CSc 120

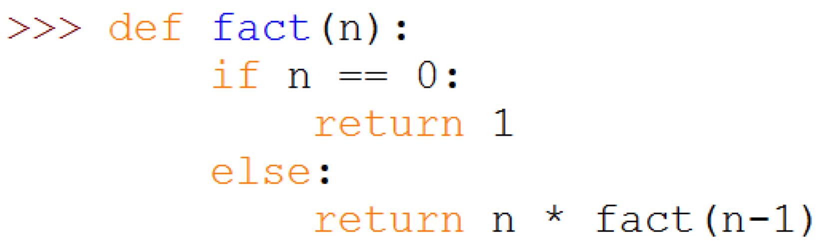
Introduction to Computer Programming II

Adapted from slides
by Dr. Saumya Debray

16: Stacks, Recursion, Search

the runtime stack

How recursion works



```
>>> def fact(n):
        if n == 0:
            return 1
        else:
            return n * fact(n-1)
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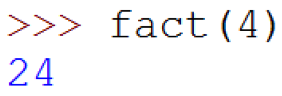

```
>>> fact(4)
24
```

How recursion works

```
>>> def fact(n):
        if n == 0:
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```

We need the value of n both before and after the recursive call

∴ its value has to be saved somewhere

"somewhere" ≡ "stack frame"

How recursion works

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```

Python's runtime system* maintains a stack:

- push a "frame" when a function is called
- pop the frame when the function returns

"frame" or "stack frame": a data structure that keeps track of variables in the function body, and their values, between the call to the function and its return

* "runtime system" = the code that Python executes to make everything work at runtime

How recursion works

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>>> def fact(n):
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Python's runtime system*
maintains a stack:

- push a "frame" when a function is called
- pop the frame when the function returns

sometimes called the "runtime stack"

---

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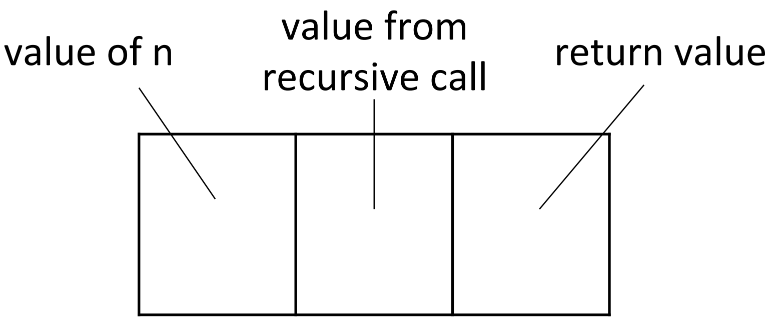

stack frame for fact()

How recursion works

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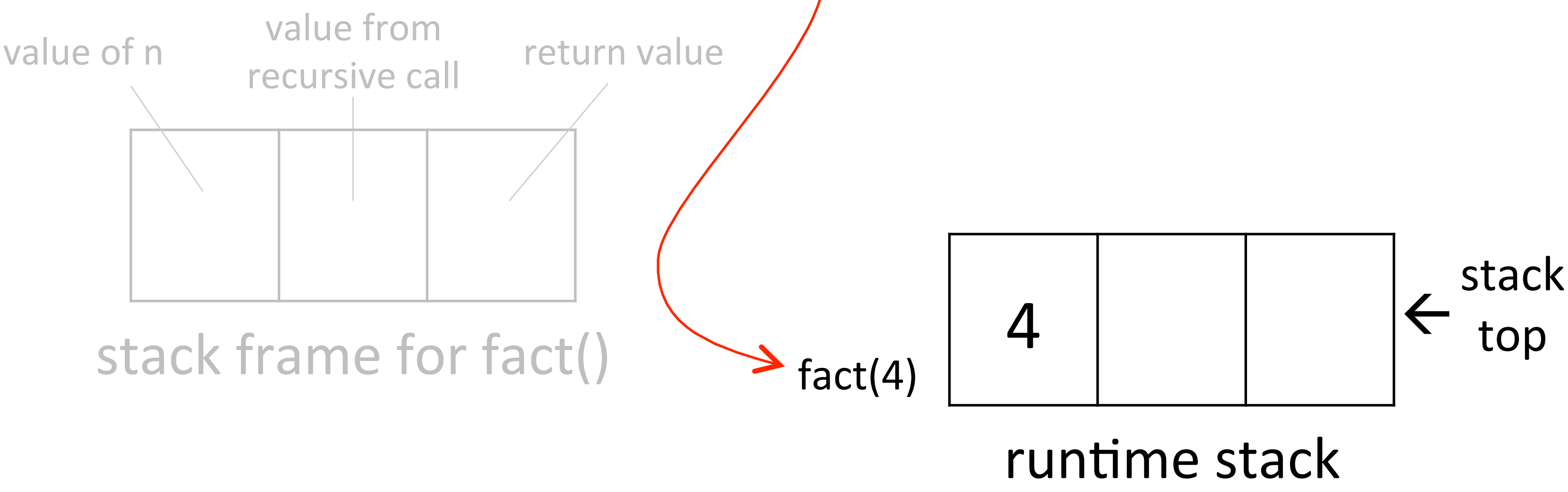

value of n

value from recursive call

return value

stack frame for fact()

fact(4)

4

stack top

runtime stack

How recursion works

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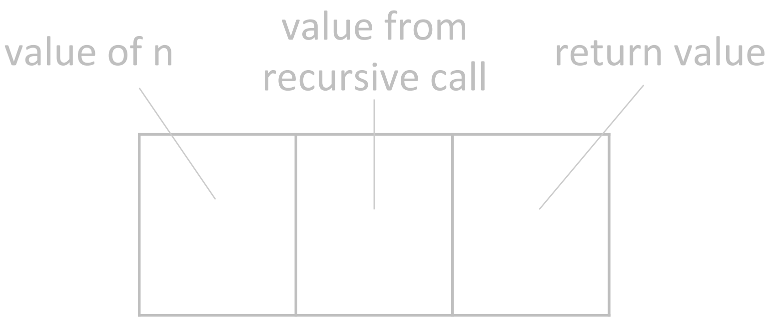
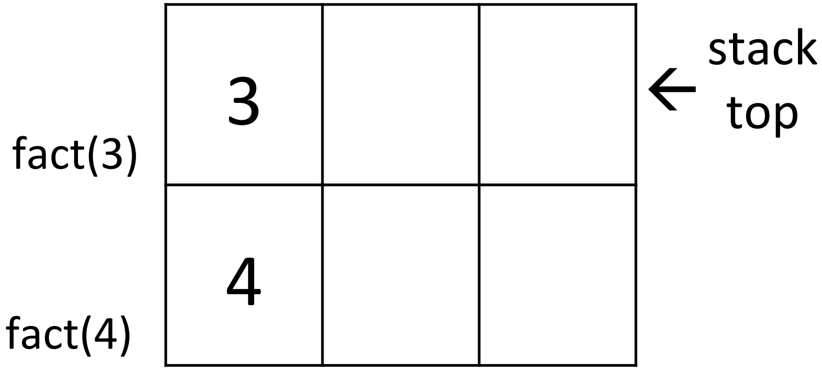


stack frame for fact()



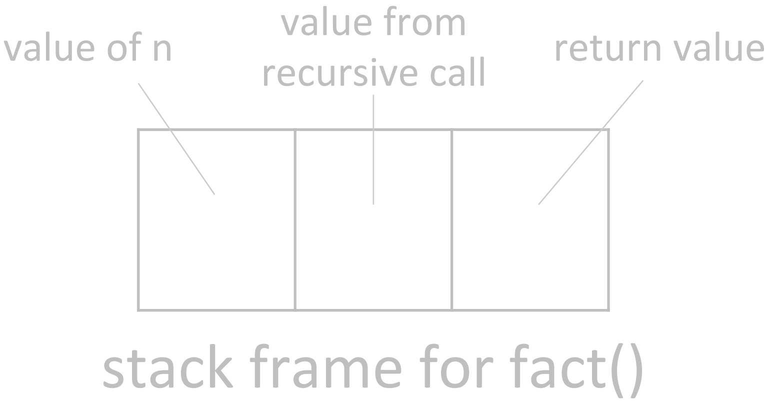
fact(3)	3		
fact(4)	4		

runtime stack

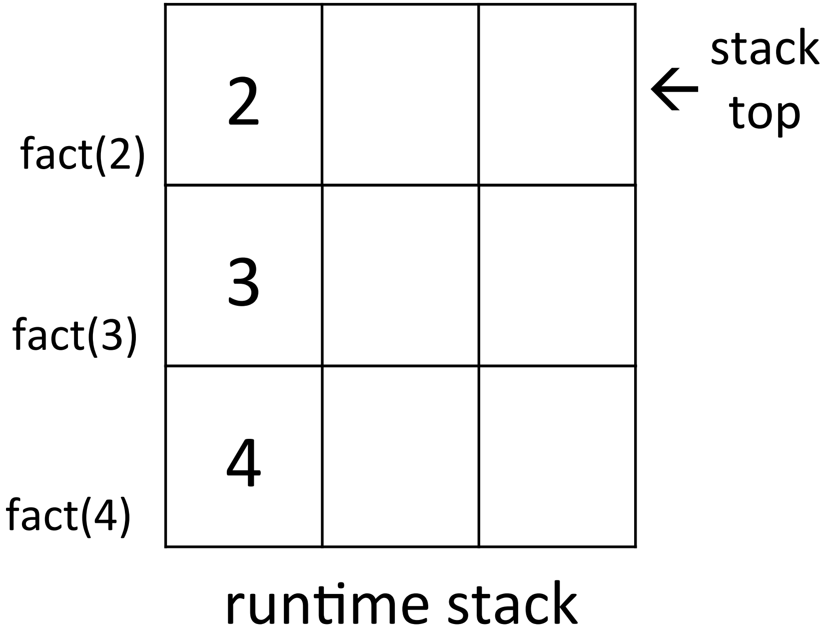
How recursion works

```
>>> def fact(n):
        if n == 0:
            return 1
        else:
            return n * fact(n-1)
```

```
>>> fact(4)
24
```



stack frame for fact()



fact(2)	2		
fact(3)	3		
fact(4)	4		

runtime stack

How recursion works

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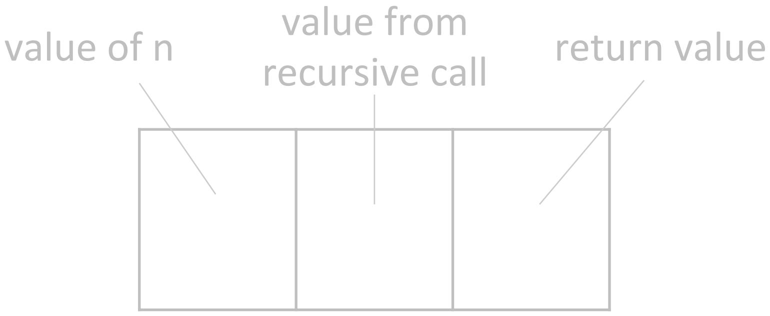
>>> def fact(n):
        if n == 0:
            return 1
        else:
            return n * fact(n-1)


