CSc 120
Introduction to Computer Programming II

Adapted from slides
by Dr. Saumya Debray

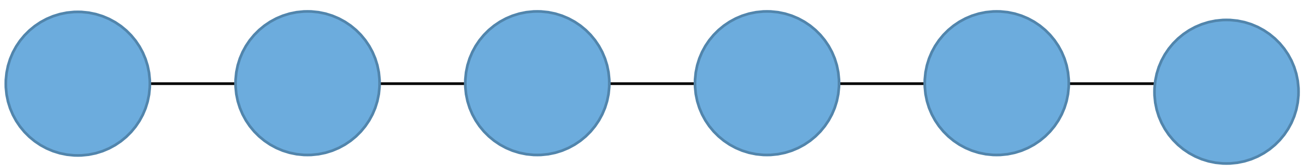
14: Stacks and Queues

linear data structures

Linear data structures

A *linear* data structure is a collection of objects with a straight-line ordering among them

- each object in the collection has a *position*
- for each object in the collection, there is a notion of the object *before* it or *after* it



Data structures we've seen

Linear	Not linear
• Python lists (aka arrays) • Linked lists	• Dictionaries • Sets

Today's topic

Linear	Not linear
• Python lists (aka arrays)	• Dictionaries
• Linked lists	• Sets
• Stacks	
• Queues	
• Dequeues	

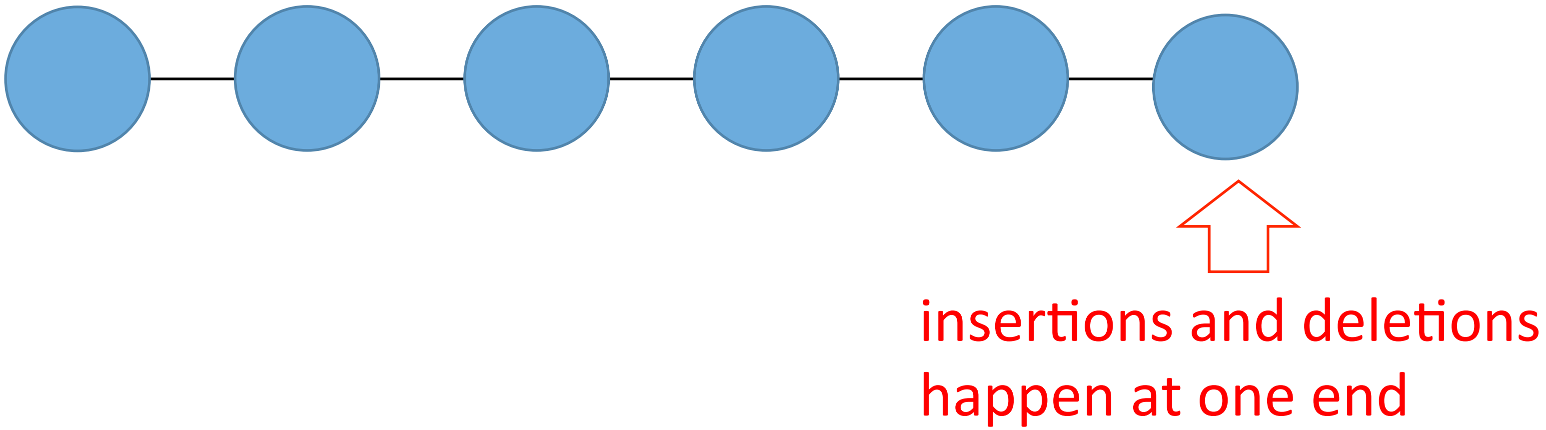
Key property: the way in which objects are added to, and removed from, the collection

stacks

Stacks

A *stack* is a linear data structure where objects are inserted or removed only at one end

- all insertions and deletions happen at one particular end of the data structure
- this end is called the *top* of the stack
- the other end is called the *bottom* of the stack



Stacks: insertion of values

Insertion of a sequence of values into a stack:

5 17 33 9 43

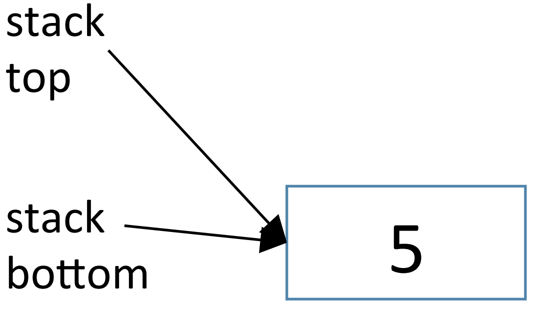
stack top None

stack bottom None

Stacks: insertion of values

Insertion of a sequence
of values into a stack:

5 17 33 9 43



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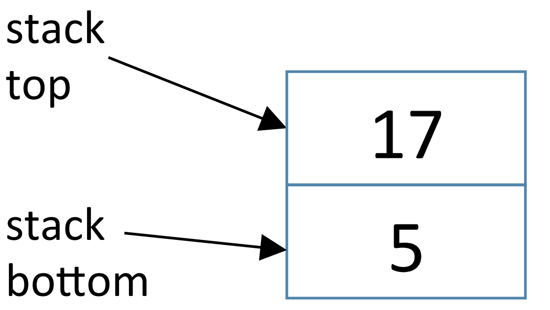

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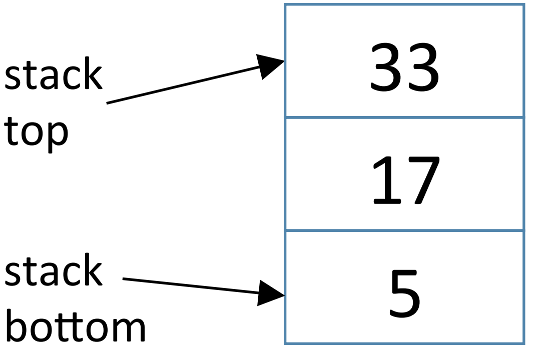

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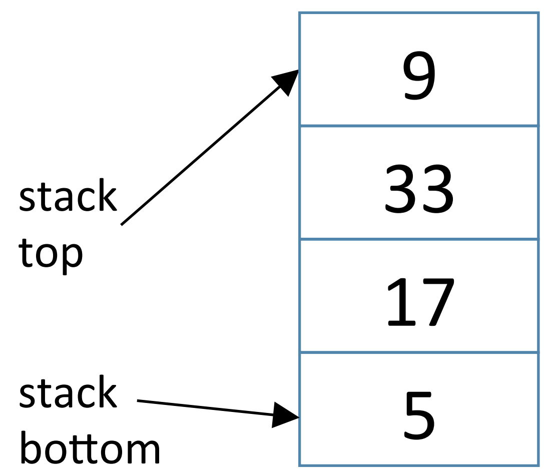

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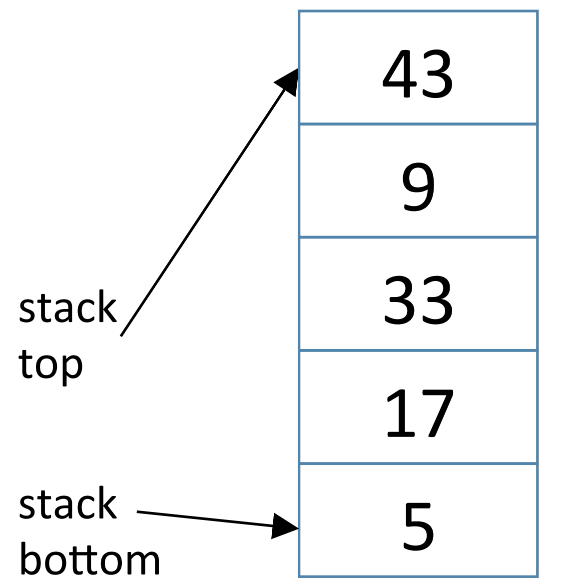
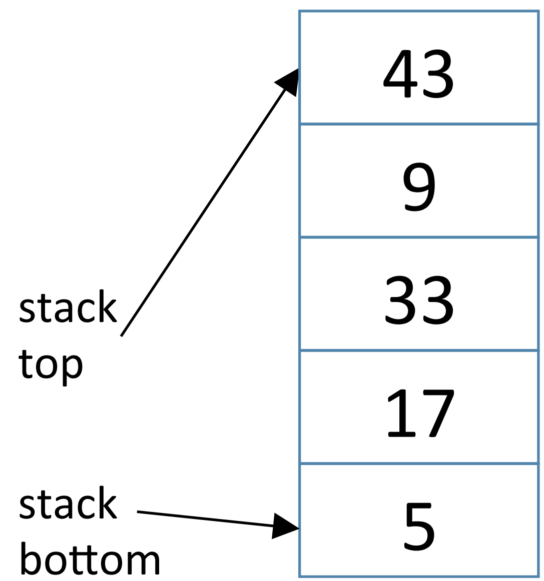


Stacks: insertion of values

5 17 33 9 43

order in which values were inserted

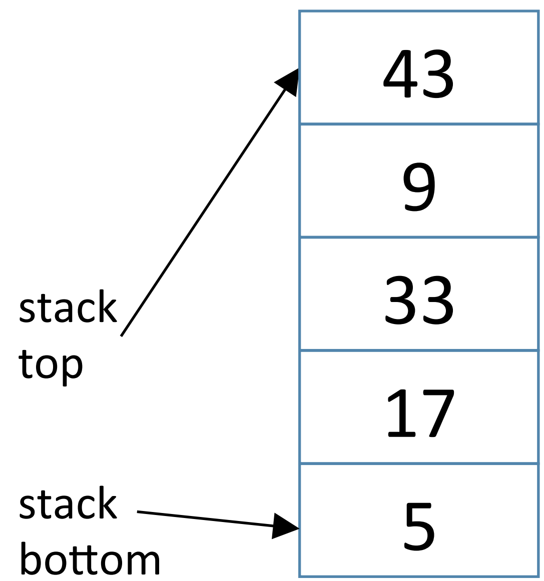


Stacks: removal of values

5 17 33 9 43

order in which values were inserted

Removing values from the stack:



