Topic 1:

Course Background

(or: Why You’re Here, and What You Learned to Get Here)

What Is Discrete Math?

**Definition: Discrete Mathematics**

Contrast this with ‘the calculus,’ which was developed by Newton and Leibniz to study objects in motion. As a result:

- ‘The Calculus’ tends to focus on real values
- Discrete Mathematics tends to focus on integer values
Sample Discrete Math Topics

Topics that fall under the umbrella of discrete math include:

- Integral Functions and Relations
- Sets
- Sequences and Summations
- Counting (Permutations/Combinations, etc.)
- Discrete Probability

To understand those, you also need:

- First-Order Logic
- Logical Arguments
- Proof Techniques
- ... and a fair amount of pre-calculus mathematics

“But Why Do I Have To Take Discrete Math?”

Discrete Structures is an ACM/IEEE core curriculum topic

- See:
  
  https://www.acm.org/binaries/content/assets/education/cs2013_web_final.pdf

DM topics underlie much of Computer Science, including:

- **Logic** → Knowledge Representation, Reasoning, Natural Language Processing, Computer Architecture
- **Proof Techniques** → Algorithm Design, Code Verification
- **Relations** → Database Systems
- **Functions** → Hashing, Programming Languages
- **Recurrence Relations** → Recursive Algorithm Analysis
- **Probability** → Algorithm Design, Simulation
Topics You May Need To Review

- Mathematical concepts, including, but not limited to:
  - Fractions
  - Rational Numbers
  - Basics of Sets
  - Associative, Commutative, Distributive, and Transitive Laws
  - Properties of Inequalities
  - Summation and Product Notation
  - Integer Division (Modulo, Divides, and Congruences)
  - Even and Odd Integers
  - Logarithms and Exponents
  - Positional Number Systems

The Math Review appendix (available from the class web page) can help you review these topics.

Notations for Sets of Values

\[
\begin{align*}
\mathbb{Z} & : \text{All integers} & \{ \ldots, -2, -1, 0, 1, 2, \ldots \} \\
\mathbb{Z}^+, \mathbb{N}^+ & : \text{All positive integers} & \{1, 2, 3, \ldots \} \\
\mathbb{Z}^*, \mathbb{N}_0 & : \text{The non–negative integers} & \{0, 1, 2, 3, \ldots \} \\
\mathbb{Z}^{\text{even}} & : \text{Even integers} & \{ \ldots, -4, -2, 0, 2, 4, \ldots \} \\
\mathbb{Z}^{\text{odd}} & : \text{Odd integers} & \{ \ldots, -3, -1, 1, 3, \ldots \} \\
\mathbb{Q} & : \text{Rational numbers} & \frac{a}{b}, a, b \in \mathbb{Z}, b \neq 0 \\
\overline{\mathbb{Q}} & : \text{Irrational Numbers} & \{i | i \not\in \mathbb{Q}\} \\
\mathbb{R} & : \text{The real values} & \{\mathbb{Q} \cup \overline{\mathbb{Q}}\}
\end{align*}
\]

Note: Avoid the term “natural numbers” and the plain \(\mathbb{N}\).
Commutativity

Assume that $\triangle$ is a binary operator on a set of values $S$.

If $x \triangle y = y \triangle x$ for any elements $x$ and $y$ in $S$, then $\triangle$ is a *commutative* operator.

**Example(s):**

Addition is commutative on $\mathbb{R}$:

Subtraction is non–commutative on $\mathbb{R}$:

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Associativity

Assume that $\triangle$ is a binary operator on a set of values $S$.

If $(x \triangle y) \triangle z = x \triangle (y \triangle z)$ for any $x, y, z$ in $S$, then $\triangle$ is an *associative* operator.

**Example(s):**

Multiplication is associative on $\mathbb{Z}$:

Subtraction is not associative on $\mathbb{Z}$:
Distributivity (1 / 2)

Assume that $\triangle$ and $\square$ are binary operators on a set $S$, and that $a, b, c$ are all values of $S$.

$\triangle$ is left–distributive over $\square$ when $a \triangle (b \square c) = (a \triangle b) \square (a \triangle c)$

$\triangle$ is right–distributive over $\square$ when $(b \square c) \triangle a = (b \triangle a) \square (c \triangle a)$

Distributivity (2 / 2)

Example(s):

Multiplication distributes over addition:

This knowledge can help you do large products by hand:
Transitivity

Assume that $\diamond$ defines a relationship on values from $S$.

For any $x, y, z$ in $S$, $\diamond$ is *transitive* if whenever $x \diamond y$ and $y \diamond z$, then $x \diamond z$.