>>> fact(4)
24
```

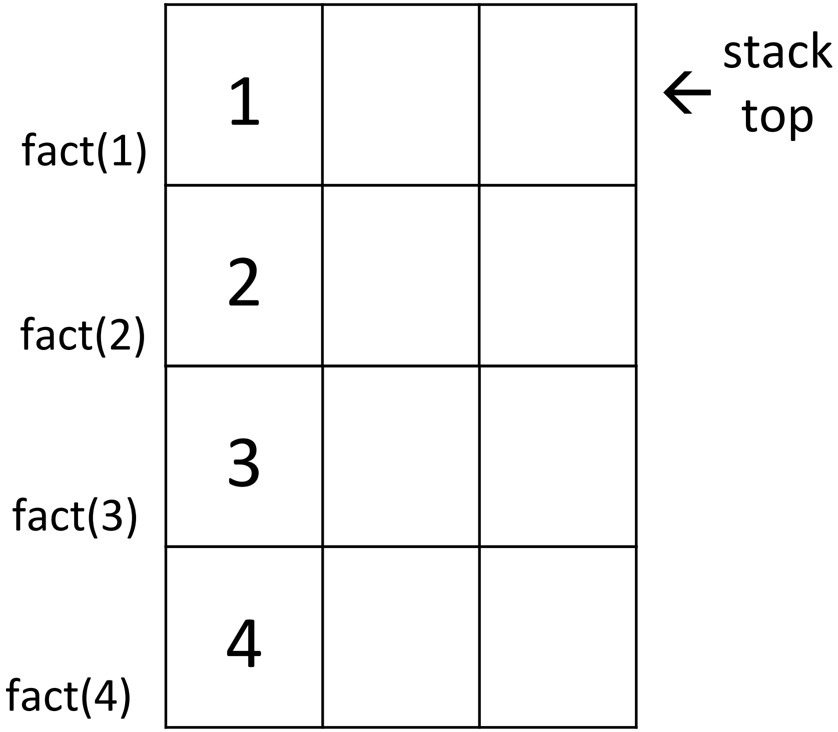


value of n

value from recursive call

return value

stack frame for fact()



stack top

fact(1)	1		
fact(2)	2		
fact(3)	3		
fact(4)	4		

runtime stack

How recursion works

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            return 1
        else:
            return n * fact(n-1)


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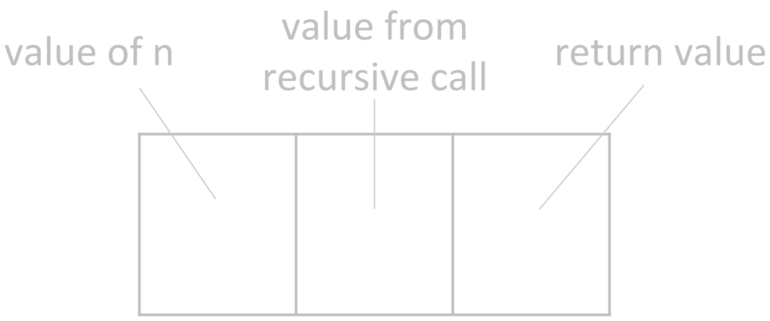
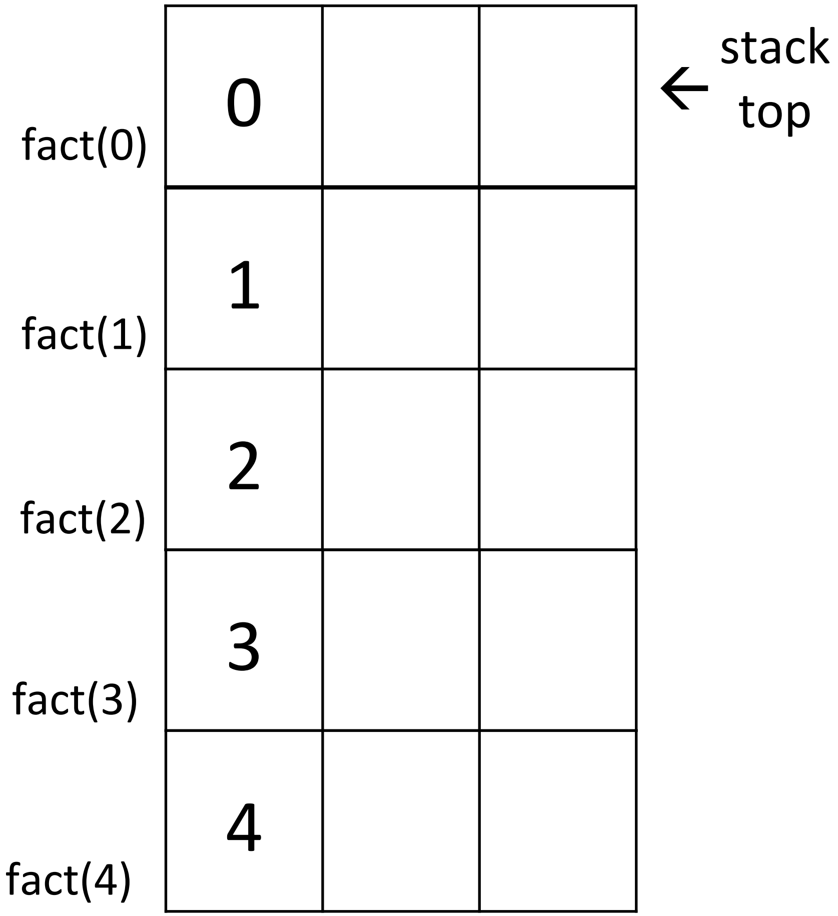
```



stack frame for fact()



fact(0)	0		
fact(1)	1		
fact(2)	2		
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runtime stack

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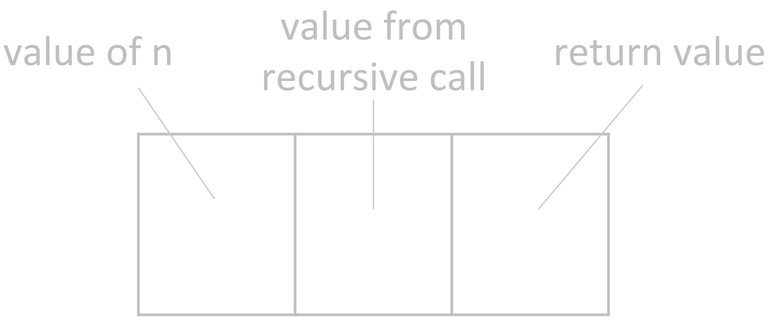
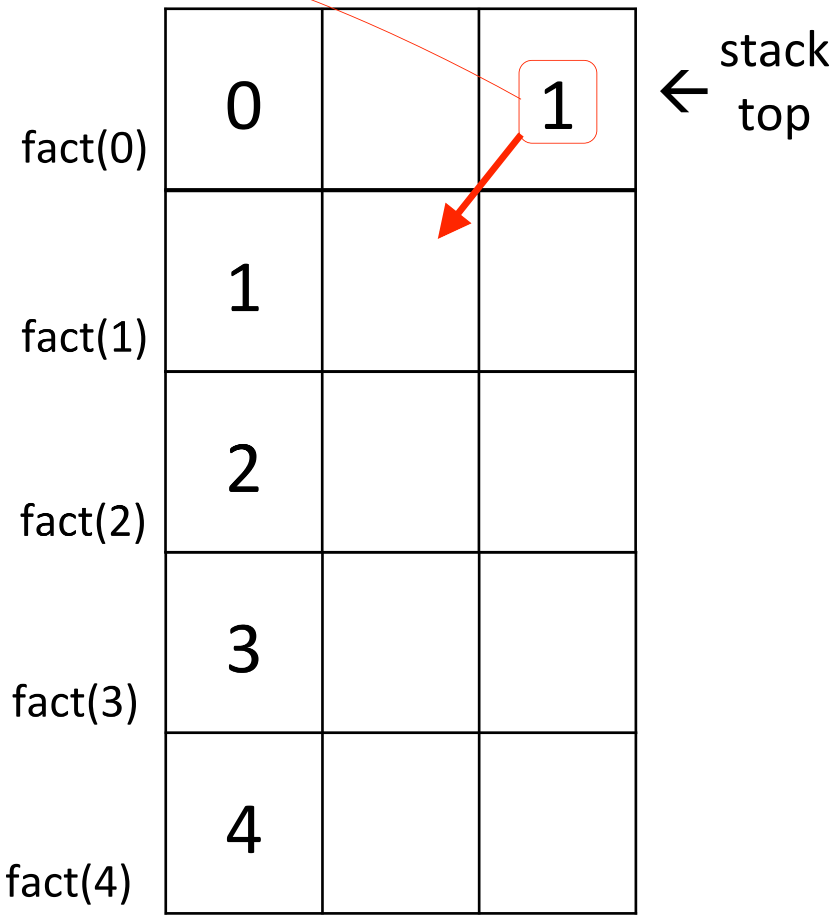


value of n

value from recursive call

return value

stack frame for fact()



stack top

fact(0)	0		1
fact(1)	1		
fact(2)	2		
fact(3)	3		
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runtime stack

How recursion works

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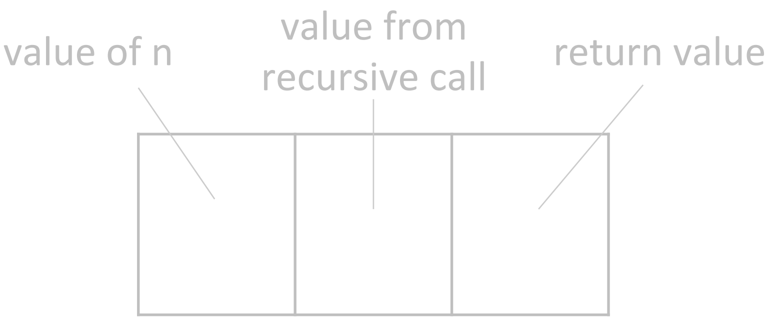
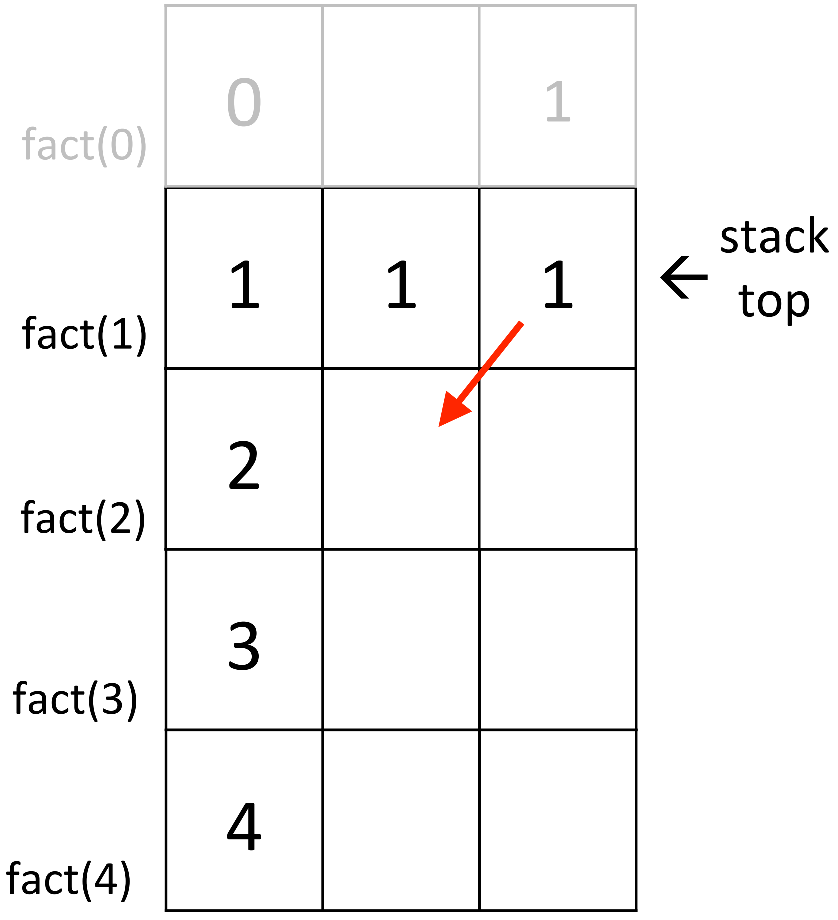


stack frame for fact()



fact(0)	0		1
fact(1)	1	1	1
fact(2)	2		
fact(3)	3		
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runtime stack

