Topic 4: Arguments

* The Logical Kind, Not The Talk Radio Kind.

Monty Python’s “The Argument Clinic”

Featuring:

- Michael Palin as “Man”
- Rita Davies as “Receptionist”
- Graham Chapman as “Mr. Barnard”
- John Cleese as “Mr. Vibrating”
- Eric Idle as “Complainer”
- Terry Jones as “Spreaders”

Definition: Argument
Inductive and Deductive Reasoning (1 / 3)

**Definition: Inductive Argument**

... 

**Definition: Deductive Argument**

... 

Inductive and Deductive Reasoning (2 / 3)

**Example(s):**
What type of argument is this?

3 is a prime number, 5 is a prime number, and 7 is a prime number. Therefore, all positive odd integers above 1 are prime numbers.

Structure of a Deductive Argument

$$(p_1 \land p_2 \land \ldots \land p_n) \rightarrow q$$
Valid and Sound Arguments (1 / 2)

Definition: Valid Argument

Example(s):

Valid and Sound Arguments (2 / 2)

Example(s):

Definition: Sound Argument
Some Rules of Inference (1 / 2)

Learn these!

1. Addition

2. Simplification

3. Conjunction

4. Modus Ponens

Some Rules of Inference (2 / 2)

Learn these, too!

5. Modus Tollens

6. Hypothetical Syllogism

7. Disjunctive Syllogism

8. Resolution
Examples of Valid Arguments (1 / 4)

#1: You accidently drop a pen. You know that the pen will fall if it is dropped. How do you know that the pen will fall?

Examples of Valid Arguments (2 / 4)

#2: If 191 is divisible by 7, then $191^2$ is divisible by 49.

191 is divisible by 7, so $191^2$ is divisible by 49.

Is this argument valid?
Examples of Valid Arguments (3 / 4)

#3: If you email me a love note, I’ll send you flowers. If you don’t, I’ll study Discrete Math. If I study Discrete Math, I’ll do well on the quiz.

Can we conclude that, if I don’t send you flowers, I’ll do well on the quiz?

#3: (cont.)

\[ p: \text{You email me a love note} \]  
\[ q: \text{I send you flowers} \]  
\[ r: \text{I study Discrete Math} \]  
\[ s: \text{I do well on the quiz} \]

\[ p \rightarrow q \]  
\[ \overline{p} \rightarrow r \]  
\[ r \rightarrow s \]

\[ \therefore \overline{q} \rightarrow s \]  

???
Four common rules that you need to know:

1. **Universal Instantiation**
   \[ \forall x \ P(x), \ x \in D / \therefore P(d) \text{ if } d \in D \]

2. **Universal Generalization**
   \[ P(d) \text{ for any } d \in D / \therefore \forall x \ P(x), \ x \in D \]

3. **Existential Instantiation**
   \[ \exists x \ P(x), \ x \in D / \therefore P(d) \text{ for some } d \in D \]

4. **Existential Generalization**
   \[ P(d) \text{ for some } d \in D / \therefore \exists x \ P(x), \ x \in D \]

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**Example(s):**

Everyone taking CSc 144 has had a programming class. Hugo is in CSc 144. **Has he had a programming class?**
Rules of Inference for Predicates (3 / 3)

Here’s a more general example using the same setup:

Someone’s taking CSc 144. Everyone in CSc 144 has had a programming class. Does there exist someone who both had a programming class and is taking CSc 144?

(1) \( \exists x \ C(x) \) (Given)
(2) \( \forall x \ (C(x) \rightarrow P(x)) \) (Given)

\[ \therefore \exists x \ (P(x) \land C(x)) \]

Fallacies (1 / 2)

Definition: Fallacy

Three classic types:

1. Affirming the Conclusion (or . . . Consequent)
2. Denying the Hypothesis (or . . . Antecedent)

3. Begging the Question (a.k.a. Circular Reasoning)

Fallacies for Fun

1. Fallacy of Interrogation

2. ‘No True Scotsman’ Fallacy
The remaining slides in this topic are some that I no longer cover in class. I won’t ask about them on a quiz or an exam, but they could be referenced on a homework or in SIs.

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Specious Reasoning: The Bear Patrol (1 / 3)

Homer: Ah, not a bear in sight. The Bear Patrol must be working like a charm!

Lisa: That’s specious reasoning, Dad. [...] By your logic, I could claim that this rock keeps tigers away!

Homer: Oh . . . and how does it work?

Lisa: It doesn’t work. [...] It’s just a stupid rock. [...] But I don’t see any tigers around here, do you?

Homer: Lisa, I want to buy your rock.

From: The Simpsons, “Much Apu About Nothing”

(Season 7, Episode 151, Production Code 3F20)
Specious Reasoning: The Bear Patrol (2 / 3)

**Definition: Specious Reasoning**

An unsupported or improperly constructed argument. (That is, an unsound or invalid argument.)

**Question: Where is the error in Homer’s logic?**

\[ b: \text{ There are bears in Springfield} \]
\[ w: \text{ The Bear Patrol is working} \]

First issue: Which of these is Homer’s argument?

\[
\begin{align*}
(1) & \quad \neg b \quad \text{(Given)} \\
(2) & \quad \therefore w \quad \text{(???)}
\end{align*}
\]

\[
\begin{align*}
(1) & \quad w \quad \text{(Given)} \\
(2) & \quad \therefore \neg b \quad \text{(???)}
\end{align*}
\]

The first seems most reasonable in context.

Specious Reasoning: The Bear Patrol (3 / 3)

**Question: Where is the error in Homer’s logic? (cont.)**

Next, what is the missing piece of Homer’s argument?

\[
\begin{align*}
(1) & \quad \neg b \\
(2) & \quad \neg b \rightarrow w \quad \text{← this is what we’re trying to show!} \\
(3) & \quad \therefore w \quad (1, 2, \text{Modus Ponens})
\end{align*}
\]

OK, then, how about . . .

\[
\begin{align*}
(1) & \quad \neg b \\
(2) & \quad w \rightarrow \neg b \quad \text{← might sound good, but . . .} \\
(3) & \quad \therefore w \quad (1, 2, \text{um . . . Abracadabra?})
\end{align*}
\]

(The second form of Homer’s argument fails similarly.)