CSc 144 — Discrete Structures for Computer Science I
(McCann)

Collected Definitions Since Exam #3

Here are the definitions that we've covered since the material for the last midterm exam. I'm not going to re–print all of the definitions for the whole semester — if you lost a previous exam's definition handout, you can print another from the class web page or D2L.

Topic 12: Counting

- I provided two definitions of the *(Generalized) Pigeonhole Principle*; learn either one:
  (a) if n items are placed in k boxes, then at least one box contains at least $\lceil \frac{n}{k} \rceil$ items.
  (b) Let $f : X \to Y$, where $|X| = n$ and $|Y| = k$, and let $m = \lceil \frac{n}{k} \rceil$. There are at least m values $(a_1, a_2, \ldots, a_m)$ such that $f(a_1) = f(a_2) = \ldots = f(a_m)$.
- The *Multiplication Principle* (a.k.a. the *Product Rule*): If there are s steps in an activity, with n_1 ways of accomplishing the first step, n_2 of accomplishing the second, etc., and n_s ways of accomplishing the last step, then there are $n_1 \cdot n_2 \cdot \ldots \cdot n_s$ ways to complete all s steps.
- The *Addition Principle* (a.k.a. the *Sum Rule*): If there are t tasks, with n_1 ways of accomplishing the first, n_2 ways of accomplishing the second, etc., and n_t ways of accomplishing the last, then there are $n_1 + n_2 + \ldots + n_t$ ways to complete one of these tasks, assuming that no two tasks interfere with one another.
- The *Principle of Inclusion-Exclusion for Two Sets* says that the cardinality of the union of sets M and N is the sum of their individual cardinalities excluding the cardinality of their intersection. That is:
  $|M \cup N| = |M| + |N| - |M \cap N|$
- The *Principle of Inclusion-Exclusion for Three Sets* says that the cardinality of the union of sets M, N, and O is the sum of their individual cardinalities excluding the sum of the cardinalities of their pairwise intersections and including the cardinality of their intersection. That is:
  $|M \cup N \cup O| = |M| + |N| + |O| - (|M \cap N| + |M \cap O| + |N \cap O|) + |M \cap N \cap O|$
- An ordering of n distinct elements is called a *permutation.*
- An ordering of an r-element subset of n distinct elements is called an *r-Permutation.*
- An r-Combination of an n-element set X is an r-element subset of X. The quantity of r-element subsets is denoted $C(n, r)$ or $\binom{n}{r}$, and is read "n choose r."
- A *combinatorial proof* is an argument based on the principles of counting.

Topic 13: Finite Probability

- The *probability* that a specific event will occur, denoted $P(E)$, equals $\frac{|E|}{|S|}$, where $|E|$ is the quantity of occurrences of interest and $|S|$ is the quantity of possible occurrences.
- Let X and Y be events. The *conditional probability* of X given Y, denoted $P(X|Y)$, is $\frac{P(X \cap Y)}{P(Y)}$.
- If $P(A|B) = P(A)$, then the events A and B are *independent.*
- A *discrete random variable* (DRV) X is a function that maps outcomes of an activity to a countable range.
- A *probability distribution* is a function that maps the elements of the sample space to their probabilities of occurrence.

(Continued . . .)

- The population *mean* (a.k.a. *expected value*) of a DRV Y, denoted μ, equals $\frac{\sum_i y_i}{n}$, where y_i is observation i and n is the cardinality of Y's sample space. (A version using probability is given next.)

- The population *mean* (a.k.a. *expected value*) of a DRV Y, denoted μ, equals $\sum yP(Y = y)$. (A version not using probability is given just above.)

- The population *variance* of a DRV Y, denoted σ^2, equals $\sum(y-\mu)^2 P(Y = y)$ and also $\sum y^2 P(Y = y)-\mu^2$.

- The population *standard deviation* (SD), denoted σ, of a DRV Y is the square root of Y's sample variance.

- A *binomial distribution* is a probability distribution whose sample space has only two possible outcomes.

- A *Bernoulli trial* is a sequence of experiments in which each experiment (a) either succeeds or fails, (b) is independent of the other experiments, and (c) has the same probability of success as the others.

- The *binomial probability formula* for a binomial distribution on a DRV Y of n trials and a probability of success p is $P(Y = y) = \binom{n}{y} p^y (1-p)^{n-y}$, where $0 \le y \le n$.