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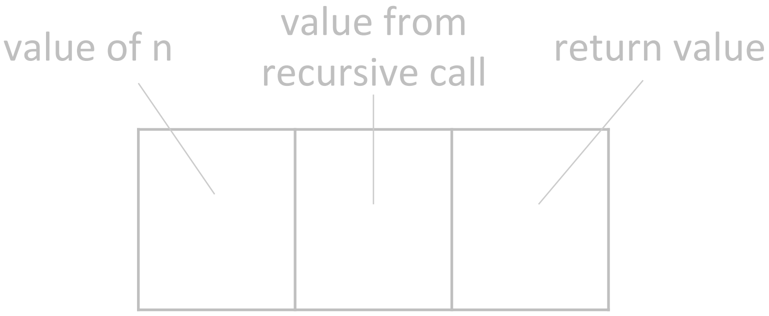

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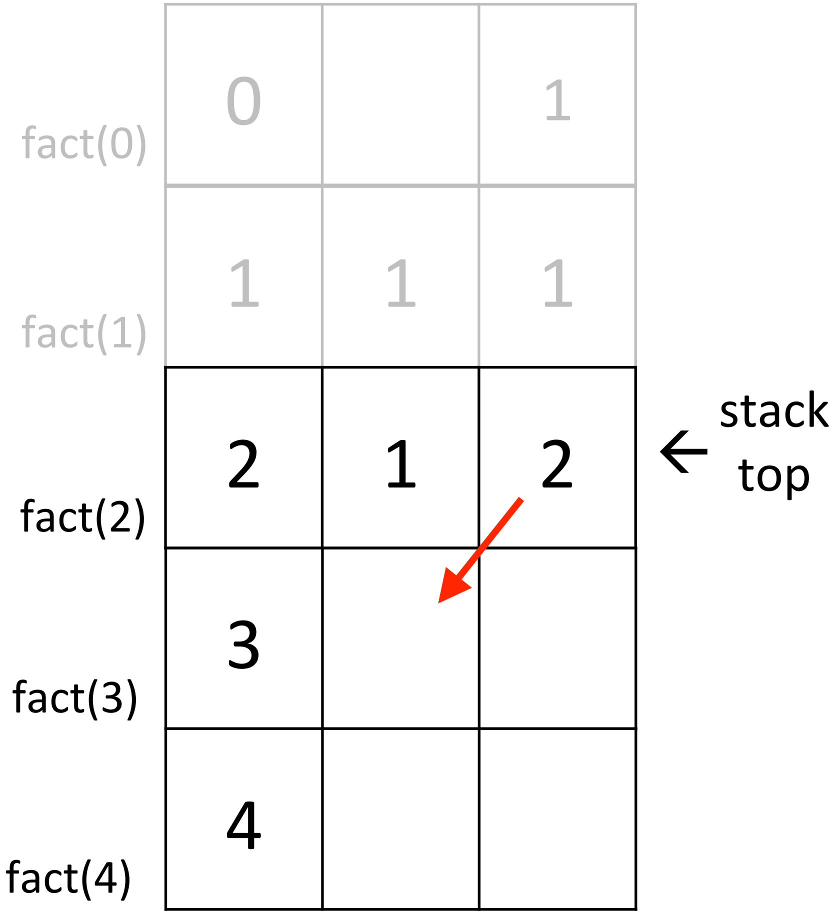

fact(0)	0		1
fact(1)	1	1	1
fact(2)	2	1	2
fact(3)	3		
fact(4)	4		

runtime stack

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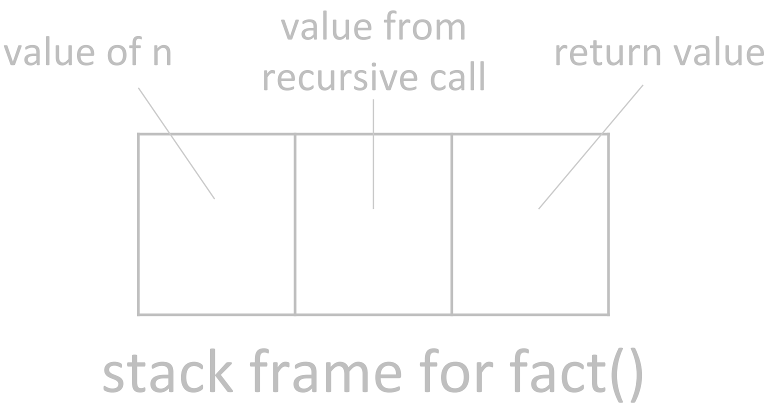

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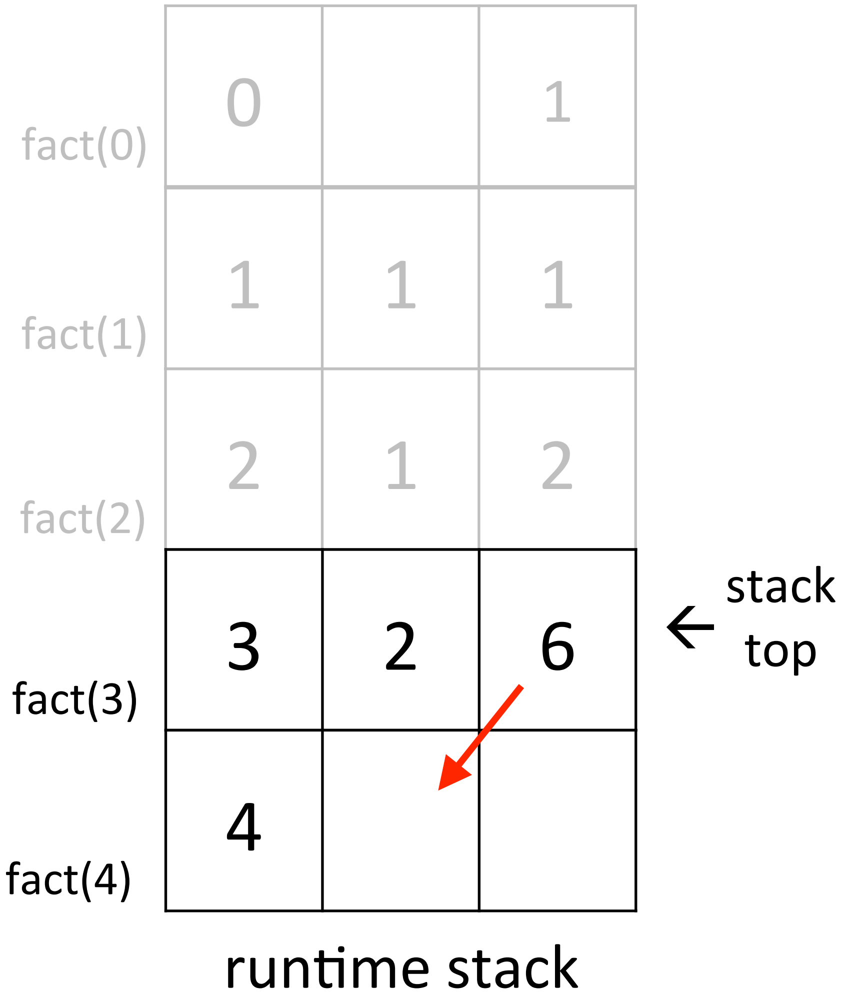

fact(0)	0		1
fact(1)	1	1	1
fact(2)	2	1	2
fact(3)	3	2	6
fact(4)	4		

← stack top

runtime stack

How recursion works

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>>> def fact(n):
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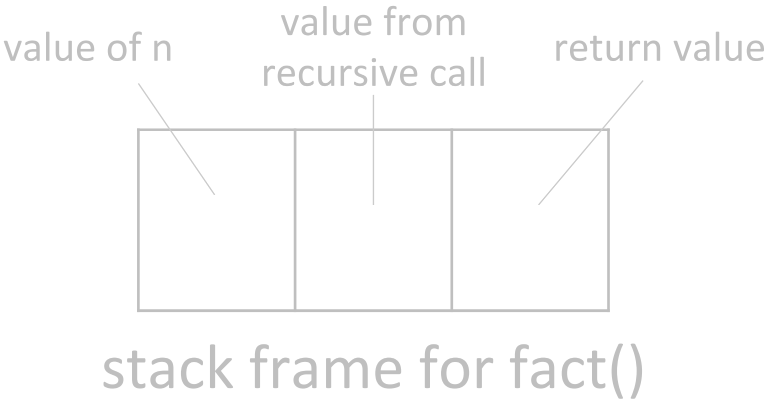

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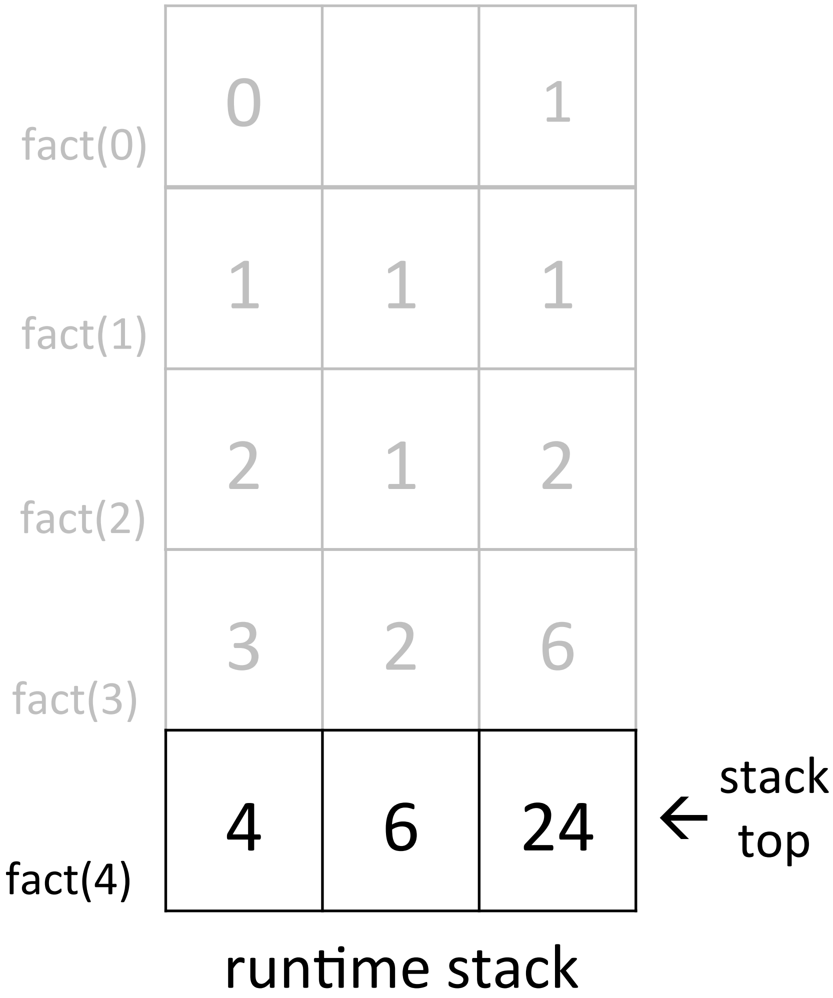