Stacks: removal of values

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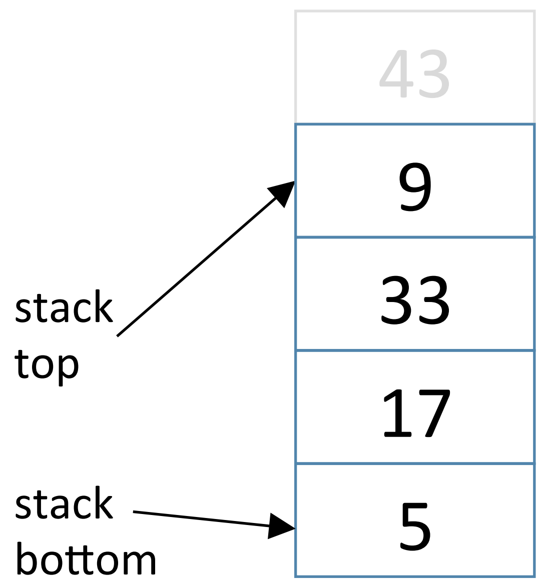
Removing values from the stack:

43



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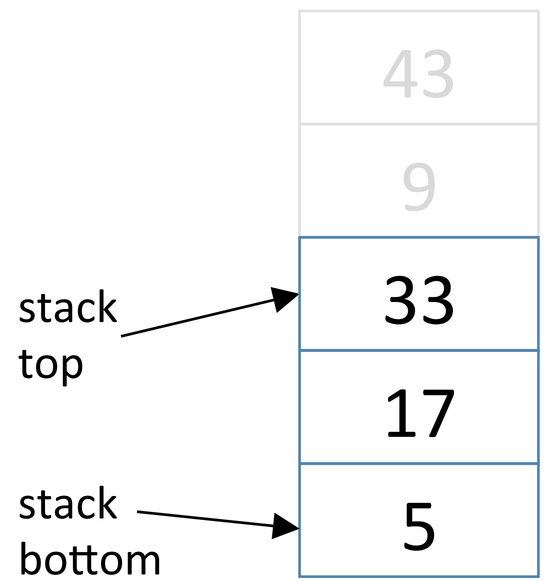
Removing values from the stack:

43 9



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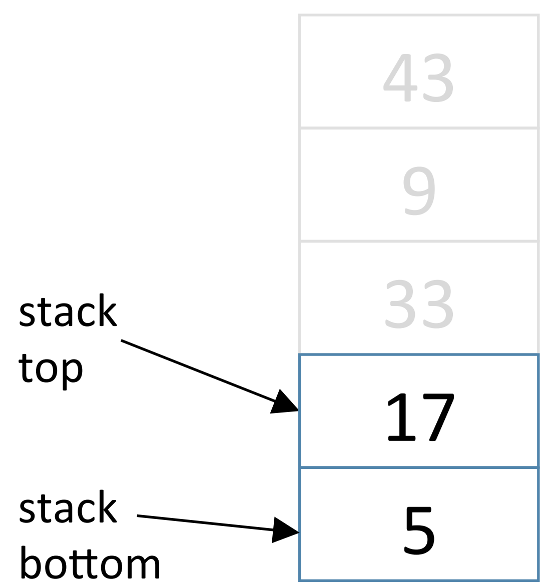


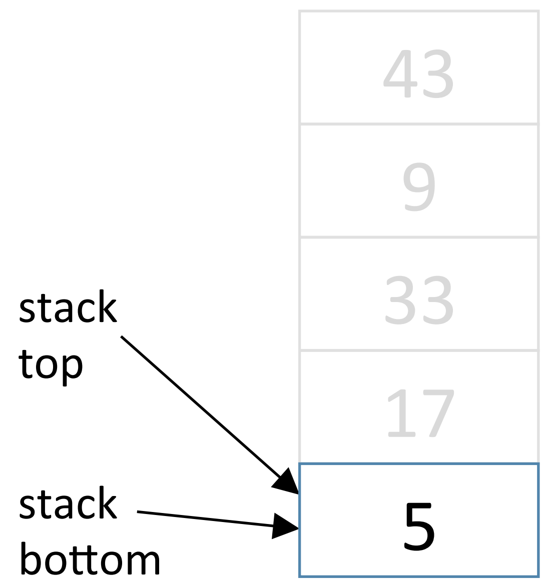
Stacks: removal of values

5   17   33   9   43

order in which values were inserted

Removing values from the stack:

43   9   33   17



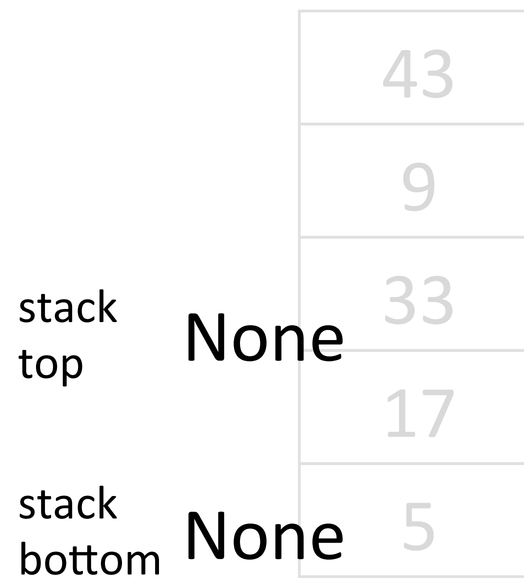
Stacks: removal of values

5 17 33 9 43

order in which values were inserted

Removing values from the stack:

43 9 33 17 5



Stacks: removal of values

5 17 33 9 43

→

order in which values were inserted

Removing values from the stack:

43 9 33 17 5

→

order in which values were removed

Stacks: LIFO property

5 17 33 9 43

→

order in which values were inserted

Removing values from the stack:

43 9 33 17 5

→

order in which values were removed

values are removed in reverse order from the order of insertion

"LIFO order"
Last in, First out

Methods for a Stack class

- Stack() : creates a new empty stack
- push(*item*) : adds *item* to the top of the stack
  - returns nothing
  - modifies the stack
- pop() : removes the top item from the stack
  - returns the removed item
  - modifies the stack
- is_empty() : checks whether the stack is empty
  - returns a Boolean

Implementing a Stack class

```
class Stack:
    # the top of the stack is the last item in the list
    def __init__(self):
        self._items = []

    def push(self, item):
        self._items.append(item)

    def pop(self):
        return self._items.pop()
```

removes and returns the last item in a list

EXERCISE

```
>>> s = Stack()
>>> s.push(4)
>>> s.push(17)
>>> s.push(5)
>>> x = s.pop()
>>> y = s.pop()
```

← *what does the stack* s *look like here?*
what are the values of x *and* y*?*

EXERCISE

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>>> y = s.pop()
>>> s.push(x)
>>> s.push(y)
```

← *what does the stack* s *look like here?*

stacks: applications

An application: balancing parens

IDLE (the Python shell) matches up left and right parens ( ), brackets [ ], and braces { }

