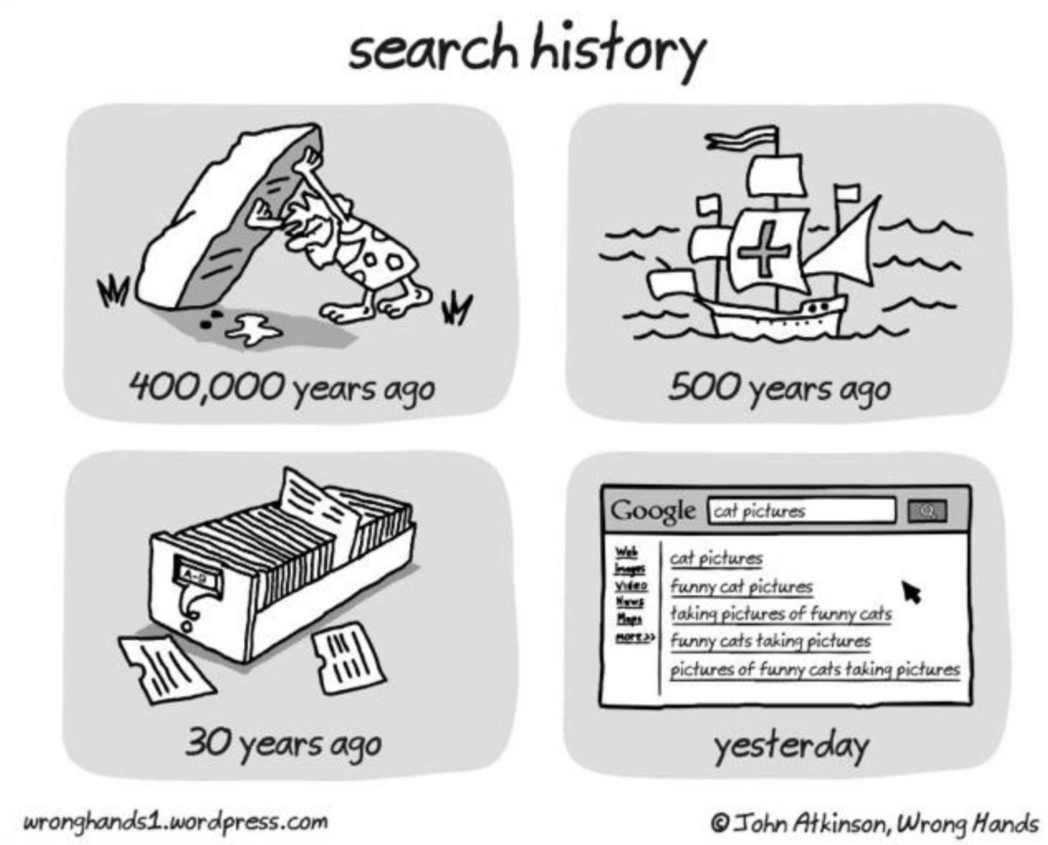
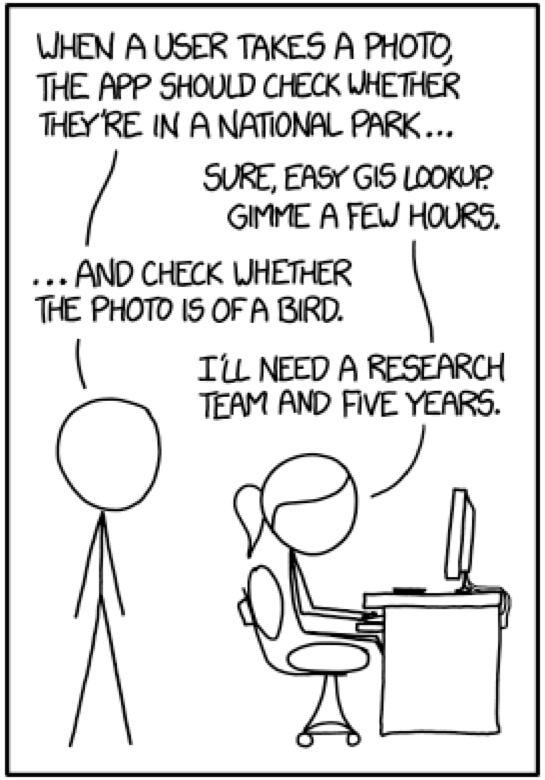
CSc 110, Spring 2017

Lecture 39: searching



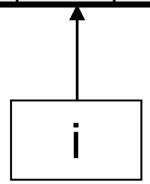


IN CS, IT CAN BE HARD TO EXPLAIN THE DIFFERENCE BETWEEN THE EASY AND THE VIRTUALLY IMPOSSIBLE.