fact(0)	0		1
fact(1)	1	1	1
fact(2)	2	1	2
fact(3)	3	2	6
fact(4)	4	6	24

runtime stack

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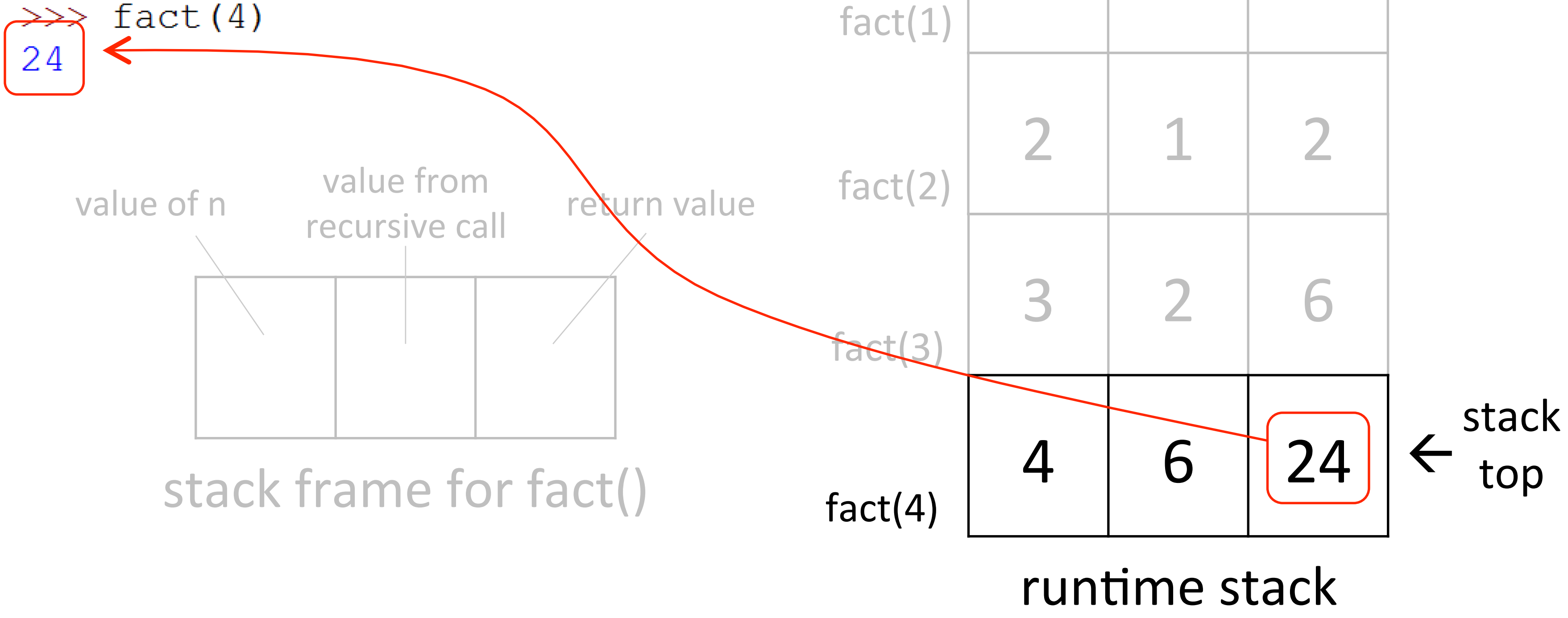
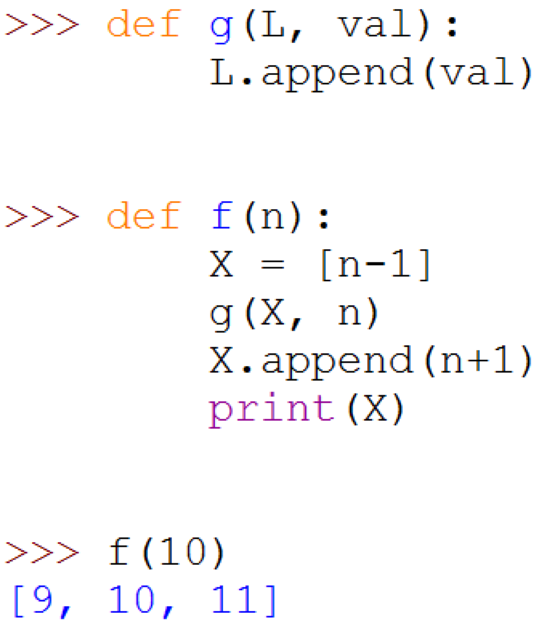


fact(0)	0		1
fact(1)	1	1	2
fact(2)	2	1	2
fact(3)	3	2	6
fact(4)	4	6	24

The runtime stack

- The use of a *runtime stack* containing *stack frames* is not specific to recursion
  - all function and method invocations use this mechanism
  - not just in Python, but other languages as well (Java, C, C++, …)

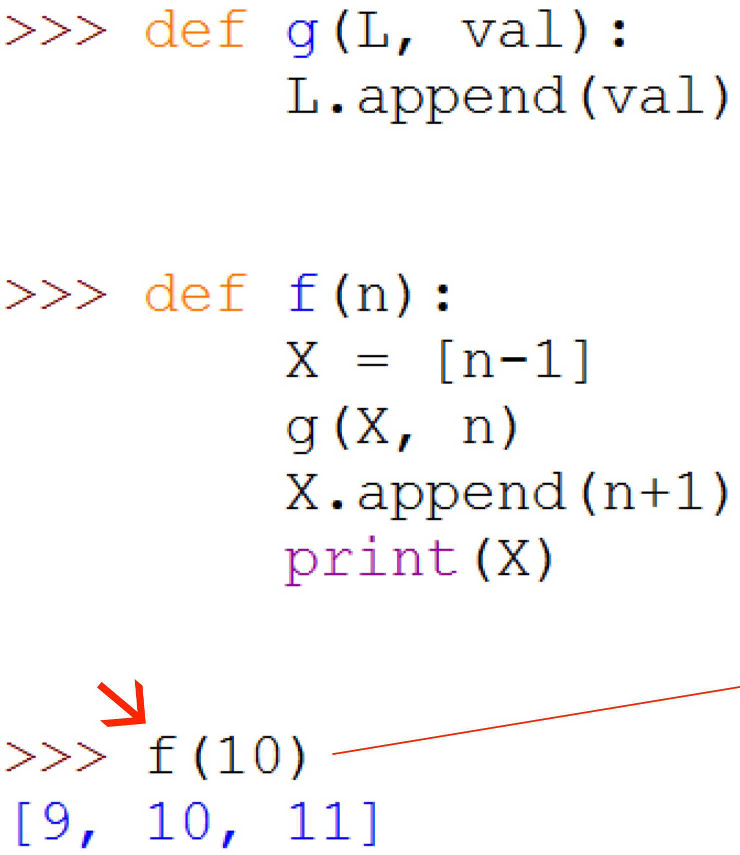


```
>>> def g(L, val):
        L.append(val)


>>> def f(n):
        X = [n-1]
        g(X, n)
        X.append(n+1)
        print(X)


>>> f(10)
[9, 10, 11]
```

The runtime stack



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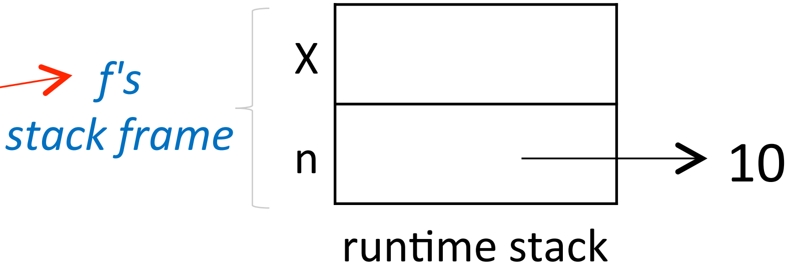

f's
stack frame

X

n

10

runtime stack

The runtime stack

```
>>> def g(L, val):
        L.append(val)


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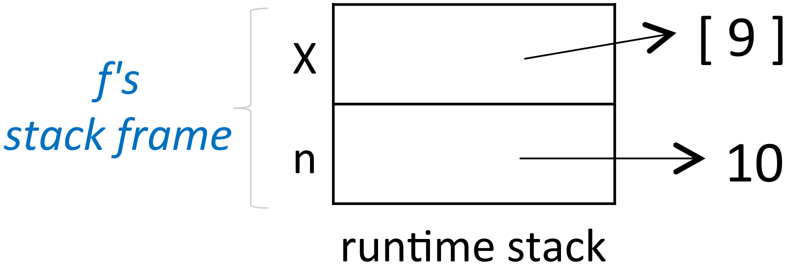


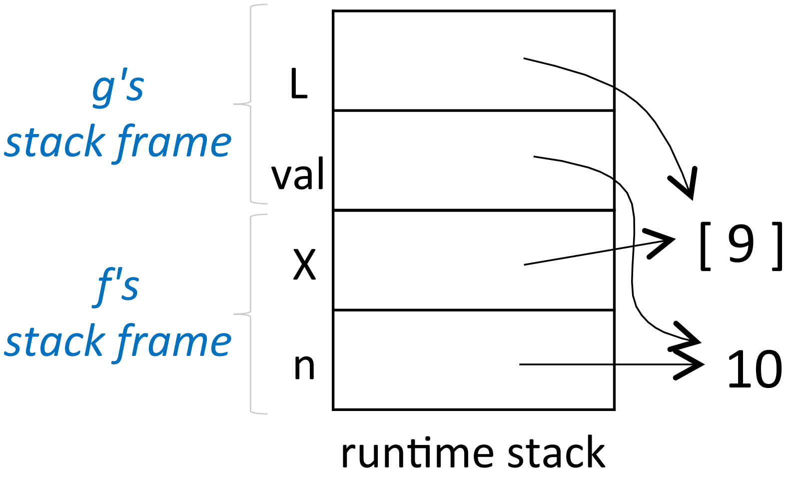
The runtime stack

```
>>> def g(L, val):
        L.append(val)
```

Argument passing: the callee is passed a reference to the argument object

```
>>> def f(n):
        X = [n-1]
    →   g(X, n)
        X.append(n+1)
        print(X)
```