```
>>> x = [1,2,[3,4,[5],7],8]
```

How does it figure out how far back to highlight?

An application: balancing parens

Basic idea: Match each] with corresponding [

- similarly for (...) and { ... } pairs

- Idea:
  - maintain a stack
  - on seeing '[' : push
  - on seeing ']' : pop the matching symbol

Example: [1, 2, [3, [4], 5 , [7]]]

Stack (empty)

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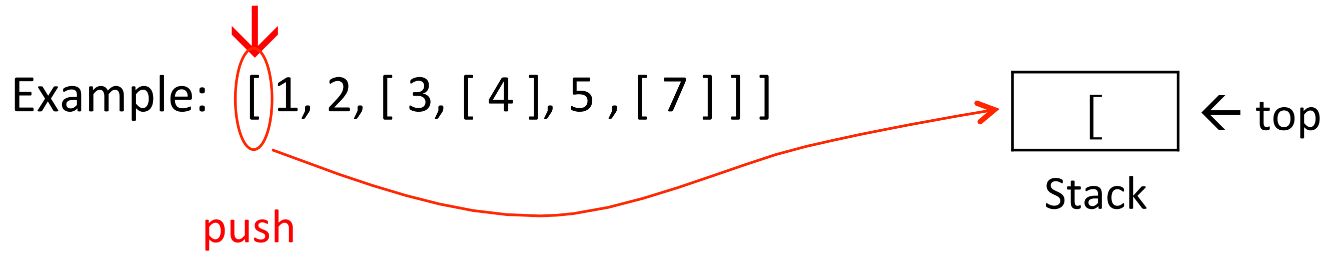

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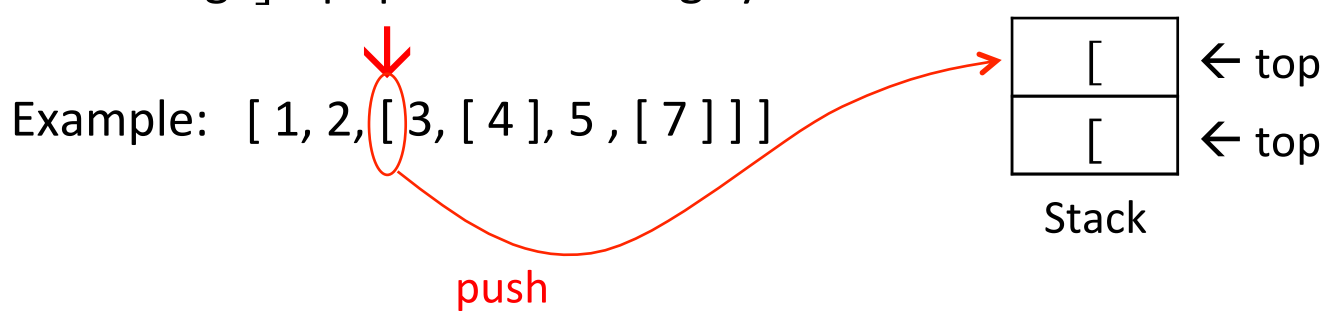

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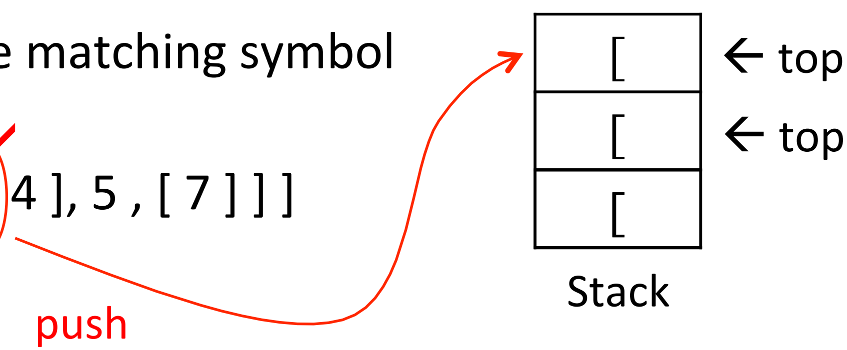

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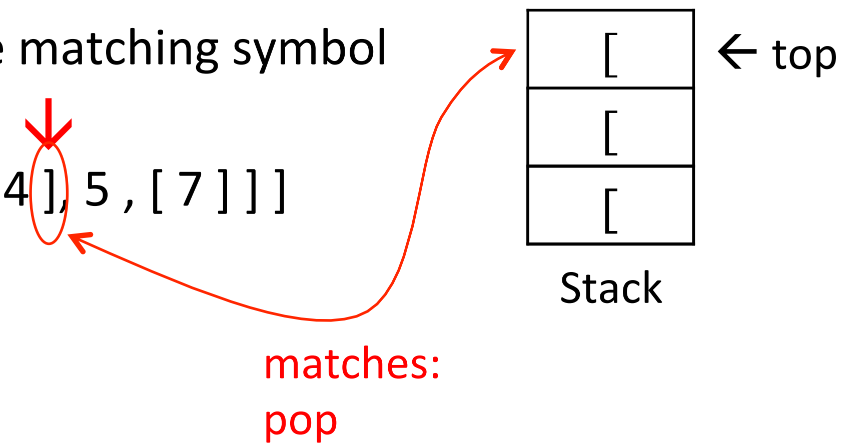

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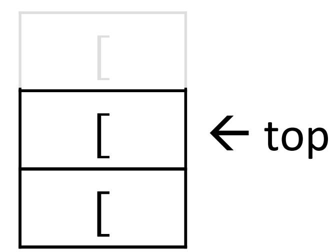

Stack

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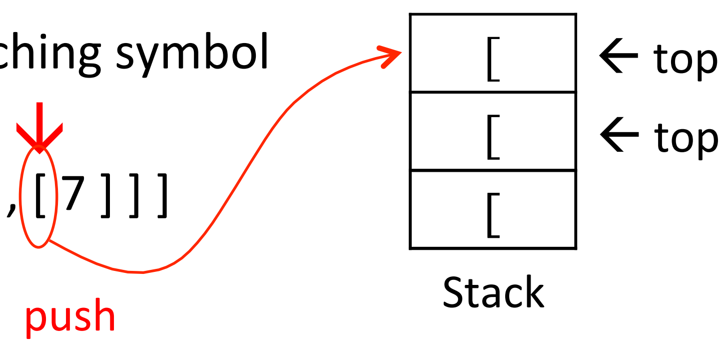

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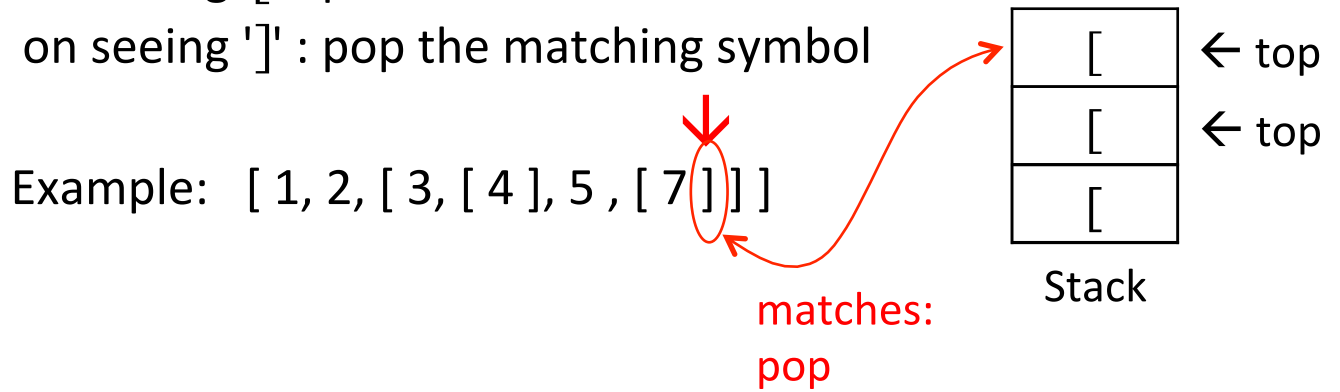

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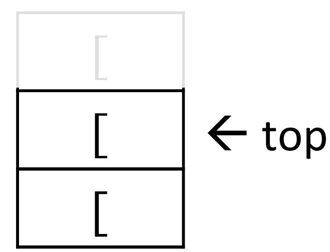

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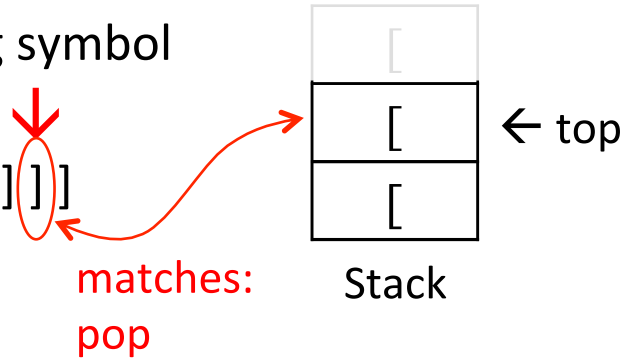

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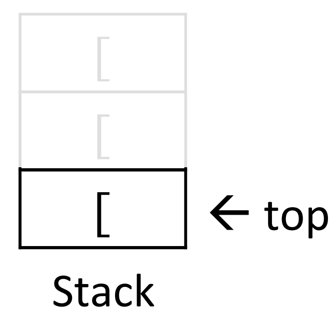

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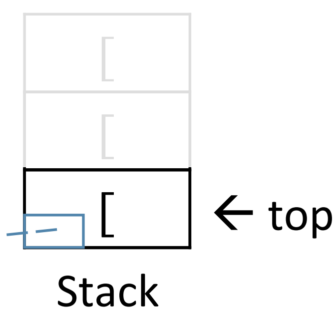

Elaboration: Have each stack element keep track of the position of its [

EXERCISE

Write a function `balanced(s)` *that returns True if the string* `s` *is balanced with respect to '[' and ']' and False otherwise.*

