Topic 2:

Logic

What Is Logic?

Definition: Philosophical Logic

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Definition: Mathematical Logic

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Propositional Logic

Propositional Logic is part of Mathematical Logic. Versions include:

- *First Order Logic* (FOL, a.k.a. *First Order Predicate Calculus* (FOPC)) includes simple term variables and quantifications.
- *Second Order Logic* allows its variables to represent more complex structures (in particular, predicates).
- *Modal Logic* adds support for modalities; that is, concepts such as possibility and necessity.

Well-Formed Formulae

Definition: Well-Formed Formula (wff)

. . . .

Example(s):

Why Are We Studying Logic?

A few of the many reasons:

- Logic is the foundation for computer operation.
- Logical conditions are common in programs:
  - Selection:

    ```
    if (score <= max) { ... }
    ```

  - Iteration:

    ```
    while (i<limit && list[i]!=sentinel) ...
    ```

- All manner of structures in computing have properties that need to be proven (and proofs that need to be understood).
  - Examples: Trees, Graphs, Recursive Algorithms, . . .
- Even programs can be proven correct!
- Computational linguistics must represent and reason about human language, and language represents thought (and thus also logic).

Simple Propositions (1 / 2)

Definition: Proposition

Definition: Simple Proposition

Simple Propositions (2 / 2)

Example(s):

Proposition Labels

To save writing, it is traditional to label propositions with lower-case letters called *proposition labels* or *statement letters*.

Example(s):

Compound Propositions

Definition: Compound Proposition

. . . .

And with what do we combine them?

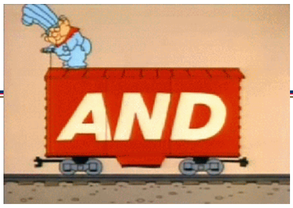
Conjunctions (1 / 2)

Remember ABC's "Schoolhouse Rock" education series?

"Conjunction Junction" (1973)

(Music/Lyrics by Bob Dorough; Performed by Jack Sheldon)

Conjunctions (2 / 2)



Conjunctions are:

- compound propositions formed in English with “and” & “but”,
- formed in logic with the caret symbol (“ $\wedge$ ”), and
- true only when both participating propositions are true.

Example(s):

Disjunctions (1 / 3)

Consider this compound proposition:

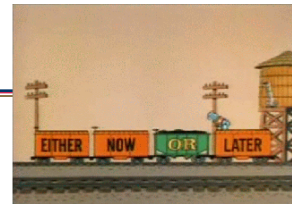
Under which circumstances is that claim true? Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If all three are acceptable, the disjunction is

________________ (   ).

Disjunctions (2 / 3)



Consider the same example and possibilities:

`3 is the number of sides of a triangle or the number of times this class meets per week.`

Possibilities:

1. The first proposition is true.
2. The second proposition is true.
3. Both of the propositions are true.

If the third possibility is not acceptable, the disjunction is

________________ (   ).

Disjunctions (3 / 3)

Example(s):

Negation

Negating a proposition simply flips its value.

Common negation notations: $\neg x$ $\overline{x}$ $\sim x$ x'

Example(s):

Notes:

Truth Tables (1 / 2)

Truth tables aid in the evaluation of compound propositions.

Structure of a Truth Table:

p	q	$p \wedge q$	$(p \wedge q) \vee p$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

Truth Tables (2 / 2)

Truth Tables of $\wedge$, $\vee$, $\oplus$, and $\neg$:

NOT ($\neg$)

p	$\neg p$

AND ($\wedge$)

p	q	$p \wedge q$

OR ($\vee$)

p	q	$p \vee q$

XOR ($\oplus$)

p	q	$p \oplus q$

Precedence of Logical Operators

Total agreement is hard to come by:

Precedence	Rosen 8/e p. 11	Gersting 5/e p. 6	Hein 2/e p. 351	Epp 1/e p. 24
Highest	$\neg$	$'$	$\neg$	$\sim$
	$\wedge$	$\wedge, \vee$	$\wedge$	$\wedge, \vee$
$\Updownarrow$	$\vee$	$\rightarrow$	$\vee$	$\rightarrow, \leftrightarrow$
	$\rightarrow$	$\leftrightarrow$	$\rightarrow$	
Lowest	$\leftrightarrow$			

(**Note:** We'll cover $\rightarrow$ and $\leftrightarrow$ soon.)