```
>>> f(10)
[9, 10, 11]
```



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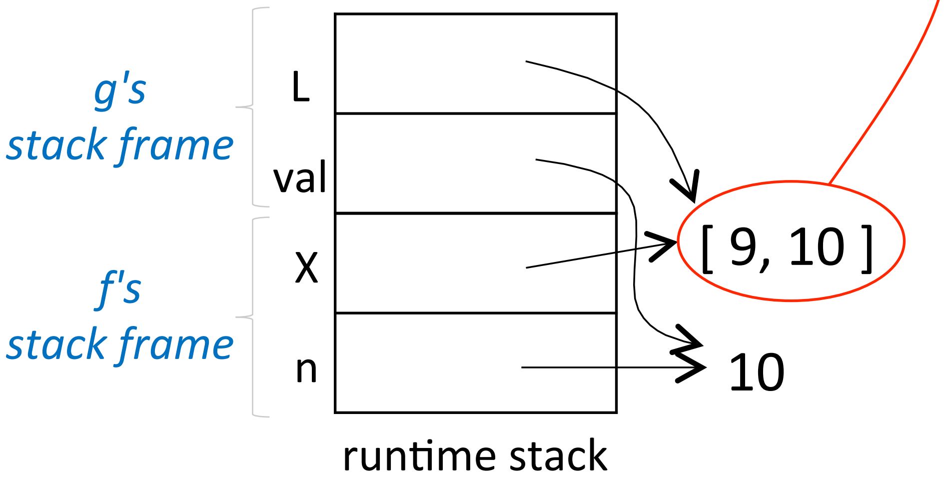
Argument passing: the callee is passed a reference to the argument object

⇒ the change made in g() is visible in f()



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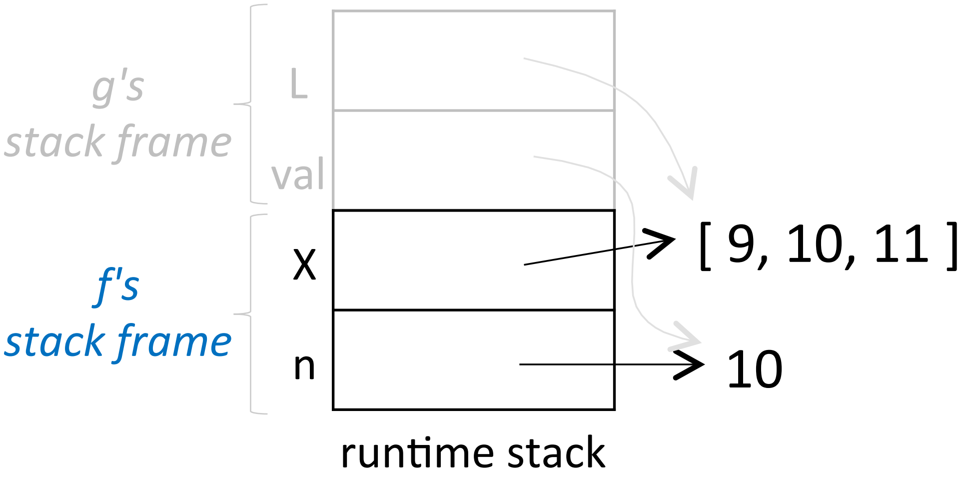

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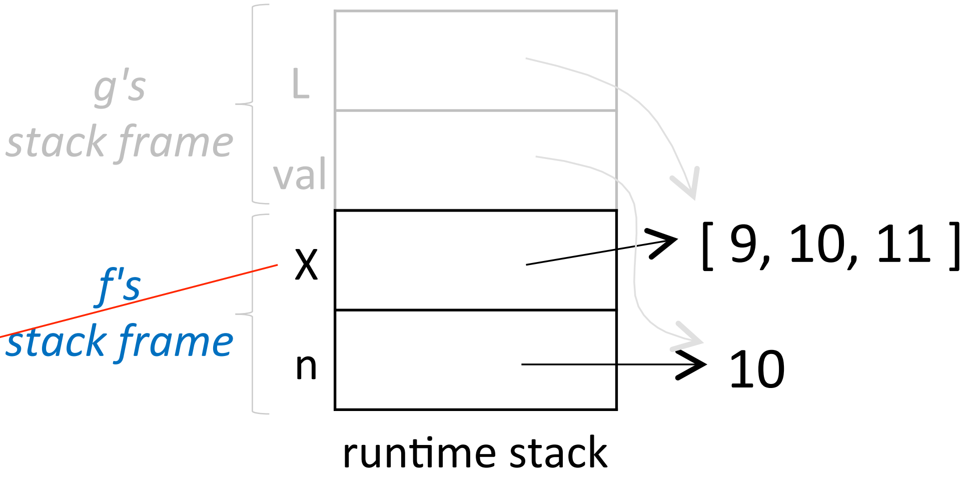


The runtime stack: summary

- *Runtime stack*: holds information about function (and method) activations
- *Stack frame* for a function:
  - holds information about the variables in the function body
  - pushed when the function is called
  - popped when it returns
- *Argument passing*: a reference to the argument value (an object) is passed
  - if the value is mutable, changes made by the callee are visible in the caller

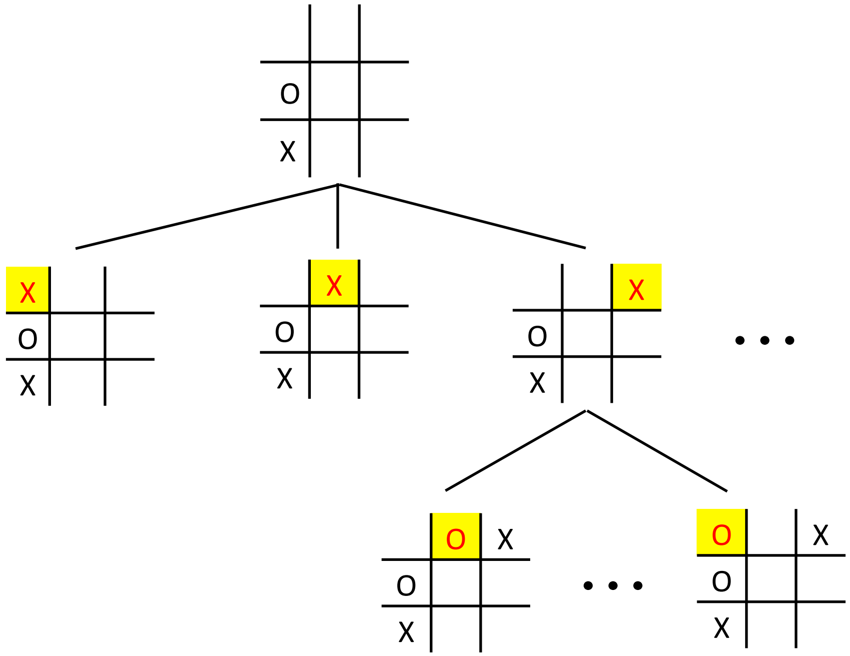
search revisited

Game tree search revisited

Recall our tic-tac-toe program

Given a starting position,

- it generates successive positions from different possible moves
- evaluates the effect of continuing play from each of these positions
- picks a move that leads to the best position after some number of turns n (n = “lookahead”)



Game tree search revisited

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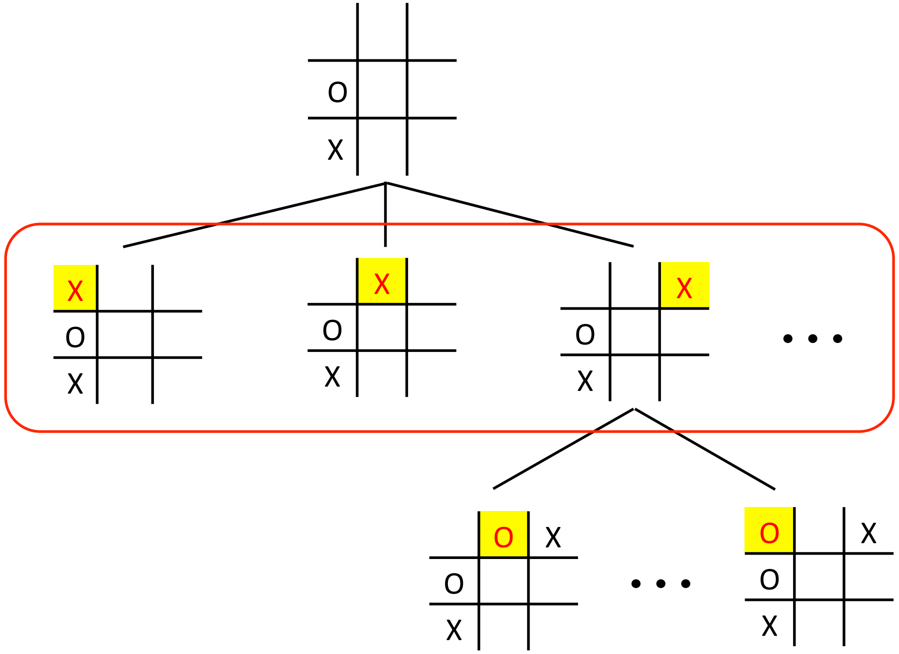


search problems: examples

Word morph

- Change one word into another by changing one letter at a time

  Examples:

  - cat → cot → cog → dog
  - head → heal → hell → hall → tall → tail

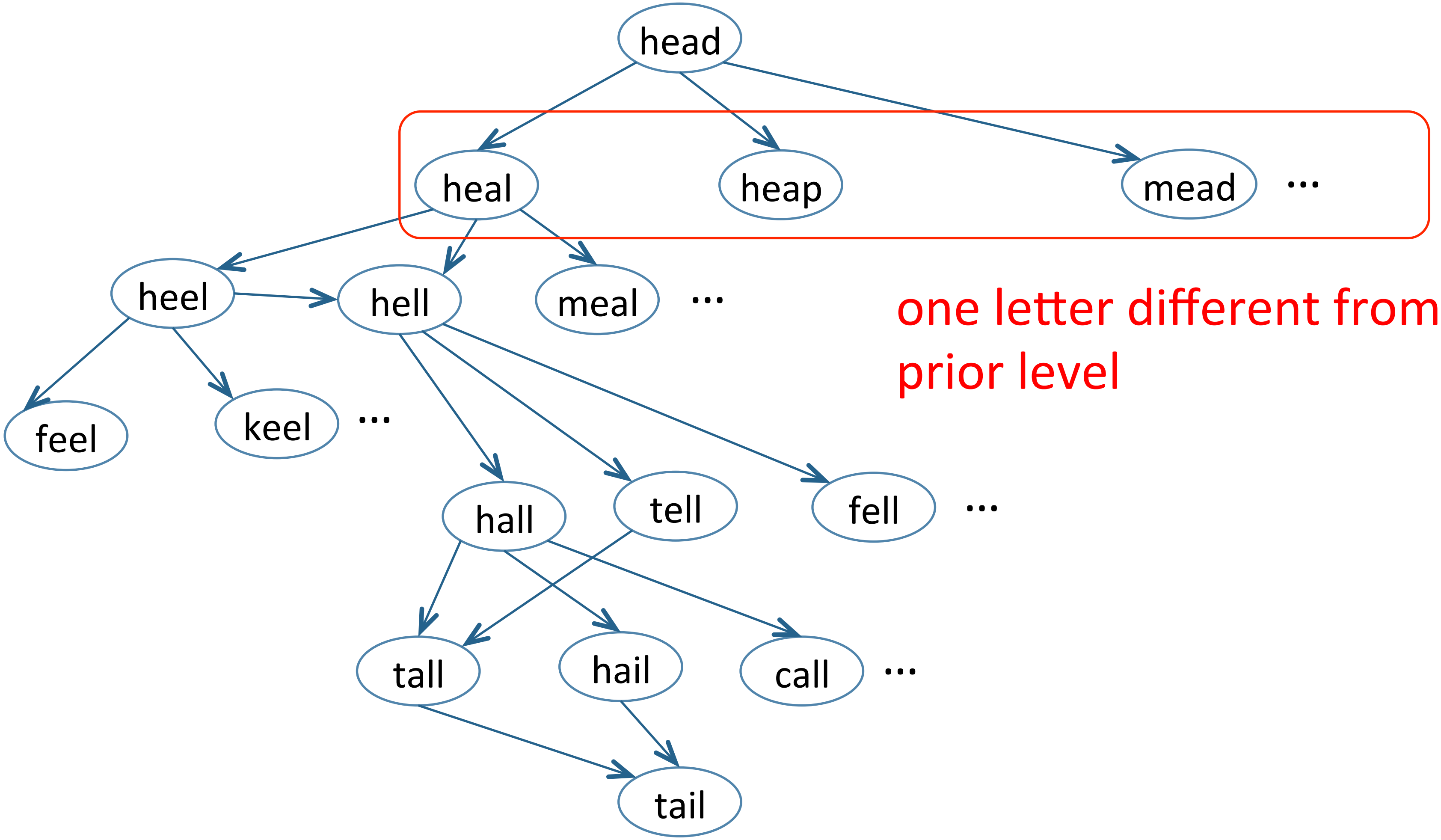
Word morph

- Change one word into another by changing one letter at a time

  Examples:
  - cat → cot → cog → dog
  - head → heal → hell → hall → tall → tail

- Imagine a tree where each level is the set of possible words created by changing one character

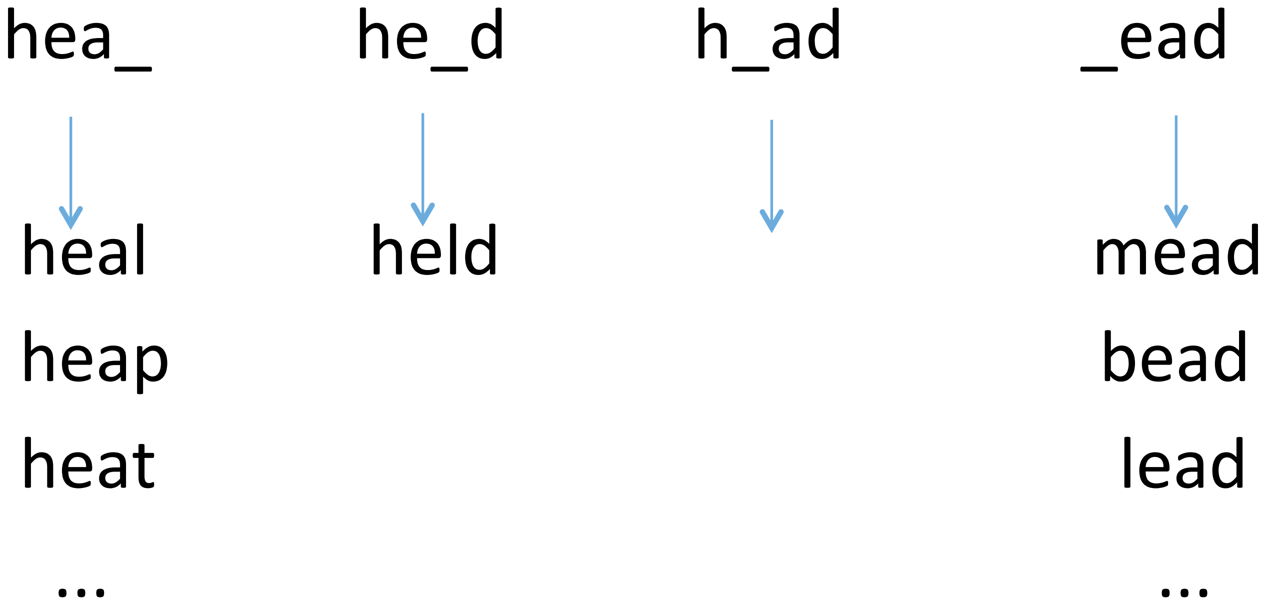
Word morph: sample tree



one letter different from prior level

Word morph

- Change one word into another by changing one letter at a time
- All of the words generated by changing one letter go in the next level of the tree.



hea_	he_d	h_ad	_ead
heal	held		mead
heap			bead
heat			lead
…			…

Word morph

- Given a dictionary of valid words
  - Generate the set of words that differ from a word *w1* by one letter
- Solution 1
  - For each position *i* in *w1*,

