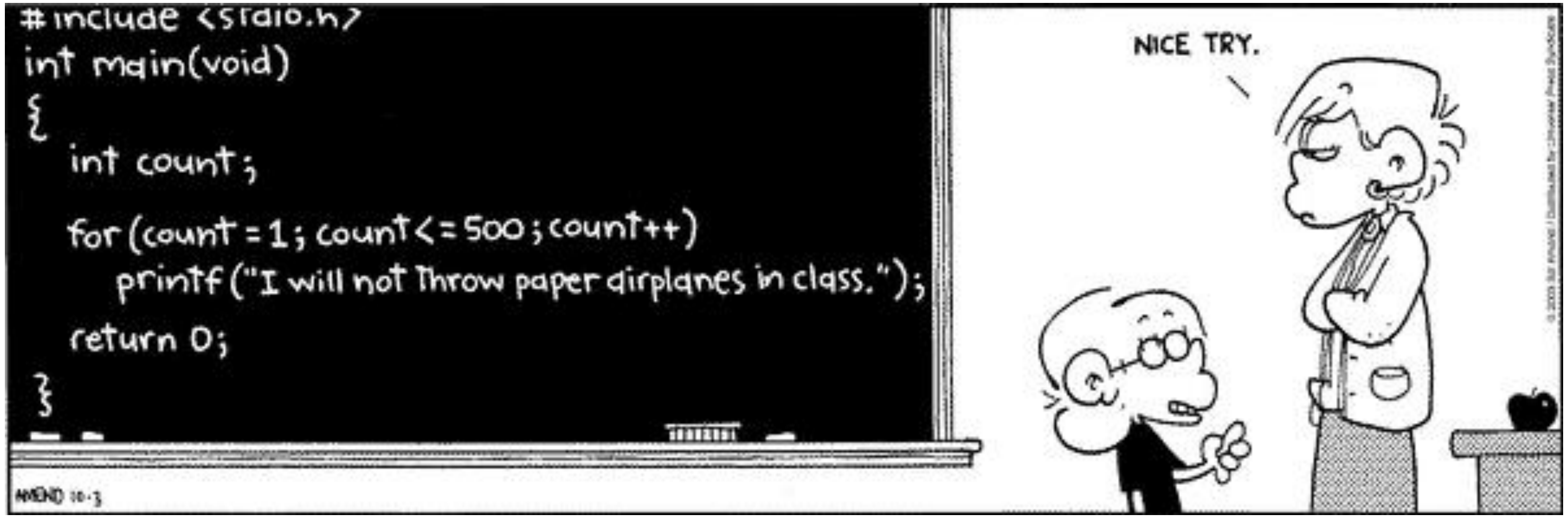
CSc 110, Spring 2017

Lecture 40: Sorting

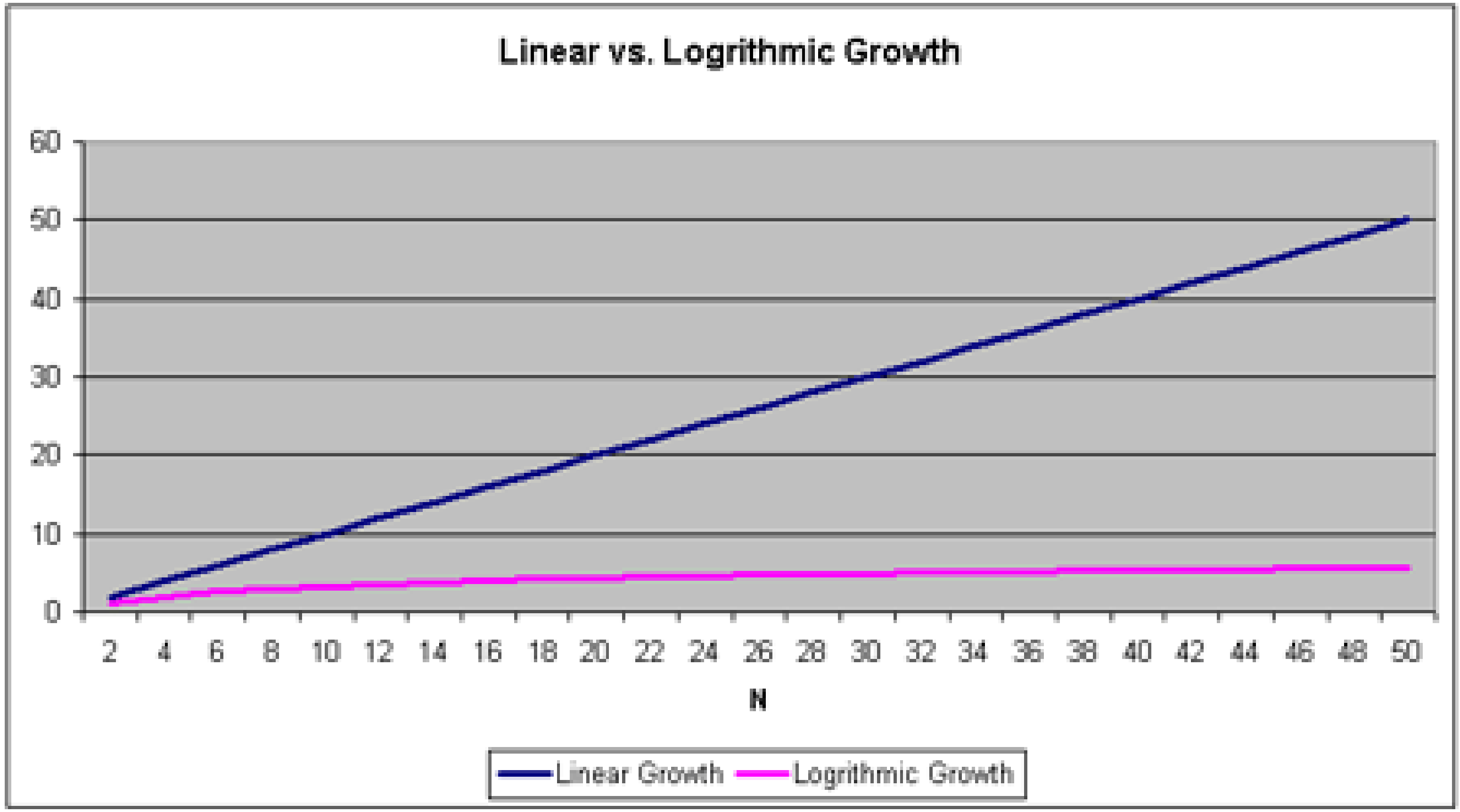
Adapted from slides by Marty Stepp and Stuart Reges



Searching

- How many items are examined worse case for sequential search?
- How many items are examined worst case for binary search?
- An algorithm's efficiency can be expressed in terms of being proportional to its ___________ size.
- Why is sequential search also known as linear search?

Linear vs. Logarithmic Growth



Sorting

- **sorting**: Rearranging the values in a list into a specific order (usually into their "natural ordering").
  - one of the fundamental problems in computer science
  - can be solved in many ways:
    - there are many sorting algorithms
    - some are faster/slower than others
    - some use more/less memory than others
    - some work better with specific kinds of data
    - some can utilize multiple computers / processors, ...
  - *comparison-based sorting* : determining order by comparing pairs of elements:
    - $<$, $>$, …

Sorting algorithms

- **bogo sort**: shuffle and pray
- **selection sort**: look for the smallest element, move to front
- **bubble sort**: swap adjacent pairs that are out of order
- **insertion sort**: build an increasingly large sorted front portion
- **merge sort**: recursively divide the list in half and sort it
- **heap sort**: place the values into a sorted tree structure
- **quick sort**: recursively partition list based on a middle value

other specialized sorting algorithms:

- **bucket sort**: cluster elements into smaller groups, sort them
- **radix sort**: sort integers by last digit, then 2nd to last, then ...
- ...

Bogo sort

- **bogo sort**: Orders a list of values by repetitively shuffling them and checking if they are sorted.
  - name comes from the word "bogus"

  The algorithm:
  - Scan the list, seeing if it is sorted. If so, stop.
  - Else, shuffle the values in the list and repeat.

- This sorting algorithm (obviously) has terrible performance!

Bogo sort code

```
# Places the elements of a into sorted order.
def bogo_sort(a):
    while (not is_sorted(a)):
        shuffle(a)

# Returns true if a's elements
#are in sorted order.
def is_sorted(a):
    for i in range(0, len(a) - 1):
        if (a[i] > a[i + 1]):
            return False
    return True
```

Selection sort

```
10   13   8   2   4   7
```

Selection sort

- **selection sort**: Orders a list of values by repeatedly putting the smallest or largest unplaced value into its final position.