```
class Stack:
    def __init__(self):
        self._items = []

    def push(self, item):
        self._items.append(item)
    def pop(self):
        return self._items.pop()
    def is_empty():
        return self._items == []
```

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CSc 120: Phylogenetic Trees

This problem brings together many different programmin
and trees. It is one of the most technically challenging pr

Background

An evolutionary tree (also called a phylogenetic tree) is a

This program involves writing code to construct phylogen
example, since programs are sequences of characters, we

Expected Behavior

Write a Python program, in a file **phylo.py**, that behaves

1. *Read in the input parameters*:
   - Read in the name of an input file using **input**
   - Read in an integer value **N** using **input('n-gr**
2. *Read in the input file*. The file format is specified un

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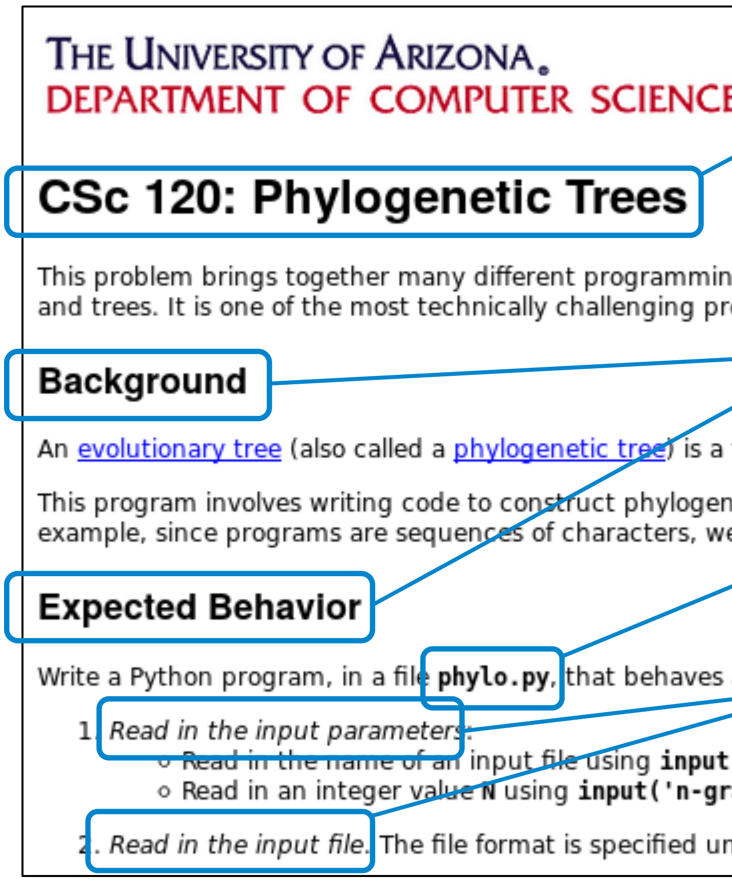

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Display considerations

main header: large font, bold

secondary header: medium font, bold

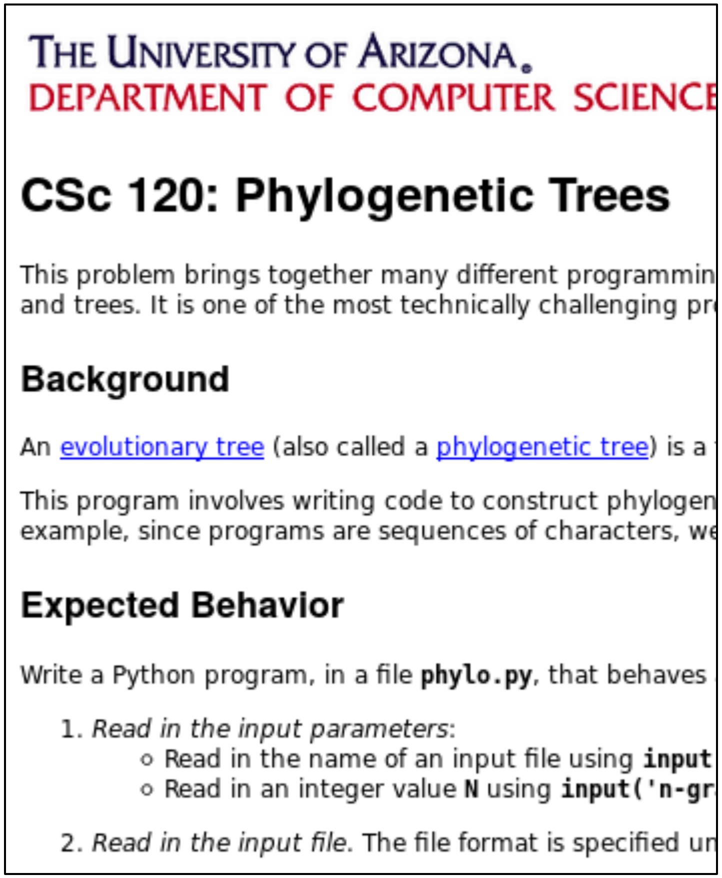
bold font

italics font