```
for each letter in the alphabet,
    create a new word by changing position i to the
        next letter in the alphabet
    if it's in the dictionary, add it to the set of words*
```

*unless it's been seen already

Word morph

- Given a dictionary of valid words
  - Generate the set of words that differ from a word *w1* by one letter
- Solution 2
  - Write a distance function that computes the number of positions in two strings where the two strings differ
    - heap --- heat: distance is 1
    - keep --- beat: distance is 2

  - For each word *w2* in the dictionary

    if the distance between *w1* and *w2* is 1, then add *w2* to the set of words*

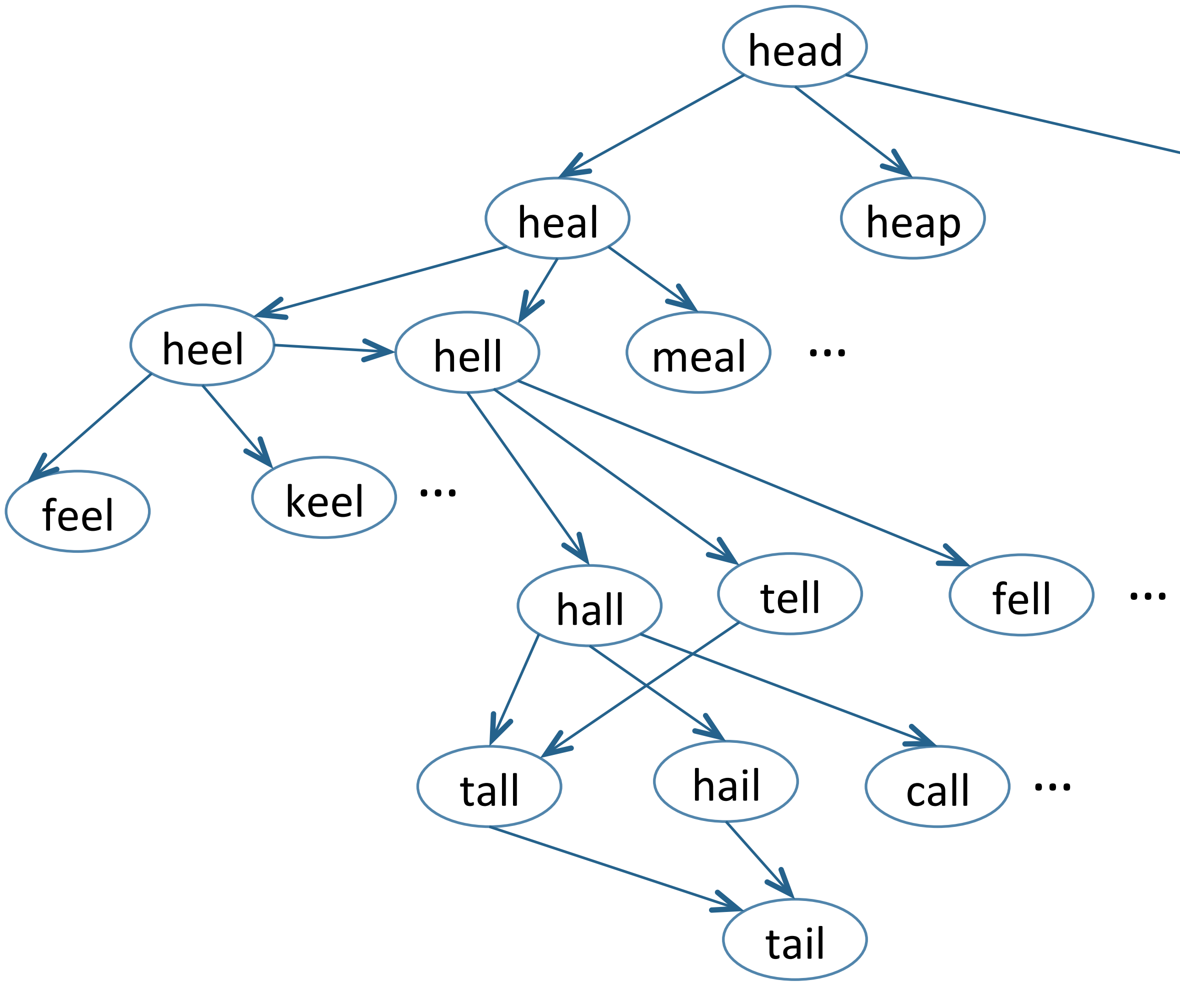
*unless it's been seen

Exercise

Write a function dist(w1, w2) that returns the number of positions where words w1 and w2 differ. It requires that len(w1) == len(w2).

Use a list comprehension.

Word morph: sample tree



- How do we traverse it?

Word morph

```
def main():
    (word1,word2,word_list) = read_input()
    morph_seq = morph(list(word1), list(word2), word_list, [])
    print_seq(morph_seq)
```

Word morph: search

```
def morph(w1, w2, word_list, Seen):
    if w1 == w2:
        return [w2]
    elif w1 in Seen:
        return []
    else:
        candidate_list = next_words(w1,w2,word_list):
        for candidate in candidate_list:
            next = update_word(w1, candidate)
            result = morph(next, w2, word_list, Seen + [w1])

            if result != []:
                return [w1] + result
        return []
```

generate the list of words to try next

Word morph: search

```
def morph(w1, w2, word_list, Seen):
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```

update the “current position”

Word morph: search

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def morph(w1, w2, word_list, Seen):
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            if result != []:
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        return []
```

search from “new position”

Word morph: search

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def morph(w1, w2, word_list, Seen):
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    else:
        candidate_list = next_words(w1,w2,word_list):
        for candidate in candidate_list:
            next = update_word(w1, candidate)
            result = morph(next, w2, word_list, Seen + [w1])

            if result != []:
                return [w1] + result
        return []
```

if a solution is found, return immediately. Otherwise, keep searching (i.e., iterating).

Word morph: utility functions

```
def next_words(wd1, wd2, word_list):
    cands = [wd for wd in word_list \
                    if len(wd) == len(wd1) and dist(wd, wd1) == 1]
    cands.sort(key = lambda wd : dist(wd, wd2))

# dist(w1, w2) returns the number of positions where words
# w1 and w2 differ.  It requires that len(w1) == len(w2).

def dist(w1, w2):
    assert len(w1) == len(w2)
    diffs = [i for i in range(len(w1)) if w1[i] != w2[i]]
    return len(diffs)
```

Word morph: code

```
# File: “morph.py”
# Author: Saumya Debray

import sys
from copy import *

DICT = 'WORDS.txt'

def read_input():
   # read the dictionary into a list
   try:
      dict_file = open(DICT)
   except IOError:
      print('ERROR: could not open file: ' + dictfilename)
      sys.exit(1)

   word_list = []
   for word in dict_file:
      word_list.append(word.strip())
   # read the two words to be morphed
   word1 = input('Word 1: ')
   word2 = input('Word 2: ')
   return (word1,word2,word_list)

# dist(w1, w2) returns the no. of positions where w1, w2
differ.
def dist(w1, w2):
   assert len(w1) == len(w2)
   diffs = [i for i in range(len(w1)) if w1[i] != w2[i]]
   return len(diffs)
```

```
def morph(w1, w2, word_list, Seen):
   if w1 == w2:
      return [w2]
   elif w1 in Seen:
      return []
   else:
      candidates = [w for w in word_list \
                    if len(w) == len(w1) and dist(w, w1) == 1]

      # consider candidates closer to w2 first
      candidates.sort(key = lambda w:dist(w, w2))

      for cand in candidates:
         result = morph(cand, w2, word_list, Seen + [w1])

         # a non-empty result means a successful morph
         if result != []:
            return [w1] + result

      return []

def print_seq(word_list):
   if word_list == []:
      print('Sorry, no morph sequence found')
   else:
      out_str = ' --> '.join(word_list)
      print(out_str)

def main():
   (word1,word2,word_list) = read_input()
   morph_seq = morph(word1, word2, word_list, [])
   print_seq(morph_seq)

main()
```

Word morph: example runs

- cat → dog
  - cat, cot, cog, dog
- head → tail
  - head, heal, ~~heel~~, hell, hall, tall, tail