**Example(s):**

"Greater than" is transitive on $\mathbb{R}$:

In sports, "defeats" is not transitive on a set of teams:

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Three Fraction Reminders

1. The product of fractions is the ratio of the products of the numerators over the products of the denominators:

\[
\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}
\]

2. One fraction divided by another equals the product of the numerator fraction and the reciprocal of the denominator:

\[
\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}
\]

3. Computing the sum of two fractions requires a common denominator, then we add the numerators:

\[
\frac{a}{b} + \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{d} + \frac{b}{b} \cdot \frac{c}{d} = \frac{ad+bc}{bd}
\]
Rational and Irrational Numbers

**Definition: Rational Number**

A real number that is not rational is **irrational**.

**Example(s):**

Basic Set Operators (1 / 2)

1. **Union** (∪): $A \cup B$ contains all elements of both set $A$ and set $B$

2. **Intersection** (∩): $C \cap D$ contains only the elements present in both sets $C$ and $D$

3. **Difference** (−): $E - F$ contains only the elements of set $E$ that are **not** also in set $F$

   *(Note: Take out the “not,” and you’ve got a definition for ∩ )*

4. **Complement** (%): Given a set $G$, $\overline{G} = \mathcal{U} - G$, the set of available items, where $\mathcal{U}$ is the *universe*.

**Note:** $X - Y = X \cap \overline{Y}$
Basic Set Operators (2 / 2)

Example(s):

\[ A = \{1, 2, 4, 9\} \]
\[ B = \{0, 2, 6, 8\} \]
\[ C = \{2, 4, 7\} \]

Summation and Product Notation

\[ \sum_{i=1}^{5} 2i = 2(1) + 2(2) + 2(3) + 2(4) + 2(5) = 30 \]

where:

- \( \sum \) is the ____________.
- \( i \) is the ____________.
- \( 1 \) is the ____________.
- \( 5 \) is the ____________.
- \( 2i \) is the ____________.
Summation and Product Notation (cont.)

Switch $\sum$ to $\Pi$ (capital Pi) for multiplication:

Example(s):

Use parentheses to eliminate confusion:

Example(s):

Nested Summations and Products

Much like nested FOR loops.

Example(s):
Modulo and Divides

Integer Division (\) produces quotients;
Modulo (%) produces remainders

Example(s):

Modulo and Divides (cont.)

Definition: Divides

Example(s):
Congruences

**Definition: Congruent Modulo** \( m \)

\( b \) is called the base, \( r \) is the residue or remainder, and \( m \) is the modulus

**Example(s):**

- \( w^{x+y} = w^x w^y \)
- \( (w^x)^y = w^{xy} \)
- \( v^x w^x = (vw)^x \)
- \( \frac{w^x}{w^y} = w^{x-y} \)
- \( \frac{v^x}{w^x} = \left(\frac{v}{w}\right)^x \)

Laws of Exponents
Laws of Logarithms

The connection between exponents and logarithms:

If \( b^y = x \), then \( \log_b x = y \).

For each of the following laws, \( a, b > 0 \) and \( a, b \neq 1 \):

1. \( \log_a x = \frac{\log_b x}{\log_b a} \)
2. If \( m > n > 0 \), then \( \log_b m > \log_b n \)
3. \( b^{\log_b x} = x \)
4. \( \log_b(x^y) = y\log_b x \)
5. \( \log_b(xy) = \log_b x + \log_b y \)
6. \( \log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y \)

Number Systems: Decimal

The Base 10 (a.k.a. Decimal, a.k.a. Arabic) System

- 10 symbols (glyphs): 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

- In a string of symbols, each position is worth the product of the symbol’s value and a power of 10, starting with \( 10^0 = 1 \) on the right.

Example(s):
Number Systems: Binary

The Base 2 (a.k.a. Binary) System

- Just 2 symbols: 0 and 1.
- Each position is valued with increasing powers of 2.

Example(s):

Converting Decimal to Binary

1. Repeated divisions by 2
   
   Example(s):

2. Sums of powers of 2
   
   Example(s):
Number Systems: Octal

(Key point: Octal is based on groups of 3 binary digits)

The Base 8 (a.k.a. Octal) System

- 8 symbols: 0 through 7, inclusive
- Each position is valued with increasing powers of 8

Example(s):

Converting Octal to . . .

. . . Decimal: Multiply digits by powers of 8:

Example(s):

. . . Binary: Convert digits to binary, and “degroup:”

Example(s):
Number Systems: Hexadecimal

(Key point: ‘Hex’ is based on groups of 4 binary digits)

The Base 16 (a.k.a. Hexadecimal) System

- 16 symbols: 0-9, inclusive, and A-F, inclusive
- Each position is valued with increasing powers of 16

Example(s):

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Why Hexadecimal Is More Common Than Octal

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What’s The Secret Message?

“Foxtrot” from January 11, 2006:

Hint: Find an ASCII table!

Remember!

The math review topics are used in this class, and direct questions about them will be asked on quizzes, Exam #1, and the Final Exam.

If you are not confident in your knowledge of them:

- Read Appendix A in “Kneel Before $\mathbb{Z}^{\text{odd}}$,”
- Attend a Supplemental Instruction (SI) session, and
- Review and self–test the topics on your own!