Question: how does the web browser figure out how much a given display format should include?
E.g., which text is in boldface, how much is in italics, etc.

Related: Displaying web pages

Web page



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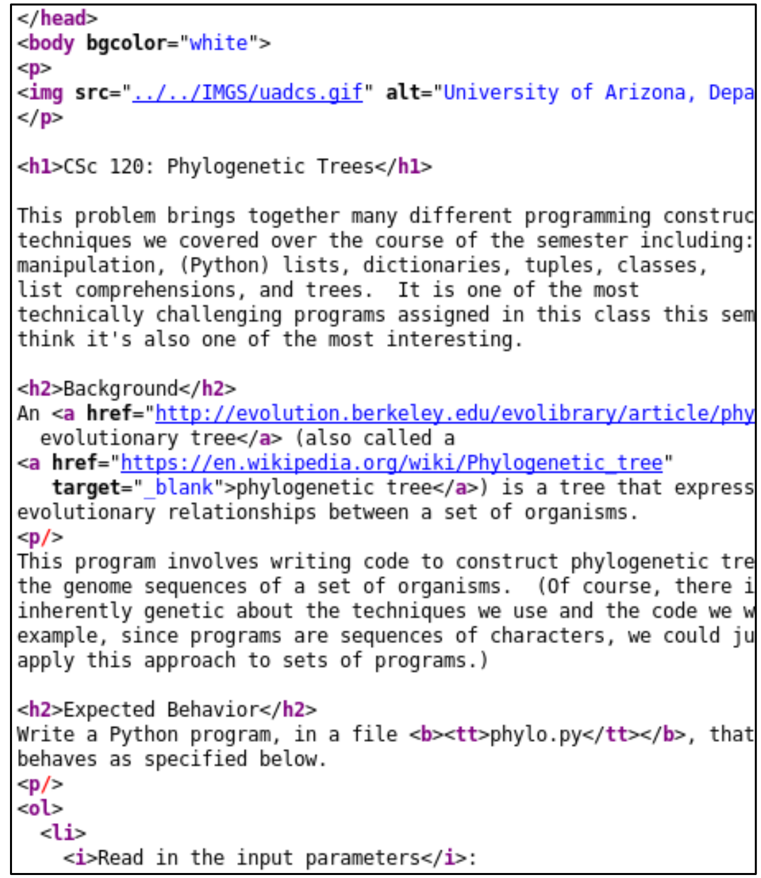

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</head>
<body bgcolor="white">
<p>
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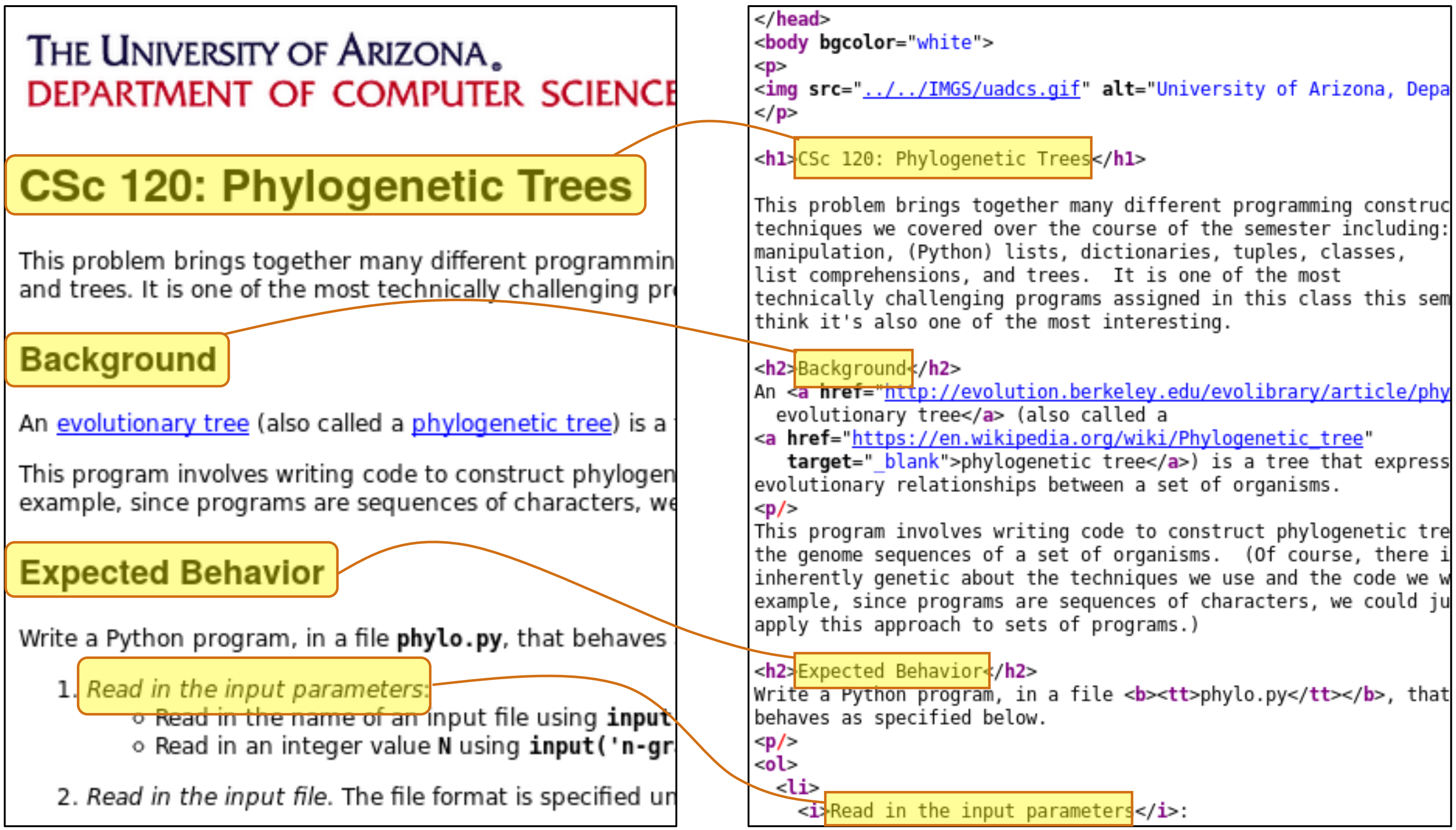
Web page | **HTML source**



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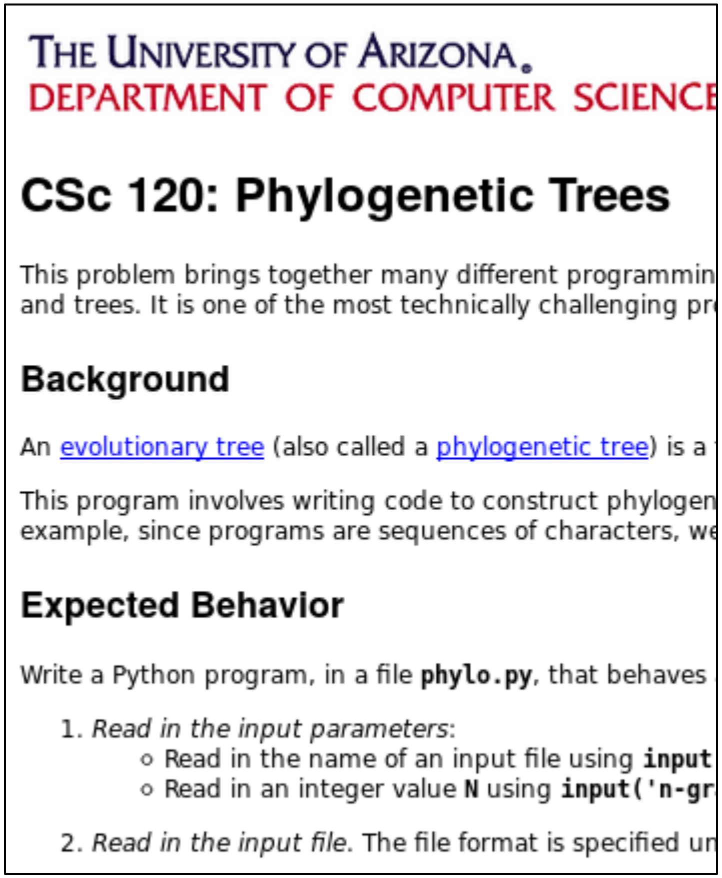
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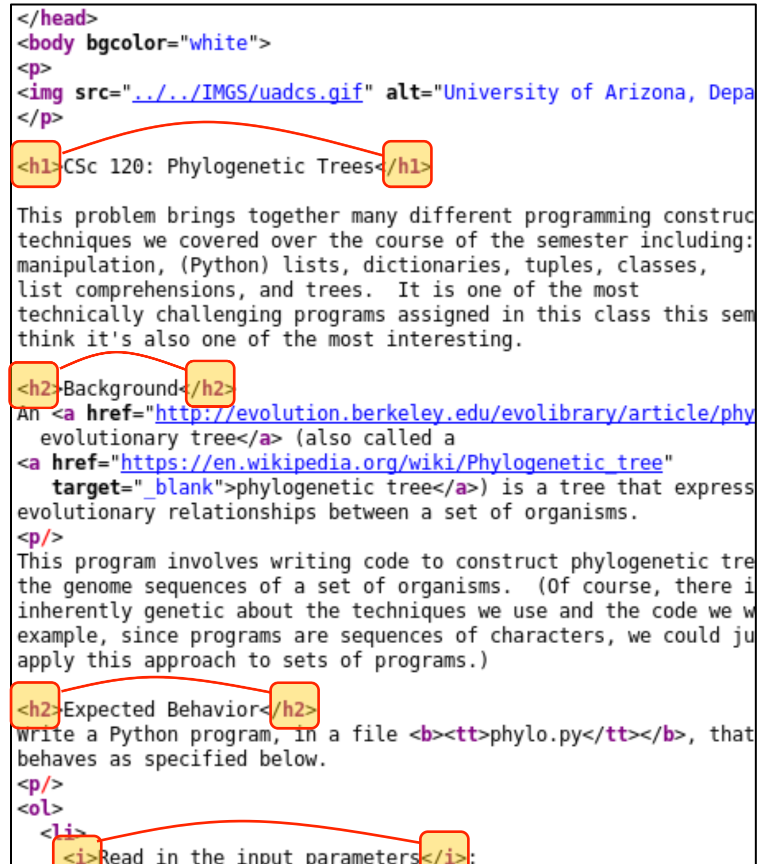
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"tags"

<h1> : "open header 1"

</h1> : "close header 1"

<h2> : "open header 2"

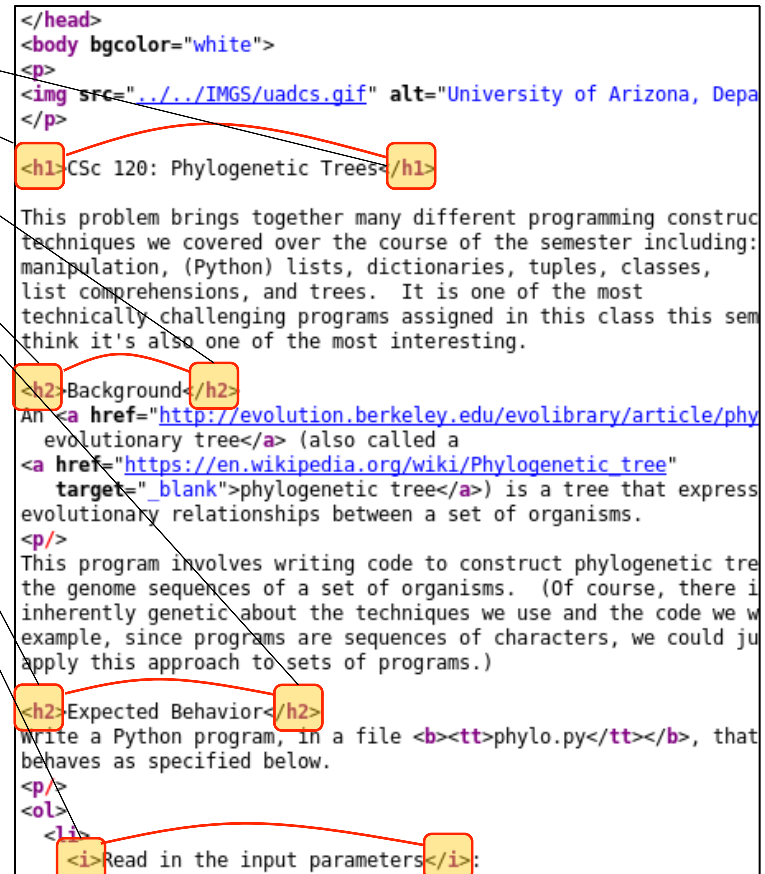
</h2> : "close header 2"

<i> : "open italics"

</i> : "close italics"

…

HTML source



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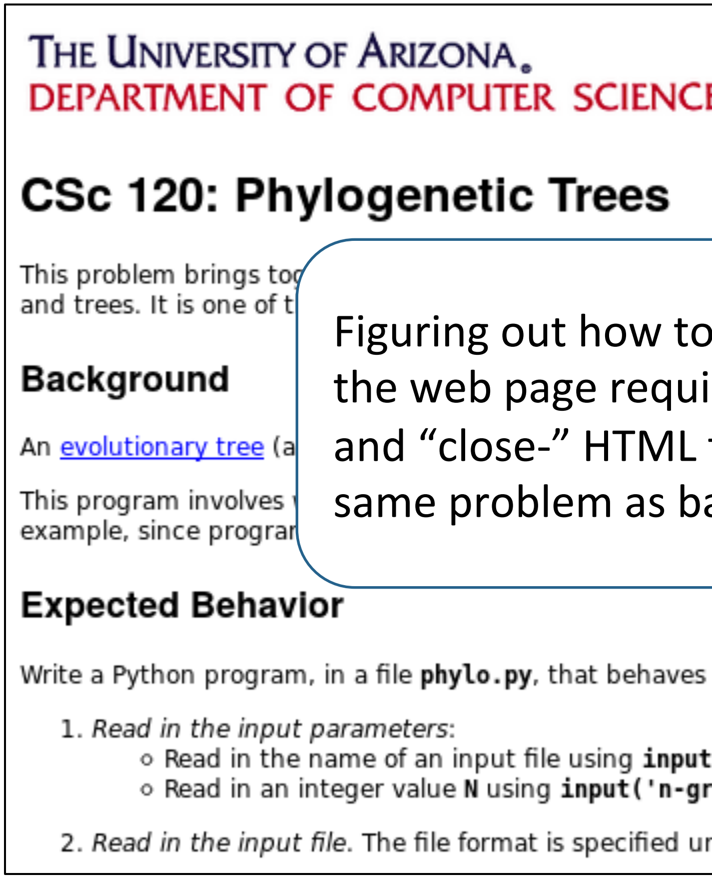
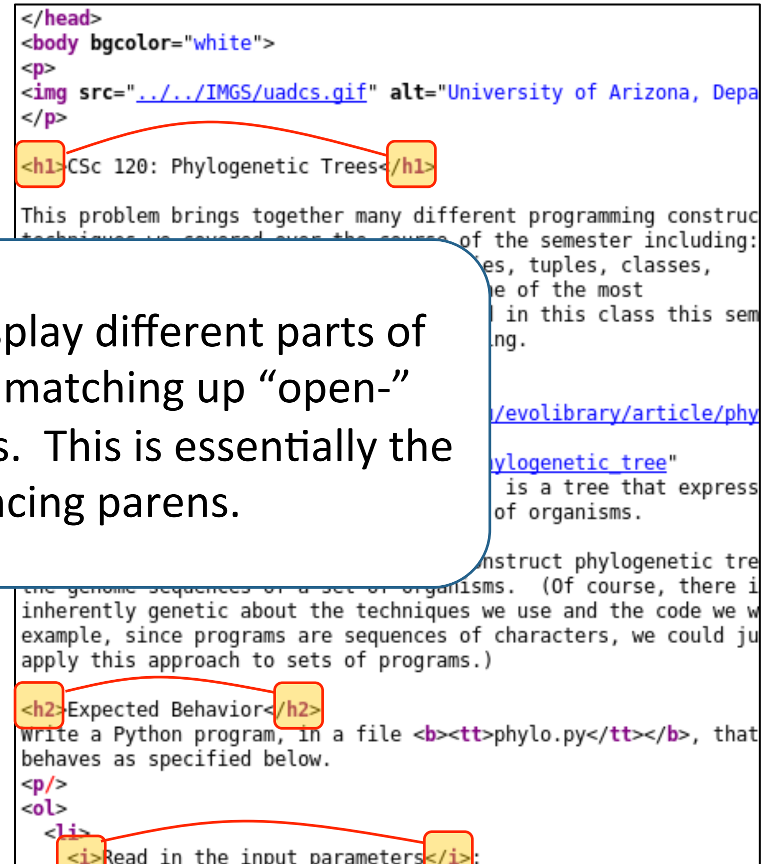


HTML source



Figuring out how to display different parts of the web page requires matching up “open-” and “close-” HTML tags. This is essentially the same problem as balancing parens.

EXERCISE