why the extra words? ☹

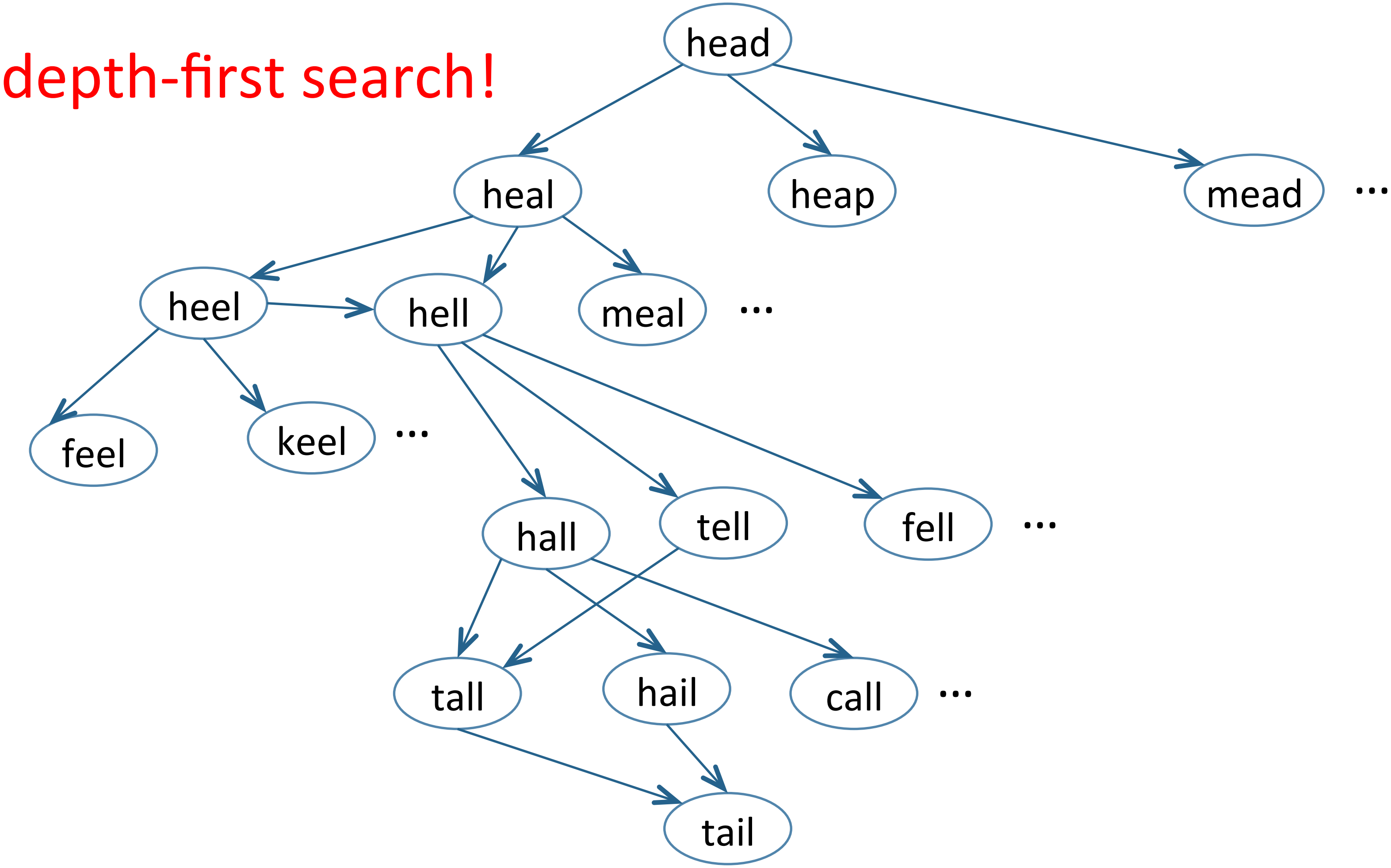
- nose → chin

dome

  - nose, ~~Bose~~, dose, dole, dale, dame, ~~came~~, ~~cage~~, ~~cake~~, ~~cape~~, ~~care~~, ~~card~~, ~~carp~~, ~~camp~~, ~~lamp~~, ~~lame~~, ~~fame~~, ~~fare~~, ~~dare~~, ~~darn~~, damn, dawn, down, ~~gown~~, sown, soon, coon, coin, chin

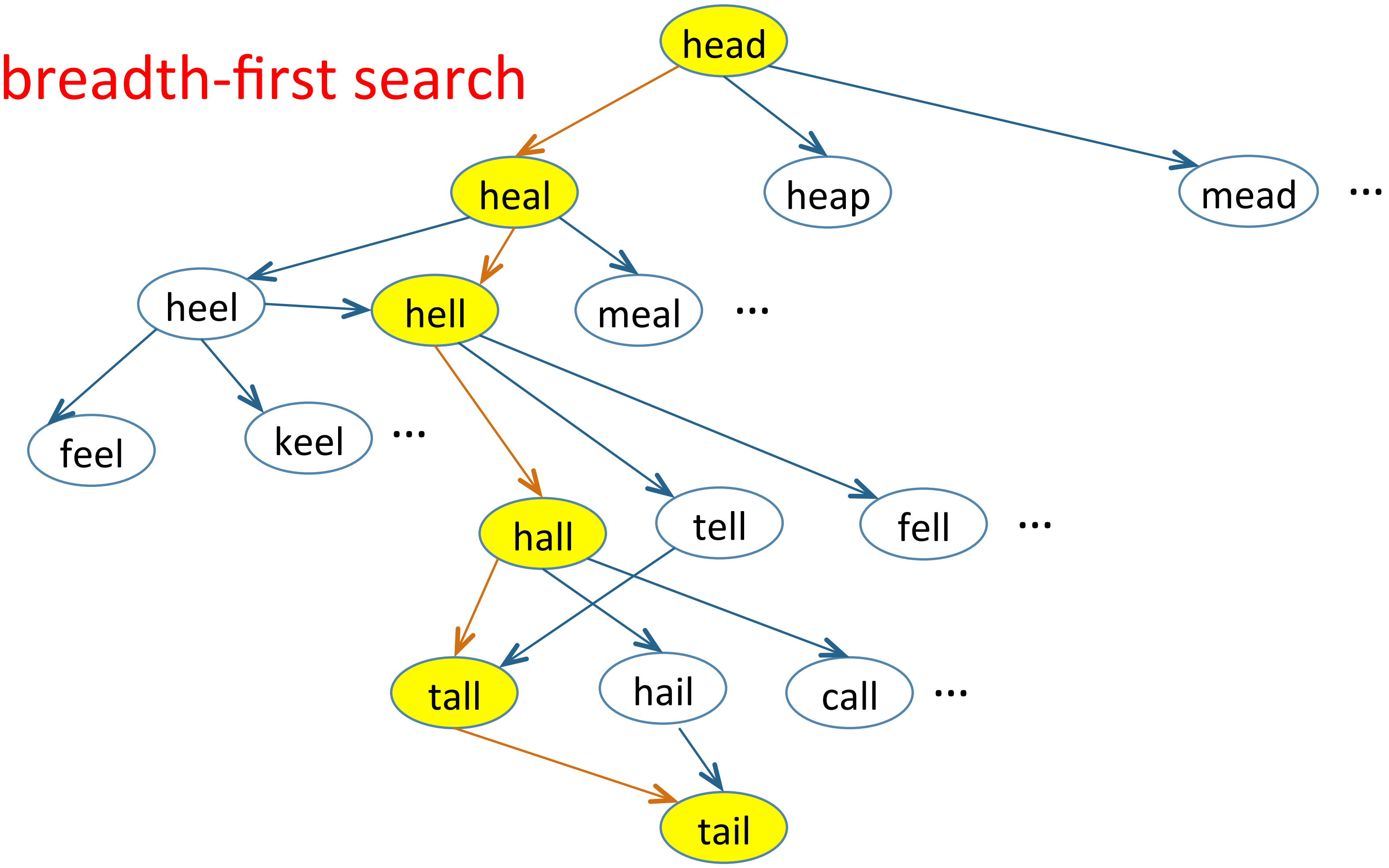
Word morph: example runs

depth-first search!



Word morph: example runs

breadth-first search



Challenge

- This version of the word morph game works with just one single word
- What would it take to let the program work with more than one word?
  - keep total length the same

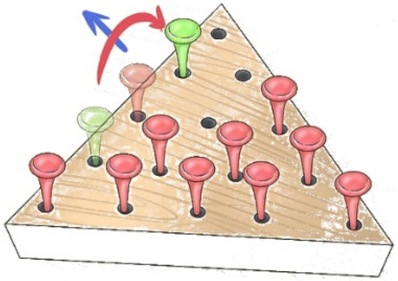
blank

e.g.: software → soft are → soft ear

- Looking for:

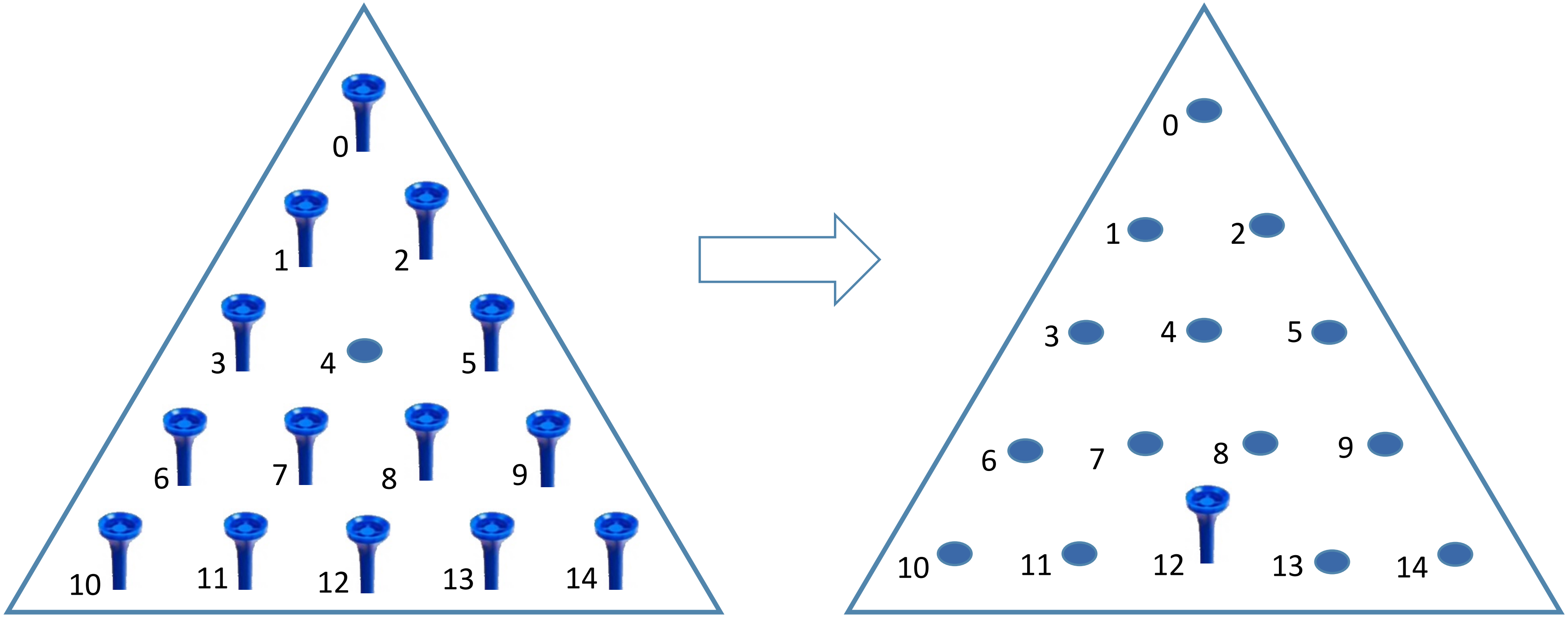
Wildcats → Beat ASU

The Cracker Barrel peg game



Repeatedly jump over and remove pegs until there is just one peg left on the board

The Cracker Barrel peg game



The Cracker Barrel peg game

```
# the recursive search

def solve(board, npins, movelist):
    if npins == 1:    # success!
        print_moves(movelist)
    else:
        mvs = [mv for mv in moves if legal_move(board, mv)]
        for mv in mvs:
            newboard = update_board(board, mv)
            solve(newboard, npins-1, movelist+[mv])
```

a list of all possible moves

which moves are legal
given the current board

generate a new position

The Cracker Barrel peg game

```
# the recursive search

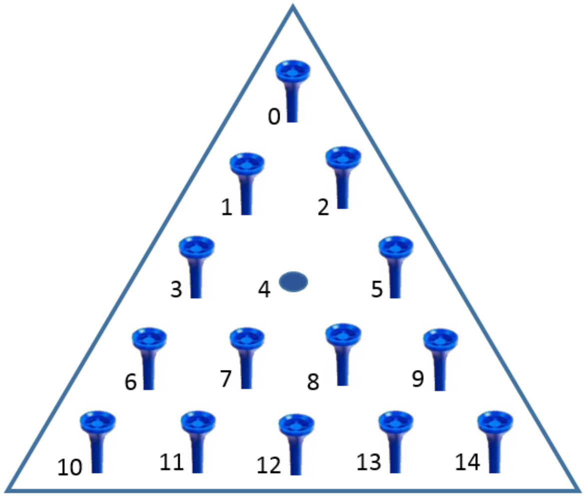
def solve(board, npins, movelist):
    if npins == 1:    # success!
        print_moves(movelist)
    else:
        mvs = [mv for mv in moves if legal_move(board, mv)]
        for mv in mvs:
            newboard = update_board(board, mv)
            solve(newboard, npins-1, movelist+[mv])
```

search from the new position

The Cracker Barrel peg game

```
# moves is a list of the possible moves.  An entry (src,mid,dst)
# indicates a move from src to dst jumping over mid.

moves = [(0,1,3),(0,2,5),(1,3,6),(1,4,8),
    (2,4,7),(2,5,9),(3,1,0),(3,4,5),(3,6,10),
    (3,7,12),(4,7,11),(4,8,13),(5,2,0),
    (5,4,3), (5,8,12),(5,9,14),(6,3,1),
    (6,7,8),(7,4,2),(7,8,9),(8,4,1),(8,7,6),
    (9,5,2),(9,8,7),(10,6,3),(10,11,12),
    (11,7,4),(11,12,13),(12,7,3),(12,8,5),
    (12,11,10),(12,13,14),(13,8,4),
    (13,12,11),(14,9,5),(14,13,12)]
```