Sequential search

- **sequential search**: Locates a target value in a list (may not be sorted) by examining each element from start to finish. Also known as *linear* search.
  - How many elements will it need to examine?
  - Example: Searching the list below for the value **42**:

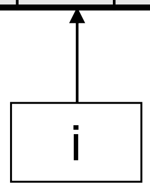
index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	2	7	10	30	56	20	68	36	-4	25	42	50	22	92	15	85	103



Sequential (linear) search

- **sequential search**: Even if the list is sorted, elements are examined in the way (one after the other).
  - Example: Searching the list below for the value **42**:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103



Sequential (linear) search

- Sequential search code:

```
def sequential_search(my_list, value):
    for i in range(0, len(my_list)):
        if (my_list[i] == value):
            return i
    return -1    # not found
```

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

- Note that -1 is returned if the element is not found.

Sequential (linear) search

- For a list of size N, how many elements will be checked worst case?
- On average how many elements will be checked?
- A list of 1,000,000 elements may require 1,000,000 elements to be examined.
- The number of elements to check grows in proportion to the size of the list, i.e., it grows linearly.

Binary Search

- **Binary search**: a method of searching that takes advantage of sorted data.
- Consider a guessing game:

  Someone thinks of a number between 1 and 100. You must guess the number.
  On each round, you are told whether your number is low, high, or correct.

- Best strategy: use a first guess of 50
  - Eliminates half of the numbers immediately
  - On each round, half the numbers are eliminated:
    - 100
    - 50
    - 25
    - …

Binary search

- **binary search**: Locates a target value in a *sorted* list by successively eliminating half of the list from consideration.
  - How many elements will it need to examine?
  - Example: Searching the list below for the value **42**:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

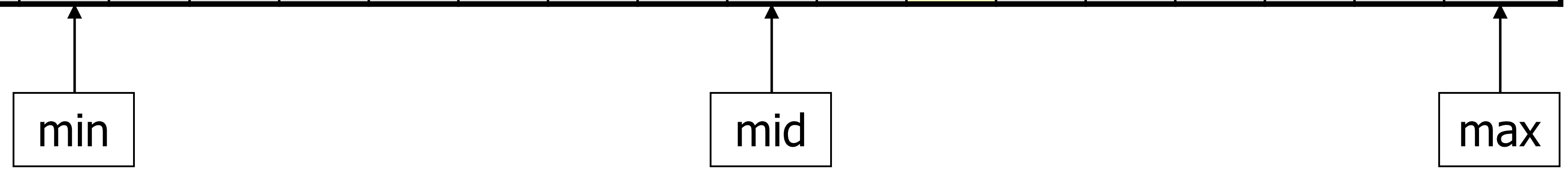
Keep track of indices for a min, mid and max.

- **Search for 42**: Round 1.

list[mid] < 42

eliminate from min to mid (left half)

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

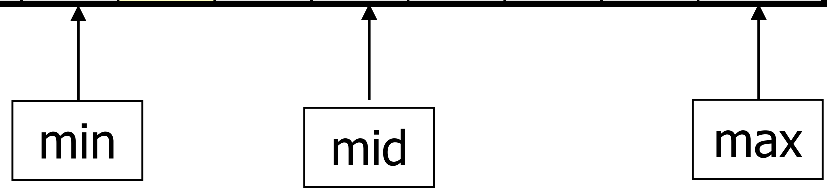


- **Search for 42**: Round 2.

list[mid] > 42

eliminate from mid to max (right half of what's left)

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103

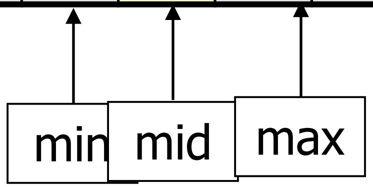


- **Search for 42**: Round 3.

list[mid] == 42

found!

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	10	15	20	22	25	30	36	42	50	56	68	85	92	103



Binary search runtime

- For a list of size N, it eliminates ½ until 1 element remains.