  The algorithm:
  - Look through the list to find the smallest value.
  - Swap it so that it is at index 0.

  - Look through the list to find the second-smallest value.
  - Swap it so that it is at index 1.

    ...

  - Repeat until all values are in their proper places.

Selection sort example

- Initial list:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	22	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

- After 1st, 2nd, and 3rd passes:

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	**-4**	18	12	**22**	27	30	36	50	7	68	91	56	2	85	42	98	25

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	**2**	12	22	27	30	36	50	7	68	91	56	**18**	85	42	98	25

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	2	**7**	22	27	30	36	50	**12**	68	91	56	18	85	42	98	25

Selection sort code

```
# Rearranges the elements of a into sorted order using
# the selection sort algorithm.
def selection_sort(a):
    for i in range(0, len(a) - 1):
        # find index of smallest remaining value
        min = i
        for j in range(i + 1, len(a)):
            if (a[j] < a[min]):
                min = j
        # swap smallest value its proper place, a[i]
        swap(a, i, min)

def swap(a, i, j):
    if (i != j):
        temp = a[i]
        a[i] = a[j]
        a[j] = temp
```

Selection sort runtime

- How many comparisons does selection sort have to do?

    First round (N-1)

    Second round (N-2)

    Third round (N-3)

    …

    2

    1

or

$$(N-1) + (N-2) + (N-3) + ... + 2 + 1 = N(N-1)/2$$

- Selection sort examines a number of elements in proportional to N^2

Similar algorithms

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	22	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	18	12	-4	22	27	30	36	7	50	68	56	2	85	42	91	25	98

22 → 50 → 91 → 98 →

- **bubble sort**: Make repeated passes, swapping adjacent values
  - slower than selection sort (has to do more swaps)

Bubble sort

- **bubble sort**: Orders a list of values by repeatedly comparing adjacent values, swapping if the values are out of order.

  The algorithm for a list of size N:
  - Compare the first two adjacent values.
  - Swap if the second is smaller than the first.
  - Repeat until the the end of the list .
  - Largest value is now at position N
  - Decrement N by 1 and repeat.

Bubble sort runtime

- How many comparisons does selection sort have to do?

    First round (N-1)

    Second round (N-2)

    Third round (N-3)

    …

    2

    1

or

$$(N-1) + (N-2) + (N-3) + ... + 2 + 1 = N(N-1)/2$$

- Bubble sort examines a number of elements in proportional to N^2

Similar algorithms

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	22	18	12	-4	27	30	36	50	7	68	91	56	2	85	42	98	25

- **insertion sort**: Shift each element into a sorted sub-list
  - faster than selection sort (examines fewer values)

index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
value	-4	12	18	22	27	30	36	50	7	68	91	56	2	85	42	98	25

sorted sub-list (indexes 0-7)