```
>>> s1 = Stack()
>>> s1.push(4)
>>> s1.push(17)
>>> s2 = Stack()
>>> s2.push(s1.pop())
>>> s2.push(s1.pop())
>>> s1.push(s2.pop())
>>> s1.push(s2.pop())
```

← *what does the stack* s1 *look like here?*

Abstract Data Types

Abstract Data Types

An *abstract data type (ADT)* describes a set of data values and associated operations that are specified independent of any particular implementation.

An ADT is a logical description of how we view the data and the operations allowed on that data.

- describes *what* the data represents
- not *how* is the data represented

The data is *encapsulated*.

Abstract Data Types

Because the data is *encapsulated* we can change the underlying implementation without affecting the way the ADT behaves.

- the logical description remains the same
- the operations remain the same

EXERCISE

Hypothetical: Python 7 has just been released and built-in lists are inefficient. In fact, all operations are $O(n^2)$.

Avoid these inefficiencies by implementing the Stack class using LinkedLists.

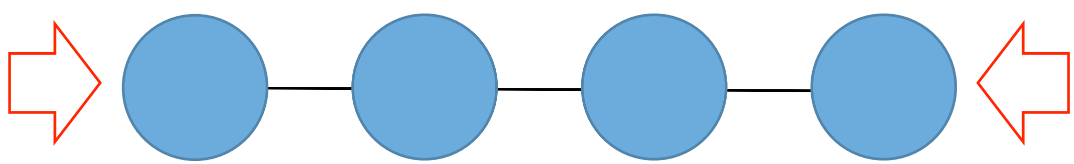
queues

A Queue ADT

A *queue* is a linear data structure where insertions and deletions happen at different ends

- insertions happen at one end (the queue's "back“, or “tail”)
- deletions happen at the other end (the queue's "front“, or “head”)

insertions occur at this end (tail)



deletions occur at this end (head)

Queues: insertion of values

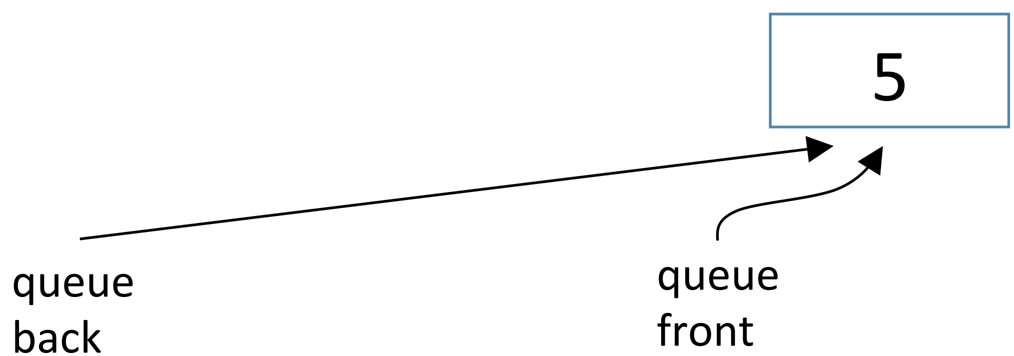
Insertion of a sequence of values into a queue: 5 17 33 9 43

queue back None

queue front None

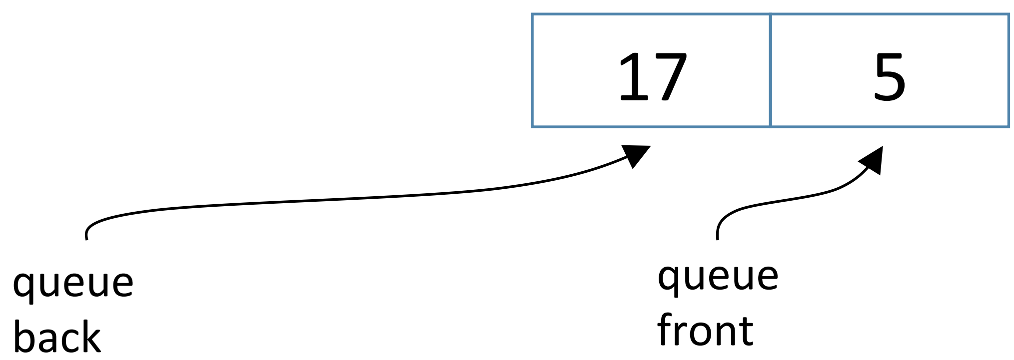
Queues: insertion of values

Insertion of a sequence of values into a queue: 5 17 33 9 43



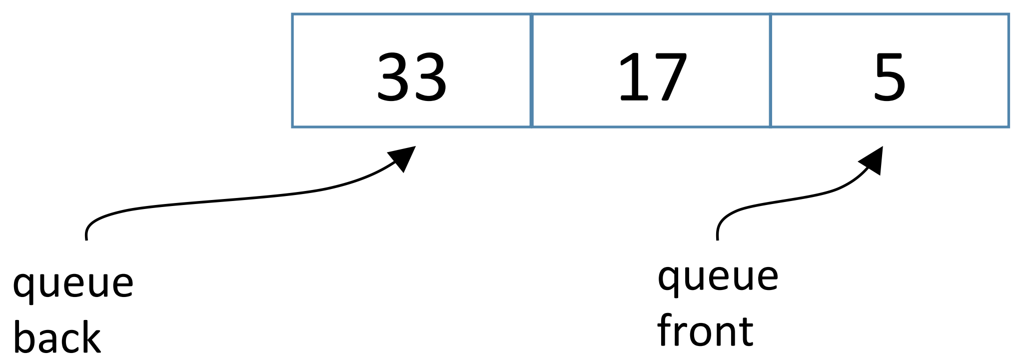
Queues: insertion of values

Insertion of a sequence of values into a queue: 5 17 33 9 43



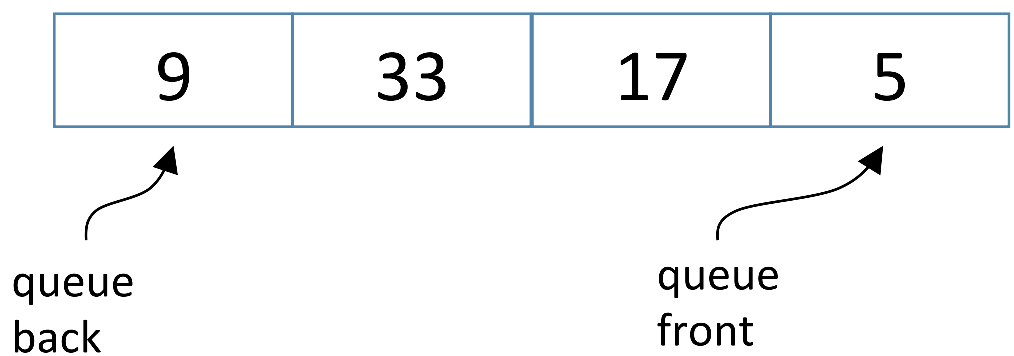
Queues: insertion of values

Insertion of a sequence of values into a queue: 5 17 33 9 43



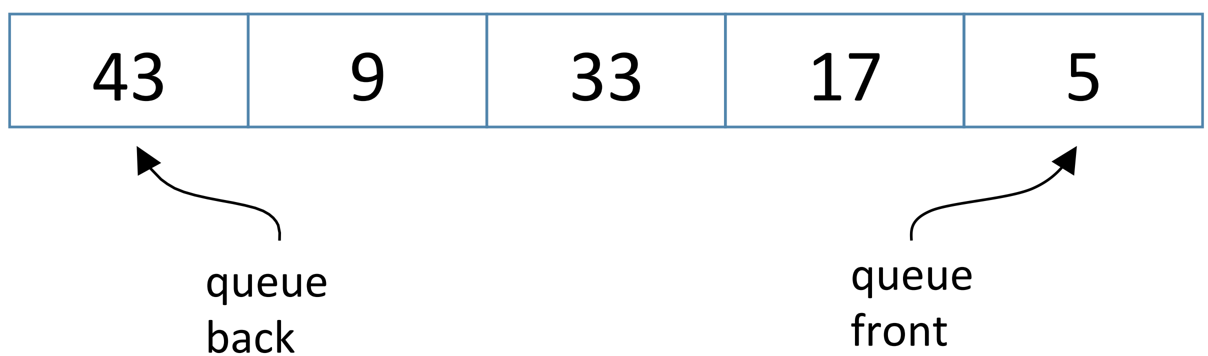
Queues: insertion of values

Insertion of a sequence of values into a queue: 5 17 33 9 43



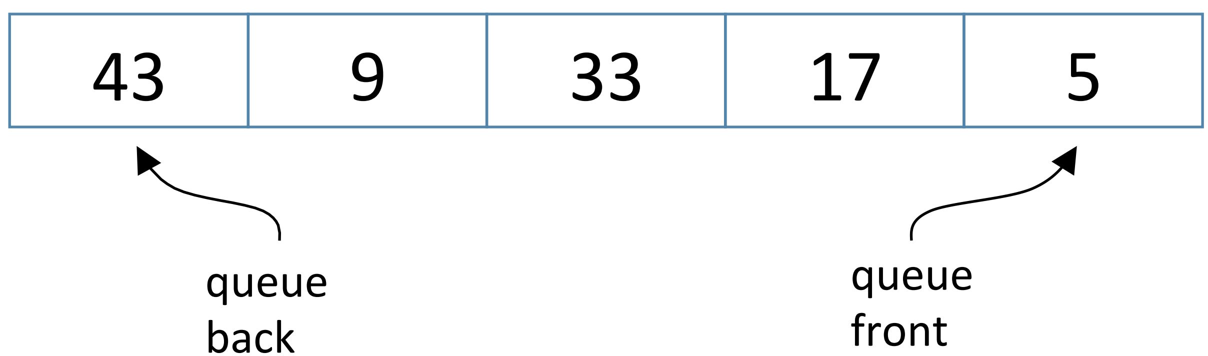