  N, N/2, N/4, N/8, ..., 4, 2, 1

  - How many divisions does it take?
  - Suppose N = 1024

    1024, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1 (10 divisions)
  - **$10 = \log_2 (1024)$**

- Suppose we double the number the number of elements.
  - How many divisions does it take?
  - Suppose N = 2048

    2048, 1024, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1 (11 divisions)
  - **$11 = \log_2 (2048)$**

Binary search runtime

- For a list of size N, it eliminates ½ until 1 element remains.

  N, N/2, N/4, N/8, ..., 4, 2, 1

  - How many divisions does it take?
  - Suppose N = 1024

    1024, 512, 256, 128, 64, 32, 16, 8, 4, 2, 1 (10 divisions)
  - Binary search examines a number of elements proportional to the number of divisions

- Think of it from the other direction:
  - How many times do I have to multiply by 2 to reach N?

    1, 2, 4, 8, ..., N/4, N/2, N
  - Call this number of multiplications "x".

  $2^x = N$

  $x = \log_2 N$

- Binary search examines a number of elements proportional to **log of N**.

Binary search code

```
# Returns the index of an occurrence of target in a,
# or a negative number if the target is not found.
# Precondition: elements of a are in sorted order
def binary_search(a, target):
    min = 0
    max = len(a) - 1

    while (min <= max):
        mid = (min + max) // 2
        if (a[mid] < target):
            min = mid + 1
        elif (a[mid] > target):
            max = mid - 1
        else:
            return mid   # target found

    return -(min + 1)    # target not found
```

Binary search

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	7	12	18	25	27	30	36	42	56	68	85	91	92	98	102

What do the following calls return when passed the above list?

```
binary_search(a, 2)
binary_search(a, 68)
binary_search(a, 12)
```

How many comparisons does each call do?

Comparing Binary vs. Sequential search

- **Binary search vs Sequential search**: number of items examined

List size	Binary search	Sequential search
1	1	1
10	4	10
1,000	11	1,000
5,000	14	5,000
100,000	18	100,000
1,000,000	21	1,000,000

bisect

```
from bisect import *

# searches an entire sorted list for a given value
# returns the index the value should be inserted at to maintain sorted order
# Precondition: list is sorted
bisect(list, value)

# searches given portion of a sorted list for a given value
# examines min_index (inclusive) through max_index (exclusive)
# returns the index the value should be inserted at to maintain sorted order
# Precondition: list is sorted
bisect(list, value, min_index, max_index)
```

Using `bisect`

```
# index 0   1   2   3    4    5    6    7    8    9   10   11   12   13   14   15
a   =  {-4, 2, 7, 9, 15, 19, 25, 28, 30, 36, 42, 50, 56, 68, 85, 92}

index1 = bisect(a, 42, 0, 16)    # index1 is 11
index2 = bisect(a, 21, 0, 16)    # index2 is 6
```

- `bisect` returns the index where the value could be inserted while maintaining sorted order
- if the value is already in the list the next index is returned

Sorting

- **sorting**: Rearranging the values in a list into a specific order (usually into their "natural ordering").
  - one of the fundamental problems in computer science
  - can be solved in many ways:
    - there are many sorting algorithms
    - some are faster/slower than others
    - some use more/less memory than others
    - some work better with specific kinds of data
    - some can utilize multiple computers / processors, ...

  - *comparison-based sorting* : determining order by comparing pairs of elements:
    - $<$, $>$, …

Sorting algorithms

- **bogo sort**: shuffle and pray
- **bubble sort**: swap adjacent pairs that are out of order
- **selection sort**: look for the smallest element, move to front
- **insertion sort**: build an increasingly large sorted front portion
- **merge sort**: recursively divide the list in half and sort it
- **heap sort**: place the values into a sorted tree structure
- **quick sort**: recursively partition list based on a middle value

other specialized sorting algorithms:

- **bucket sort**: cluster elements into smaller groups, sort them
- **radix sort**: sort integers by last digit, then 2nd to last, then ...
- ...

Bogo sort

- **bogo sort**: Orders a list of values by repetitively shuffling them and checking if they are sorted.
  - name comes from the word "bogus"

  The algorithm:
  - Scan the list, seeing if it is sorted. If so, stop.
  - Else, shuffle the values in the list and repeat.

- This sorting algorithm (obviously) has terrible performance!

Bogo sort code

```
# Places the elements of a into sorted order.
def bogo_sort(a):
    while (not is_sorted(a)):
        shuffle(a)

# Returns true if a's elements
#are in sorted order.
def is_sorted(a):
    for i in range(0, len(a) - 1):
        if (a[i] > a[i + 1]):
            return False
    return True
```