight 17.

#4: A Counting We Will Go (cont.)

- The resulting SQL query:

```
  select distinct sno
    from spj, p
   where spj.pno = p.pno and weight = 17
group by sno
  having count(distinct p.pno) =
             (select count (distinct pno)
              from p
              where weight = 17);
```

- No negations at all!
- Not surprisingly, students like it quite well.

Two Division Pitfalls

1. As “All” / “For All” queries need division, does that mean division $\equiv \forall$? No!
   - Consider this query:

     What are the names of the students taking all of the Computer Science seminar classes?

   - We need operand relations like these:

enroll	name	class		*seminar*	class

   - But . . . what if *seminar* is empty?

Two Division Pitfalls (cont.)

1. (cont.)
   - One can say that, if no seminar classes are offered, then all students are taking all seminars!
   - Of course, the real meaning of the query was:

     What are the names of the students taking all of the Computer Science seminar classes, ***assuming that at least one is being offered****?*

   - Students need to realize that the divisor . . .
     - . . . is usually the result of a subquery, and
     - . . . may well contain no tuples

Two Division Pitfalls (cont.)

2. Queries that give the same result as division are not replacements for division
   - Consider this variation of our ‘all parts of weight 17’ query:

     Find the sno values of the suppliers that supply all parts of weight equal to 19.
   - If students inspect Date’s sample data, they learn the answer is suppliers S4 and S5 . . .
   - . . . which also is the result of this query:

     Find the sno values of the suppliers that supply parts of weight equal to 19.

Two Division Pitfalls (cont.)

2. (cont.)
   - That query can be answered with a simple join of the division operands:

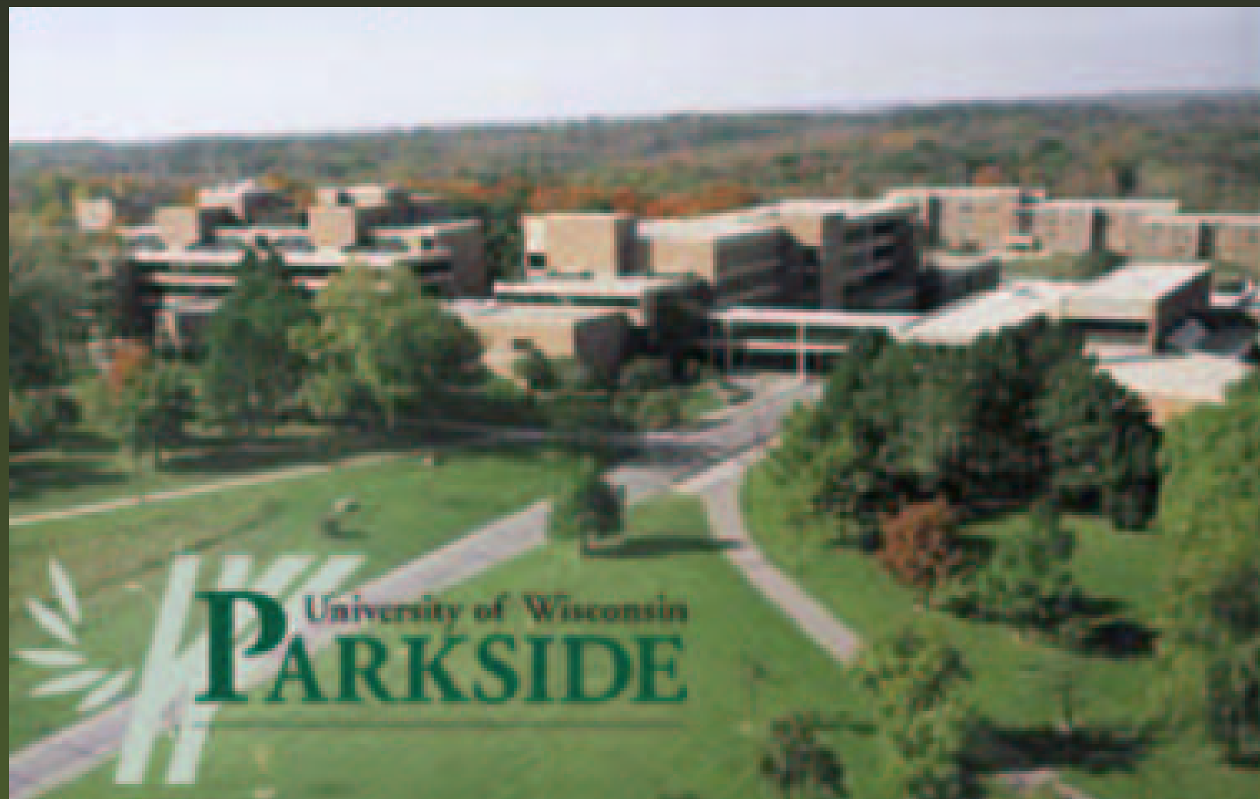
```
select distinct sno
from (select sno, pno from spj) as one,
     (select pno from p where weight = 19) as two
where one.pno = two.pno;
```

   - To help students avoid temptation, select a divisor relation that contains more than one tuple.
     - Only one part has weight 19, but two parts have weight 17.
     - Attempting the join on the ‘weight 17’ query would produce S2, S3, and S5 — all three supply at least one of the parts of weight 17.

Conclusion

- Division is as important in SQL as it is in Relational Algebra
- Students can understand division in both languages if we give them a chance
- A variety of possible implementations of division are possible in SQL
- Looking for shortcuts to division doesn’t work

Any Questions?



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