Queues: insertion of values

Insertion of a sequence of values into a queue: 5 17 33 9 43



Queues: insertion of values

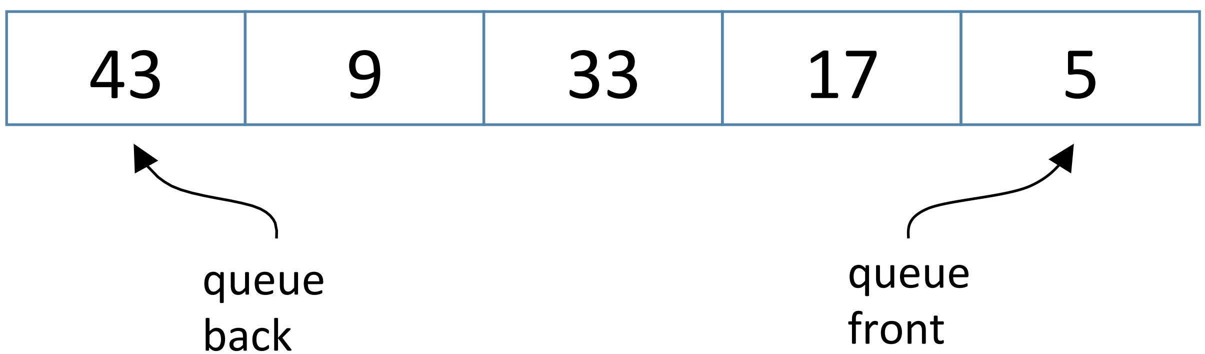
order of insertion → 5 17 33 9 43



Queues: removal of values

order of insertion → 5 17 33 9 43

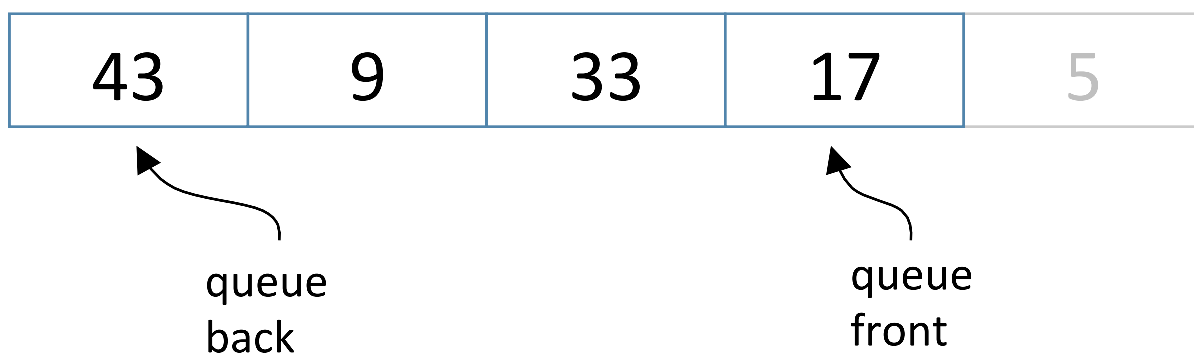
Removing values
from this queue:



Queues: removal of values

order of insertion → 5 17 33 9 43

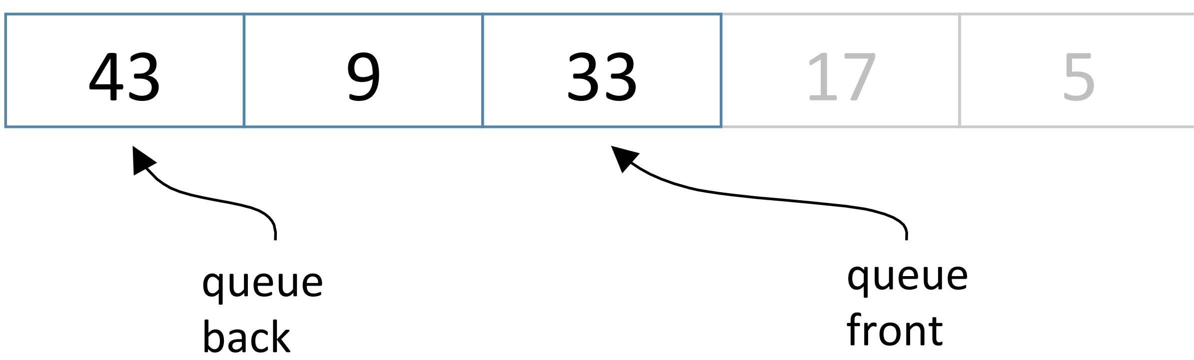
Removing values from this queue: 5



Queues: removal of values

order of insertion → 5 17 33 9 43

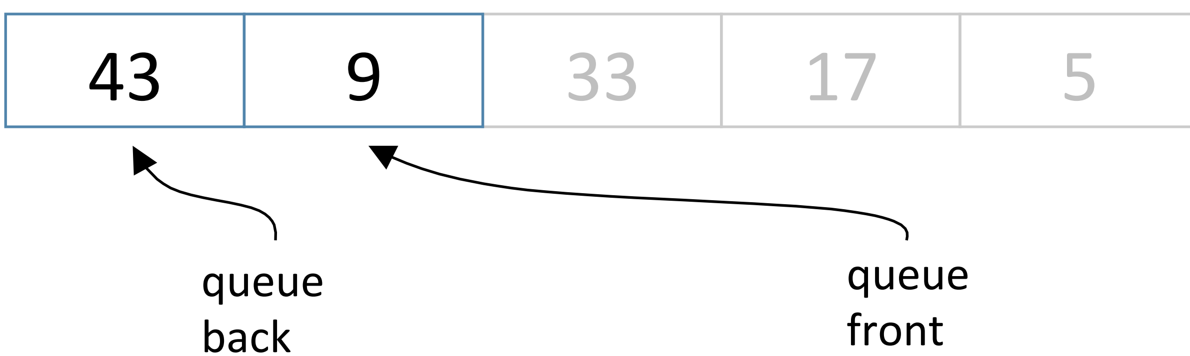
Removing values from this queue: 5 17



Queues: removal of values

order of insertion → 5 17 33 9 43

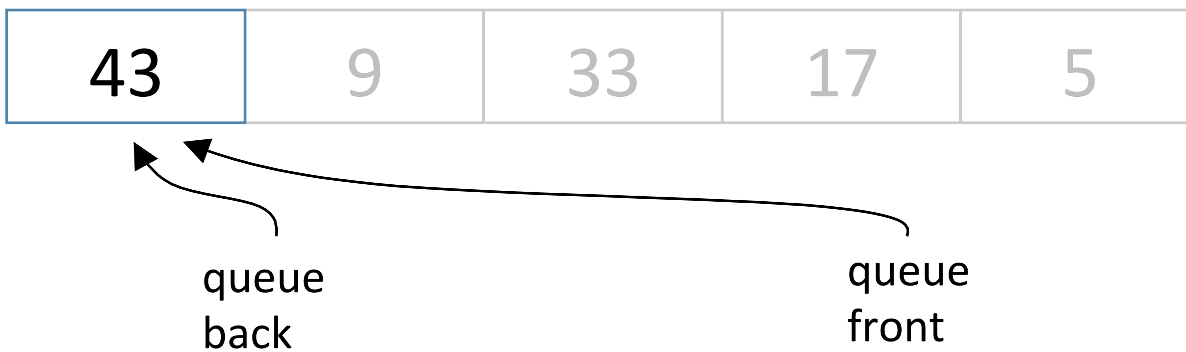
Removing values from this queue: 5 17 33



Queues: removal of values

order of insertion → 5 17 33 9 43

Removing values from this queue: 5 17 33 9



Queues: removal of values

order of insertion → 5 17 33 9 43

Removing values from this queue: 5 17 33 9 43

43	9	33	17	5

queue back None

queue front None

Queues: removal of values

order of insertion →

5 17 33 9 43

5 17 33 9 43

order of removal →

Queues: FIFO property

order of insertion →

5 17 33 9 43

5 17 33 9 43

order of removal →

values are removed in order in which they are inserted

"FIFO order"
First in, First out

Methods for a queue class

- Queue(): creates a new empty queue
- enqueue(*item*): adds *item* to the back of the queue
  - modifies the queue
  - returns nothing
- dequeue(): removes and returns the item at the front of the queue
  - returns the removed item
  - modifies the queue
- is_empty(): checks whether the queue is empty
  - returns a Boolean
- size(): returns the size of the queue
  - returns an integer

Implementing a queue class

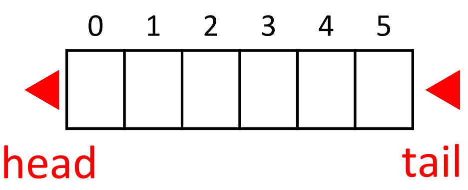
- Use a built-in list for the internal representation
  - Python lists can be added at to the front or at the end
- First implementation:
  - the head is the 0^{th} element
  - the tail is the nth element
- Second implementation
  - the head is the nth element
  - the tail is the 0^{th} element

Implementing a Queue class I

