CSc 144–002 — Discrete Mathematics for Computer Science I — Fall 2023 (McCann)
https://cs.arizona.edu/classes/cs144/fall23-002/

Homework #5

(50 points)

Due Date: November 3^{rd}, 2023, at the beginning of class

Directions

1. **This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**
2. Start early! Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via piazza.com and our office hours.
3. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.
4. When your PDF is ready to be turned in, do so on gradescope.com. Be sure to assign pages to problems after you upload your PDF. Need help? See "Submitting an Assignment" on https://help.gradescope.com/.
5. **Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**

Topic: Relations

1. (6 points) Which of these relations are equivalence relations? Explain which of the equivalence relation definition's properties are possessed by the relation (if any) and which are not (again, if any).
   (a) $\{(a,a),(a,d),(b,d),(c,c),(c,d),(d,a),(d,b),(d,c),(d,d)\}$ on $\{a,b,c,d\}$
   (b) $\{(e,f) \mid e$ and f have the same color fur$\}$ on the domain of 'puppies.'
2. (6 points) Determine which variety of partial order (weak, strict, or neither) correctly describes each of the following relations.
   (a) $\{(2,1),(3,2),(3,1)\}$ on $\{1,2,3\}$
   (b) $\{(a,a),(a,c),(a,d),(b,b),(b,c),(b,d),(c,c),(c,d),(d,c),(d,d)\}$ on $\{a,b,c,d\}$
   (c) $\{(1,1),(1,4),(2,2),(2,3),(3,3),(4,4)\}$ on $\{1,2,3,4\}$
3. (4 points) Each of the following relations is a weak partial order. Are they also total orders? If yes, explain why. If not, what must be added to the relation to make it a total order?
   (a) $\{(4,4),(4,3),(4,2),(3,3),(2,3),(2,2),(1,4),(1,3),(1,2),(1,1)\}$ on $\{1,2,3,4\}$
   (b) $\{(a,a),(b,b),(c,c)\}$ on $\{a,b,c\}$

(Continued on the back . . .)

Topic: Functions

4. (4 points) For each of the following, is it a function from $\mathbb{Z}$ to $\mathbb{R}$? If the answer is 'No,' explain why.

   (a) $f(x) = \log_2 x$

   (b) $q(x) = \frac{z}{z+1}$, where $z = \sum_{i=x}^{|x|} i$

5. (4 points) For each of these functions, what are the domains and the ranges? (To find the domains, figure out what is legal input to the function as described.)

   (a) The function that returns the integer part of a real number.

   (b) The function that returns the total number of even digits in a day of a month. (In the date March 15, 2000, "15" is the day of the month.)

6. (4 points) Evaluate each of these functions.

   (a) $\lfloor 4.999 \rfloor$

   (b) $\lceil -0.12 \rceil$

   (c) $\lfloor \lceil \frac{3}{2} \rceil \rfloor$

   (d) $\lceil \frac{8}{3} + \lfloor \frac{8}{3} \rfloor - \lceil \frac{8}{3} \rceil \rceil$

7. (4 points) By hand, draw a graph of each of the following functions. (We know that it is tempting to use a function plotting app or website to do these, but you won't have access to one on quizzes or on exams, so it's best to do these yourself.)

   (a) $f(x) = x - x^2$ on the domain of integers $\{-2 \ldots 2\}$, inclusive.

   (b) $f(x) = \lfloor x - 1 \rfloor + \lceil x - 2 \rceil$ on the domain of reals $\{-4 \ldots 4\}$, inclusive.

Topic: Proof by Contraposition

8. (6 points) Consider this conjecture: if x is irrational, then $3x + 2$ is irrational. Prove this conjecture using a proof by contraposition.

9. (6 points) Consider this conjecture: If the product xyz is even, then x, y, and z are not all odd integers. Prove this conjecture using a proof by contradiction.

10. (6 points) Prove, using your choice of proof by contraposition or proof by contradiction: If $x > -3$ and $y > -3$, the $x + y > -5$, where $x, y \in \mathbb{Z}$. *Reminder: The negation of $x > y$ is $x \leq y$.*

(For a little more proof practice, try doing each of 8, 9, and 10 using the other 'contra' proof technique!)