← 7

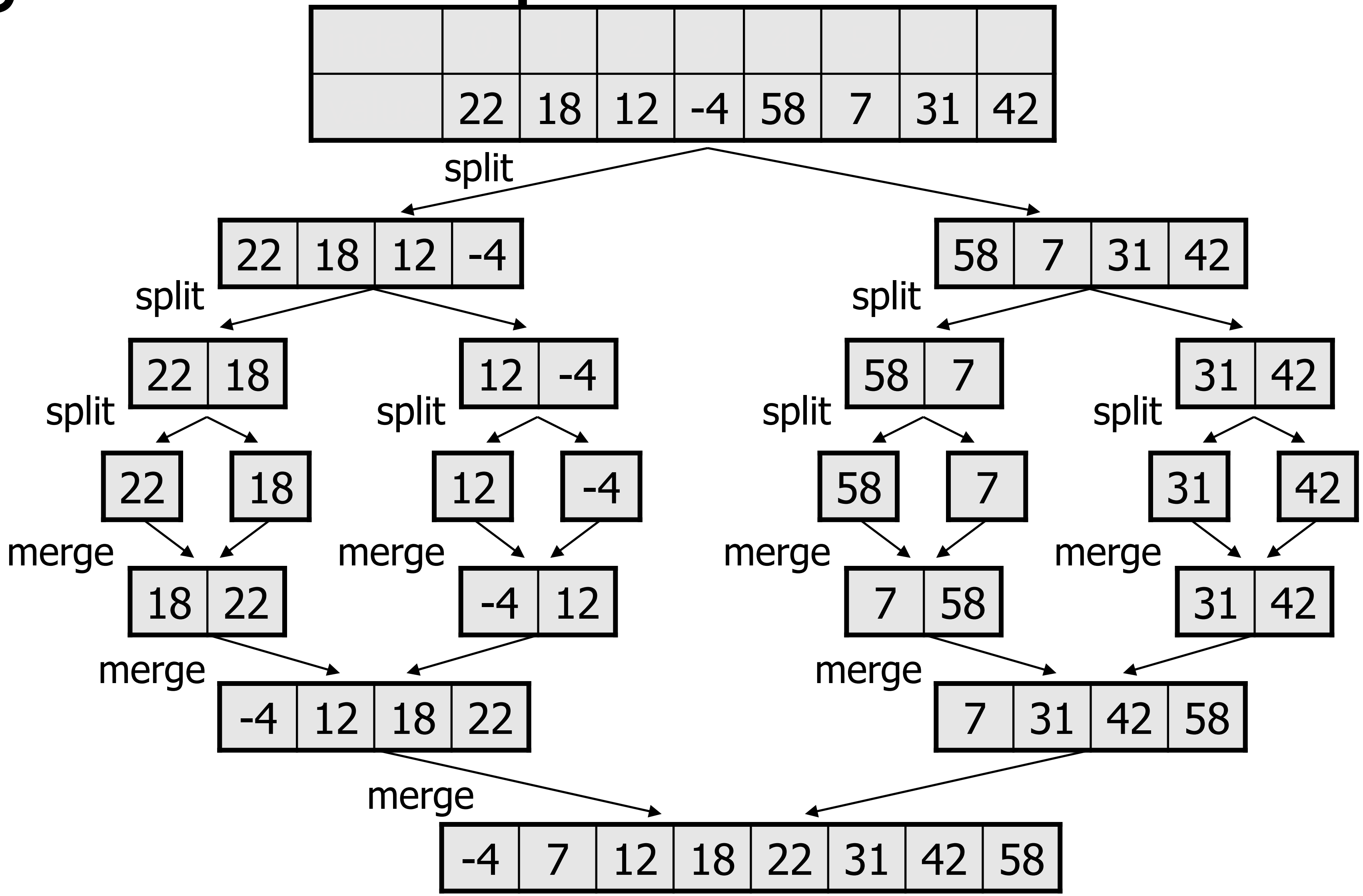
Merge sort

- **merge sort**: Repeatedly divides the data in half, sorts each half, and combines the sorted halves into a sorted whole.

  The algorithm:
  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.

  - Often implemented recursively.
  - An example of a "divide and conquer" algorithm.
    - Invented by John von Neumann in 1945

Merge sort example



Merge halves code

```
# Merges the left/right elements into a sorted result.
# Precondition: left/right are sorted
def merge(result, left, right):
    i1 = 0   # index into left list
    i2 = 0   # index into right list

    for i in range(0, len(result)):
        if (i2 >= len(right) or (i1 < len(left) and left[i1] <= right[i2])):
            result[i] = left[i1]    # take from left
            i1 += 1
        else:
            result[i] = right[i2]   # take from right
            i2 += 1
```

Merge sort code

```
# Rearranges the elements of a into sorted order using
# the merge sort algorithm.
def merge_sort(a):
    if (len(a) >= 2):
        # split list into two halves
        left  = a[0, len(a)//2]
        right = a[len(a)//2, len(a)]

        # sort the two halves
        merge_sort(left)
        merge_sort(right)

        # merge the sorted halves into a sorted whole
        merge(a, left, right)
```

Merge sort runtime

- How many comparisons does merge sort have to do?

N	Runtime (ms)
1000	0
2000	0
4000	0
8000	0
16000	0
32000	15
64000	16
128000	47
256000	125
512000	250
1e6	532
2e6	1078
4e6	2265
8e6	4781
1.6e7	9828
3.3e7	20422
6.5e7	42406
1.3e8	88344