```
class Queue:
    # the front of the queue is the first item in the list
    def __init__(self):
        self._items = []

    def enqueue(self, item):
        self._items.append(item)

    def dequeue(self):
        return self._items.pop(0)
```



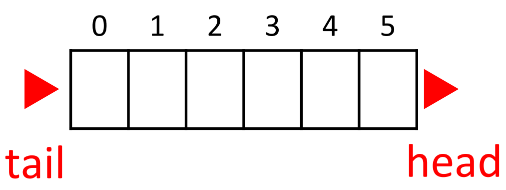
removes and returns item 0 from the list

Implementing a Queue class II

```
class Queue:
    # the front of the queue is the last item in the list
    def __init__(self):
        self._items = []

    def enqueue(self, item):
        self._items.insert(0, item)

    def dequeue(self):
        return self._items.pop()
```



removes and returns the last item in the list

EXERCISE

```
>>> q = Queue()
>>> q.enqueue(4)
>>> q.enqueue(17)
>>> x = q.dequeue()
>>> q.enqueue(5)
>>> y = q.dequeue()
```

← *what are the values of* x *and* y*?*

EXERCISE

```
>>> q = Queue()
>>> q.enqueue(4)
>>> q.enqueue(17)
>>> x = q.dequeue()
>>> y = q.dequeue()
>>> q.enqueue(y)
>>> q.enqueue(x)
>>> q.enqueue(y)
```

← *what does the queue* q *look like here?*

queues: applications

Application 1: Simulation

- Typical applications simulate problems that require data to be managed in a FIFO manner
  - Hot potato
    - Kids stand in a circle and pass a "hot potato" around until told to stop. The person holding the potato is taken out of the circle. The process is repeated until only one person remains.
  - Generalized: Given n elements, eliminate every kth element repeatedly until only 1 element is left. What was the original position of the remaining element?

- Use a *simulation* to determine which element remains.

EXERCISE

Problem: Given n elements, eliminate every kth element repeatedly until only 1 element is left. What was the original position of the remaining element?

use a queue to simulate the circle

n is the number of elements to put into the queue

while there is more than one element in the queue

eliminate every kth element

What operations take an element from the front of the queue and place it at the back of the queue?

General solution for k=2

- Given n elements, eliminate every kth element repeatedly until only 1 element is left. What was the original position of the remaining element?
- When k = 2, the original position can be derived from the binary representation of n.

  Take the first digit of the binary representation.

  Move it to the end

  The result is the original position.

Ex: n = 41, k=2

In binary

n = 101001

Therefore, the original position (in binary) is

010011

and 010011 = $2^4 + 2^1 + 2^0 = 19$

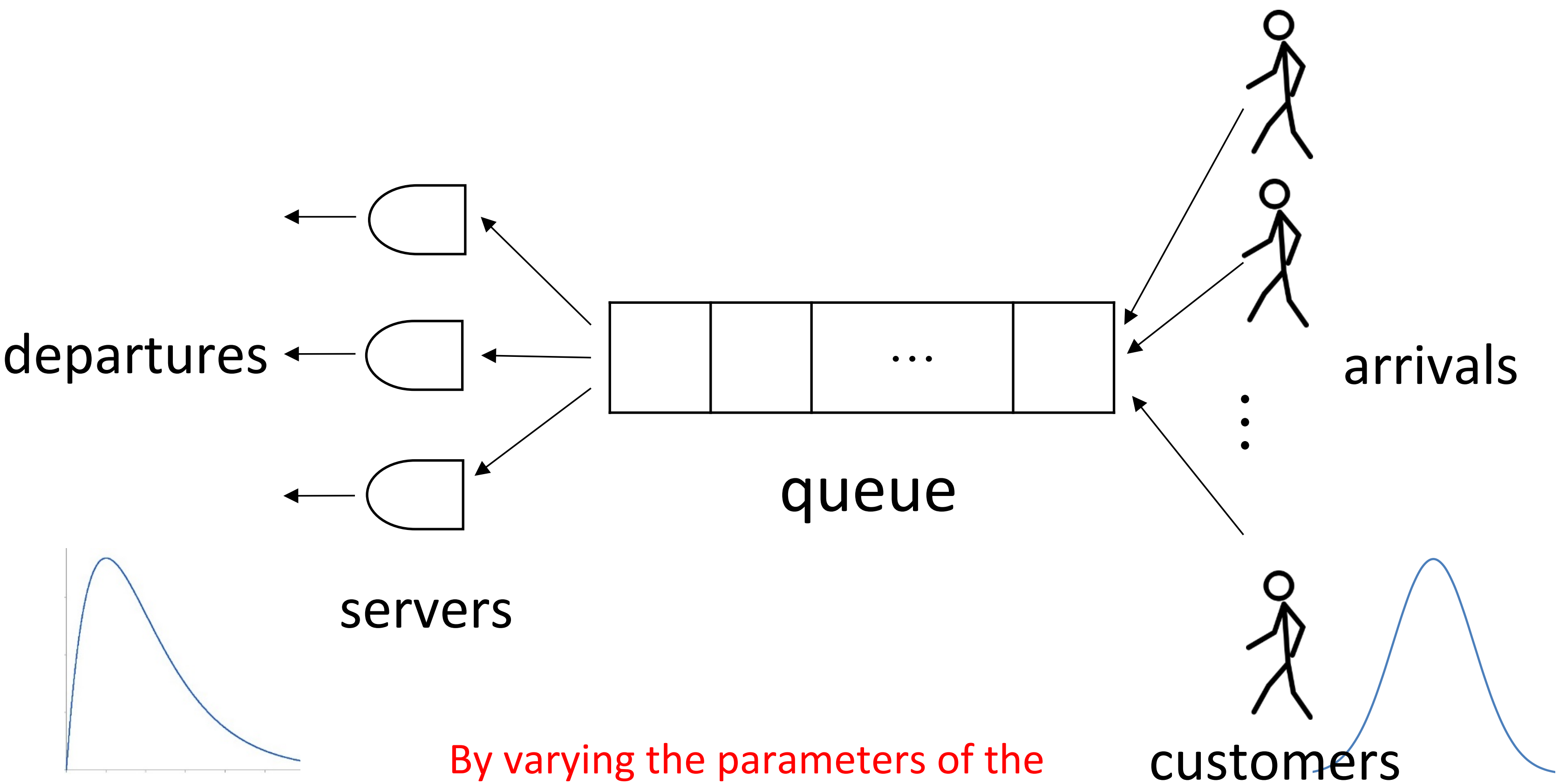
https://en.wikipedia.org/wiki/Josephus_problem#CITEREFDowdyMays1989

Application 2 : Simulation

- Suppose we are opening a grocery store. How many checkout lines should we put in?
  - too few $\Rightarrow$ long wait times, unhappy customers
  - too many $\Rightarrow$ wasted money, space

- Use *simulations* of the checkout process to guide the decision
  - study existing stores to figure out typical shopping and checkout times
  - estimate no. of customers expected at the new location
  - run simulations to determine customer wait time and checkout line utilization under different scenarios

Discrete event simulation



departure rate distribution

arrival rate distribution

By varying the parameters of the simulation (arrival and departure rates, no. of servers) we can try out different scenarios

Summary

- Stacks and queues are abstract data types (ADTs)
  - similar in that they are both *linear* data structures
  - items can be thought of as arranged in a line
  - each item has a position and a before/after relationship with the other items
- They differ in the way items are added and removed
  - stacks: items added and removed at one end
    - results in LIFO behavior
  - queues: items added at one end, removed at the other
    - results in FIFO behavior
- They find a wide range of applications in computer science

A Deque ADT

A *deque* is a linear data structure where insertions and deletions happen at both ends