In this class:

Operator Associativity

Consider evaluating: $\mathtt{a} = \mathtt{b} = -2 * 3 * 7;$ in Python

Example(s):

Equivalence of Propositions

Definition: Logically Equivalent

. . . .

. . . .

Example(s):

Natural Language Stmts → Propositions (1 / 4)

Review: Is There isn't a cloud in the sky a proposition?

Question: Is the following sentence a proposition?

Natural Language Stmts → Propositions (2 / 4)

Step 1: Identify the simple propositions.

> Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 2: Assign easy-to-remember statement labels.

Natural Language Stmts $\rightarrow$ Propositions (3 / 4)

Step 3: Identify the logical operators.

Either Walter deposits his mortgage payment or else he will lose his house and move in with Donna.

Step 4: Construct the matching logical expression.

Natural Language Stmts $\rightarrow$ Propositions (4 / 4)

So . . . what's the point? Three examples:

- Expressing Program Conditions

- Natural Language Understanding

- Proof Setup

Three Categories of Propositions (1 / 2)

Definition: Tautology

Definition: Contradiction

Definition: Contingency

Three Categories of Propositions (2 / 2)

Example(s): Which of those is $d \oplus (\neg k \wedge m)$?

Example(s):

Aside: Logical Bit Operations in Python/Java

Operator	Name	Example (Dec.)	Example (Bin.)
$\sim$	Complement	$\sim 12 = -13$	$\sim 00001100 = 11110011$
&	AND	$12 \,\&\, 10 = 8$	1100 & 1010 1000
$\vert$	OR	$12 \vert 10 = 14$	1100 $\vert$ 1010 1110
$\wedge$	XOR	$12 \wedge 10 = 6$	1100 $\wedge$ 1010 0110
>>	Shift Right	$33 >> 1 = 16$	$00100001 >> 1 = 00010000$
<<	Shift Left	$33 << 2 = 132$	$00100001 << 2 = 10000100$

Example: Default Linux File Permissions

```
$ ls -l
-rw-rw-r-- 1 mccann mccann 3561 Oct 28 1929 stocktosell
```

Conditional Propositions (1 / 3)

Example:

Definition: Conditional Proposition

. . . .

. . . .

Conditional Propositions (2 / 3)

In "if p, then q", p and q are known by various names:

Common forms of "if p, then q" (Rosen 8/e, p. 7):

- if p, then q
- if p, q
- p implies q
- p only if q
- p is sufficient for q
- a necessary condition for p is q
- q unless $\neg p$
- q if p
- q when p
- q whenever p
- q follows from p
- q is necessary for p
- a sufficient condition for q is p
- q provided that p

Conditional Propositions (3 / 3)

Example(s):

Truth of Conditional Propositions (1 / 2)

When should this be considered ‘true’?

If you make it through *voir dire*, you will serve on the jury.

The possibilities:

1. Antecedent true, Consequent true; statement is: ____.
2. Antecedent true, Consequent false; statement is: ____.
3. Antecedent false, Consequent true; statement is: ____.
4. Antecedent false, Consequent false; statement is: ____.

Truth of Conditional Propositions (2 / 2)

Not satisfied? Maybe this Python if statement will help:

```
if y < x :
    temp = x
    x = y
    y = temp
```


Inverse, Converse, and Contrapositive

Definition: Inverse

Definition: Converse

r	s
T	T
T	F
F	T
F	F

Contraposition

Definition: Contrapositive

r	s	
T	T	
T	F	
F	T	
F	F	

Examples: English Translation (1 / 2)

Examples: English Translation (2 / 2)

Example: English $\longrightarrow$ Logic

Another Example: English $\rightarrow$ Logic

Political Example: "Push" Polling

"What would you think of Elizabeth Colbert Busch if she had done jail time?"

— Asked in telephone calls by Survey Sampling International in the 2013 South Carolina 1st Congressional District special election

Biconditional Propositions and *iff* (1 / 2)

What is the meaning of:

A triangle is equilateral if and only if all three angles are equal.

