Topic 13:

Finite Probability

Probability (1 / 2)

Definition: Probability

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. . . .

- The occurrences of interest are called ____________________.
- The set of possible occurrences is the ____________________.
- These are finite sets, hence the term *finite* probability.
- The occurrence probability of an interest event:

Probability (2 / 2)

Please note: (a) $\forall e \in S,\ P(e) > 0$ (b) $\sum_{e \in S} P(e) = 1$

Example(s):

Applications of Counting to Probability (1 / 2)

1. Probability of Winning the Powerball Lottery

Applications of Counting to Probability (2 / 2)

2. Principle of Inclusion-Exclusion

Recall: $|E_1 \cup E_2| = |E_1| + |E_2| - |E_1 \cap E_2|$

Example(s):

Conditional Probability (1 / 2)

Example(s):

Conditional Probability (2 / 2)

Definition: Conditional Probability

. . . .

Example(s):

Independence of Events (1 / 3)

Recall: $P(A|B) = \frac{P(A \cap B)}{P(B)}$

Definition: Independent

Example(s):

Independence of Events (2 / 3)

Example(s):

Independence of Events (3 / 3)

Random Variables (1 / 2)

Oddly, random variables are neither random nor variables!

Definition: Discrete Random Variable (DRV)

. . . .

Example(s):

Random Variables (2 / 2)

Example(s):

DRVs and Probabilities (1 / 2)

Each of 12 students count the number of pencils they have.
The results: $3, 0, 3, 2, 3, 1, 1, 5, 2, 3, 3, 1$.
What is the probability of occurence of each quantity?

		—— probabilities ——	
# pencils	frequency	fraction	decimal
0			
1			
2			
3			
4			
5			

DRVs and Probabilities (2 / 2)

Definition: Probability Distribution

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Example(s):

Mean of a Discrete Random Variable (1 / 3)

Definition: Population Mean (Version 1)

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Example(s):

Mean of a Discrete Random Variable (2 / 3)

We can also compute the mean using probabilities.

Example(s):

The quiz scores of 6 students are 10, 4, 8, 8, 10, 10.

By our mean definition, $\mu = \frac{10+10+10+8+8+4}{6} = 8.\overline{3}$

Observe:

Mean of a Discrete Random Variable (3 / 3)

Example(s): (Continues) Let Y be the quiz score DRV.

Definition: Population Mean (Version 2)

. . . .

Variance & Standard Deviation of a DRV (1 / 4)

A little motivation:

Example(s):

The mean of 0 and 100 is

The mean of 45 and 55 is

Variance & Standard Deviation of a DRV (2 / 4)

However: Squaring changes the units of measure from, for example, "points" to "points squared."

No problem! Taking the square root of $(x_i - \mu)^2$ gives us both un–squared units and positive values.

Variance & Standard Deviation of a DRV (3 / 4)

Their expressions build on that of the expected value.

Definition: Variance of a DRV

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Definition: Standard Deviation of a DRV

. . . .

Variance & Standard Deviation of a DRV (4 / 4)

Example(s): Recall: $\mu = 8\frac{1}{3}$ for quizzes 10, 4, 8, 8, 10, 10.

y	frequency	$P(Y = y)$	y^2	$y^2 P(Y = y)$
10	3	3/6		
8	2	2/6		
4	1	1/6		

Binomial Distribution

Recall: A probability distribution maps outcomes to probabilities.

Definition: Binomial Distribution

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Many real–world experiments have this distribution, such as:

Bernoulli Trial (1 / 5)

Definition: Bernoulii Trial

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. . . .

Example(s):

Bernoulli Trial (2 / 5)

Example(s): (Continued from last slide)

The probabilities of each of the four outcomes are:

ss:

sf:

fs:

ff:

Bernoulli Trial (3 / 5)

Observation: We had [1] way to get two successes, [2] ways to get one, and [1] way to get none. [1] [2] [1] – ring a bell?

Bernoulli Trial (4 / 5)

Time to generalize!

Definition: The Binomial Probability Formula

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. . . .

Does that formula's structure remind you of anything?

Bernoulli Trial (5 / 5)

Example(s):

Mean & Std. Deviation of a Binomial DRV (1 / 2)

We can still use our $\mu = \sum x P(X = x)$ and $\sigma = \sqrt{\sum y^2 P(Y = y) - \mu^2}$ expressions, but thanks to the two–outcome limit of Binomial distributions, they can be greatly simplified.

Mean & Std. Deviation of a Binomial DRV (2 / 2)

Example(s):

Probabilistic Reasoning (1 / 6)

Each drawer of a 3x2 dresser holds either a red or a blue UA T-shirt. One row of drawers has two red shirts, one row has two blue, and one row has one of each. You open one drawer and see a red T-shirt. What is the probability that the shirt in the other drawer in the same row is also red?

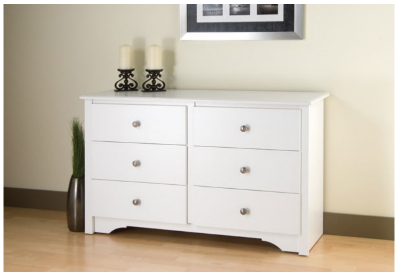

Photo Credit: overstock.com

Probabilistic Reasoning (2 / 6)

One solution approach: Enumerate the possibilities. WLOG:

Dresser

R_1	R_2
B	B
R_3	B

Open Drawer Containing	Shirt Color in Other Drawer?
R_1	
R_2	
R_3	

Probabilistic Reasoning (3 / 6)

A more famous, more recent, example:

“Suppose you’re on a game show, and you’re given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say No. 1, and the host, who knows what’s behind the doors, opens another door, say No. 3, which has a goat. He then says to you, ‘Do you want to pick door No. 2?’ Is it to your advantage to switch your choice?”

From “Ask Marilyn”, Parade, Sept. 9, 1990.

Reference:

`www.marilynvossavant.com/game-show-problem/`

Care to Play?

Probabilistic Reasoning (4 / 6)

But . . . why? Three views:

1. Enumerate the Possibilities

Probabilistic Reasoning (5 / 6)

2. ‘Car / Not Car’

Probabilistic Reasoning (6 / 6)

3. Conditional Probability