The Cracker Barrel peg game

```
# The board is a dictionary {k_1:v_1, ..., k_n:v_n} where the k_i are
# positions and the v_i are 0 or 1 indicating whether the
# position is occupied.

def update_board(board, move):
    (src,mid,dst) = move
    newboard = copy(board)
    newboard[src] = 0        # pin moved away from this position
    newboard[mid] = 0        # pin removed from this position
    newboard[dst] = 1        # pin lands on this position
    return newboard
```

create a (shallow) copy of the board

The Cracker Barrel peg game

```
def main():
    if len(sys.argv) < 2:
        print("Usage: crackerbarrel.py config_string")
        sys.exit(1)

    # "011111111111111" --> 29,760 solutions
    initial_config = sys.argv[1]
    board = build_board(initial_config)
    npins = initial_config.count('1')
    solve(board, npins, [])
```

The Cracker Barrel peg game: code

```
# File: crackerbarrel.py
# Author: Saumya Debray

import sys
from copy import *

""" This program computes all solutions the "Cracker-Barrel Problem,
   ignoring symmetries.
"""

# moves is a list of the possible moves. An entry (src,mid,dst) indicates
# a move from src to dst jumping over mid.

moves = [(0,1,3),(0,2,5),(1,3,6),(1,4,8),(2,4,7),(2,5,9),
         (3,1,0),(3,4,5),(3,6,10),(3,7,12),(4,7,11),(4,8,13),
         (5,2,0),(5,4,3),(5,8,12),(5,9,14),(6,3,1),(6,7,8),
         (7,4,2),(7,8,9),(8,4,1),(8,7,6),(9,5,2),(9,8,7),
         (10,6,3),(10,11,12),(11,7,4),(11,12,13),
         (12,7,3),(12,8,5),(12,11,10),(12,13,14),
         (13,8,4),(13,12,11),(14,9,5),(14,13,12)]


# build_board(config) returns a board corresponding to the
# configuration  string config.  The board is a dictionary
# {k1:v1, ..., kn:vn} where the ki are positions and the vi are 0 or 1        #
indicating whether that position is occupied.

def build_board(config):
  n = len(config)
  occupancy_list = [int(k) for k in list(config)]
  return dict(zip(range(n), occupancy_list))


# update_board(board, (src,mid,dst)) returns a new board that gives
# the result of making a move (src,mid,dst), i.e., moving a pin from
# position src to position dst and thereby removing the pin at position
# mid, in the given board.

def update_board(board, move):
    (src,mid,dst) = move
    newboard = copy(board)
    newboard[src] = 0
    newboard[mid] = 0
    newboard[dst] = 1
    return newboard
```

```
# legal_move(board, (src,mid,dst)) returns True if (src,mid,dst) is a
# legal move in the given board; False otherwise.
def legal_move(board, mov):
    (src,mid,dst) = mov
    return (board[src] == 1 and board[mid] == 1 and board[dst] == 0)

# solve() performs a brute-force exploration of the search space.
def solve(board, npins, movelist):
    if npins == 1:    # success!
        print_moves(movelist)
    else:
        mvs = [mv for mv in moves if legal_move(board, mv)]
        for mv in mvs:
            newboard = update_board(board, mv)
            solve(newboard, npins-1, movelist+[mv])

def print_moves(L):
    line = ""
    for i in range(len(L)):
        (src,mid,dst) = L[i]
        line = line + "[" + str(src) + "->" + str(dst) + "]"
    print(line)

def main():
    if len(sys.argv) < 2:
        print("Usage: crackerbarrel.py config_string")
        sys.exit(1)

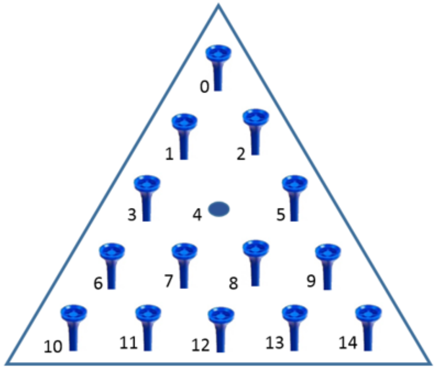
    # "011111111111111" --> 29,760 solutions
    initial_config = sys.argv[1]
    board = build_board(initial_config)
    npins = initial_config.count('1')
    solve(board, npins, [])

main()
```

The Cracker Barrel peg game

```
# moves is a list of the possible moves.  An entry (src,mid,dst)
# indicates a move from src to dst jumping over mid.

moves = [(0,1,3),(0,2,5),(1,3,6),(1,4,8),
    (2,4,7),(2,5,9),(3,1,0),(3,4,5),(3,6,10),
    (3,7,12),(4,7,11),(4,8,13),(5,2,0),
    (5,4,3), (5,8,12),(5,9,14),(6,3,1),
    (6,7,8),(7,4,2),(7,8,9),(8,4,1),(8,7,6),
    (9,5,2),(9,8,7),(10,6,3),(10,11,12),
    (11,7,4),(11,12,13),(12,7,3),(12,8,5),
    (12,11,10),(12,13,14),(13,8,4),
    (13,12,11),(14,9,5),(14,13,12)]
```

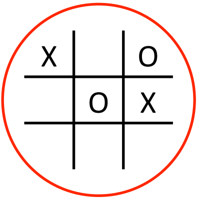


- This formulation of the set of possible moves is specific to a game with 5 rows of pins
- What would a generalization to n rows look like?

Game tree search revisited

```
def eval_pos(pos, turn):
    if game_over(pos):
        return win_or_loss(pos, turn)
    else:
        while next_pos != None:
            next_pos = generate_next_pos(pos, turn)
            if next_pos != None:
                result = eval_pos(next_pos, next[turn])
                ...
def generate_next_pos(pos, turn):
    for i in range(3):
        for j in range(3):
            if pos[ i ][ j ] == ' ':
                pos[ i ][ j ] = turn
                return pos
    return None
```

'X' or 'O'



next = { 'X':'O', 'O':'X' }

Game tree search revisited

```
def eval_pos(pos, turn):
    if game_over(pos):
        return win_or_loss(pos, turn)
    else:
        while next_pos != None:
            next_pos = generate_next_pos(pos, turn)
            if next_pos != None:
                result = eval_pos(next_pos, next[turn])
                ...
def generate_next_pos(pos, turn):
    for i in range(3):
        for j in range(3):
            if pos[ i ][ j ] == ' ':
                pos[ i ][ j ] = turn
                return pos
    return None
```

updates the position

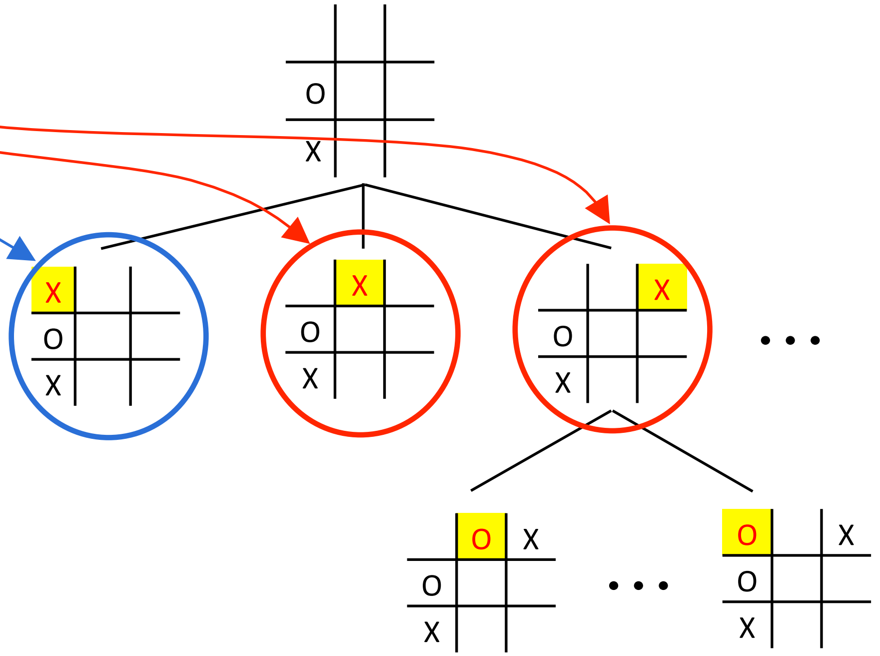
Game tree search revisited

Because arguments are passed by object reference:

- changes made to the board here will be visible here

- but these board positions are supposed to be independent!

☹

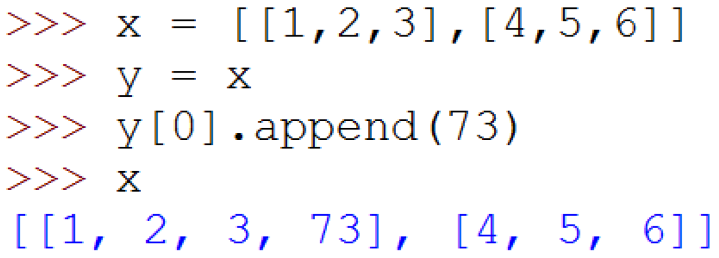


Game tree search revisited

Solution: create a copy of the board position

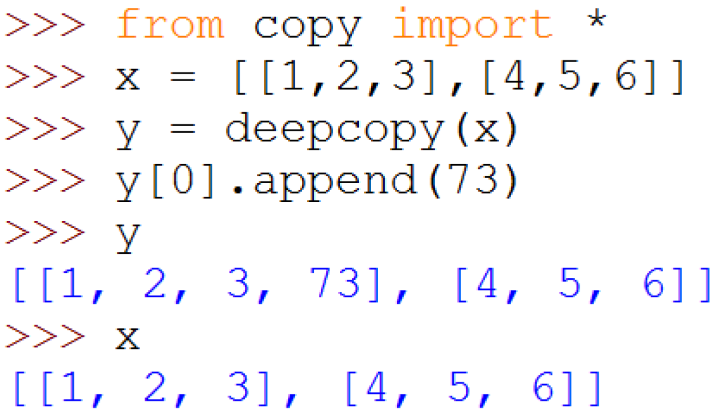
A refresher on copying:

without copying



```
>>> x = [[1,2,3],[4,5,6]]
>>> y = x
>>> y[0].append(73)
>>> x
[[1, 2, 3, 73], [4, 5, 6]]
```

with deep copying



```
>>> from copy import *
>>> x = [[1,2,3],[4,5,6]]
>>> y = deepcopy(x)
>>> y[0].append(73)
>>> y
[[1, 2, 3, 73], [4, 5, 6]]
>>> x
[[1, 2, 3], [4, 5, 6]]
```

Game tree search revisited

Solution: create a copy of the board position

```
from copy import *
...
def generate_next_pos(pos, turn):
    new_pos = deepcopy(pos)
    for i in range(3):
        for j in range(3):
            if new_pos[i][j] == ' ':
                new_pos[i][j] = turn
                return new_pos
    return None
```

updates to new_pos don't change pos