Biconditional Propositions and *iff* (2 / 2)

Definition: Biconditional Proposition

. . . .

. . . .

r	s	
T	T	
T	F	
F	T	
F	F	

Biconditionals and Logical Equivalence

Definition: Logically Equivalent (2)

. . . .

Example(s):

De Morgan's Laws

Example(s):

Example: De Morgan's Laws and Programming

Checking to see if a 0–100 numeric score is not a 'B':

Common Logical Equivalences (1 / 3)

Table I: Some Equivalences using AND ($\wedge$) and OR ($\vee$):

(a)	$p \wedge p \equiv p, \quad p \vee p \equiv p$	Idempotent Laws
(b)	$p \vee \mathbf{T} \equiv \mathbf{T}, \quad p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination Laws
(c)	$p \wedge \mathbf{T} \equiv p, \quad p \vee \mathbf{F} \equiv p$	Identity Laws
(d)	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative Laws
(e)	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative Laws
(f)	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive Laws
(g)	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	Absorption Laws

Table II: Some More Equivalences (adding $\neg$):

(a)	$\neg(\neg p) \equiv p$	Double Negation
(b)	$p \vee \neg p \equiv \mathbf{T}, \quad p \wedge \neg p \equiv \mathbf{F}$	Negation Laws
(c)	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws

Common Logical Equivalences (2 / 3)

Table III: Still More Equivalences (adding $\rightarrow$):

(a)	$p \rightarrow q \equiv \neg p \vee q$	Law of Implication
(b)	$p \rightarrow q \equiv \neg q \rightarrow \neg p$	Law of the Contrapositive
(c)	$\mathbf{T} \rightarrow p \equiv p$	"Law of the True Antecedent"
(d)	$p \rightarrow \mathbf{F} \equiv \neg p$	"Law of the False Consequent"
(e)	$p \rightarrow p \equiv \mathbf{T}$	Self-implication (a.k.a. Reflexivity)
(f)	$p \rightarrow q \equiv (p \wedge \neg q) \rightarrow \mathbf{F}$	Reductio Ad Absurdum
(g)	$\neg p \rightarrow q \equiv p \vee q$	
(h)	$\neg(p \rightarrow q) \equiv p \wedge \neg q$	
(i)	$\neg(p \rightarrow \neg q) \equiv p \wedge q$	
(j)	$(p \rightarrow q) \vee (q \rightarrow p) \equiv \mathbf{T}$	Totality
(k)	$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exportation Law (a.k.a. Currying)
(l)	$(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$	
(m)	$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$	
(n)	$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$	
(o)	$p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$	
(p)	$p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \rightarrow r)$	Commutativity of Antecedents

Common Logical Equivalences (3 / 3)

Table IV: Yet More Equivalences (adding $\oplus$ and $\leftrightarrow$):

(a)	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Definition of Biimplication
(b)	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	
(c)	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$	
(d)	$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$	Definition of Exclusive Or
(e)	$p \oplus q \equiv \neg(p \leftrightarrow q)$	
(f)	$p \oplus q \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q$	

Remember: You **do not** need to memorize these tables . . .

. . . But you **do** need to know how to use them!

Applications of Logical Equivalences (1 / 5)

Question: Is $(p \wedge q) \rightarrow p$ a tautology? (1)

By use of a Truth Table; we've seen this before:

p	q	$p \wedge q$	p	$(p \wedge q) \rightarrow p$
T	T	T	T	**T**
T	F	F	T	**T**
F	T	F	F	**T**
F	F	F	F	**T**

Because the expression evaluates to true for all possible arrangements of truth values, the expression is a tautology.

Applications of Logical Equivalences (2 / 5)

Question: Is $(p \wedge q) \rightarrow p$ a tautology? (2)

Applications of Logical Equivalences (3 / 5)

Question: Is $(p \wedge q) \rightarrow p$ a tautology? (3)

Applications of Logical Equivalences (4 / 5)

Example(s):

```
if ((games <= 10 || ties > 2) && games >= 11) ...
```

Applications of Logical Equivalences (5 / 5)

Question: Are $(p \wedge q) \vee (p \wedge r)$ and $p \wedge \overline{(\overline{q} \wedge \overline{r})}$ logically equivalent?