CSc 144–002 — Discrete Mathematics for Computer Science I — Fall 2023 (McCann)
https://cs.arizona.edu/classes/cs144/fall23-002/

Homework #7

(50 points)

Due Date: December 1st, 2023, at the beginning of class

Directions

1. **This is an INDIVIDUAL assignment; do your own work! Submitting answers created by computers or by other people is NOT doing your own work.**
2. Start early! Getting help is much easier n days before the due date/time than it will be n hours before. Help is available from the class staff via piazza.com and our office hours.
3. Write complete answers to each of the following questions, in accordance with the given directions. Create your solutions as a PDF document such that each answer is clearly separated from neighboring answers, to help the TAs easily read them. Show your work, when appropriate, for possible partial credit.
4. When your PDF is ready to be turned in, do so on gradescope.com. Be sure to assign pages to problems after you upload your PDF. Need help? See "Submitting an Assignment" on https://help.gradescope.com/.
5. **Solutions submitted more than five minutes late will cost you a late day. Submissions more than 24 hours late are worth no points.**

Topic: Pigeonhole Principle

1. (2 points) Consider the set $\{11, 13, 15, 17, 19, 21\}$. How many numbers must be selected from the set to ensure that two of the selected numbers sum to exactly 32?
2. (4 points) The midpoint of the line segment between two two–dimensional points at coordinates (a, b) and (c, d) is the point having coordinates $(\frac{a+c}{2}, \frac{b+d}{2})$. Using the Pigeonhole Principle, show that any set of five different 2–D points that all have integer coordinates must have a pair of points whose midpoint has only integer coordinates. *HINT: All integers are either even or odd.*

Topic: Multiplication and Addition Principles

3. (4 points) The following questions are based on Base 3 numbers. Base 3 uses the digits 0, 1, and 2.
   (a) How many Base 3 numbers have 8 digits? Assume that all 8 positions hold a digit, and leading zeroes are acceptable.
   (b) How many Base 3 numbers of length n start with a '2' and end with a '2'? We're looking for an answer in terms of n, not an arbitrary constant.
4. (4 points) A pre–school class has 12 children, plus a teacher and an assistant. In how many ways can a parent take a picture of four of them in a row against the background of the school's logo, if . . .
   (a) the teacher is in the picture?
   (b) both the teacher and the assistant are in the picture?

(Continued on the back . . .)

Topic: Permutations and Combinations

5. (4 points) Consider a set with nine elements.
   (a) How many subsets with three elements does the set have?
   (b) How many subsets with more than two elements does the set have?
6. (4 points) The U. of Arizona men's basketball team has 18 players on the 2023-24 roster.
   (a) In how many ways can Coach Lloyd select a group of five players to start a game, without worrying about the position each will play?
   (b) Of the players on the roster, three are from Arizona. In how many ways can five players be selected to start the game, regardless of position, if at least one of them must be from Arizona?
7. (6 points) The golf course pro shop from our in–class example (see the slides from Nov 20th) has added more containers of 'experienced' golf balls. Now there are seven containers: red, green, blue, orange, yellow, indigo, and violet, each with plenty of balls. In how many ways can a golfer buy ...
   (a) 10 balls?
   (b) 20 balls with at least two of each color?
   (c) 20 balls with no more than two green balls?
8. (4 points) How many distinguishable strings can be created using the letters in the word ...
   (a) STRING
   (b) SENSELESSNESS

Topic: Algebraic and Combinatorial Proofs

9. (18 points) Lynn, member of the $\Upsilon\Pi\mathrm{E}$ honor society, is having a birthday. Lynn's parents sent enough money for 6 people to go to dinner to celebrate, so Lynn needs to invite five companions, and decides to select those five from the other 18 society members. Unfortunately, Lynn doesn't have a car, so one of those five needs to drive. Assume that all of the other society members have cars.
   (a) One way for Lynn to decide who gets to go to dinner is to first select the five companions, and then choose one of them to drive. What is the expression that results from this approach, **and** how many ways (we want an integer answer!) are there to select the companions and then the driver from them?
   (b) Lynn realizes that there's another way to select the dinner companions: Lynn can pick the driver from the society's membership first, then add four more from the rest of the members. What is the expression that results from this approach, **and** how many ways (we want an integer answer!) are there to select the driver and the remaining guests? (Hint: You should have the same integer answer as you got for (a). If not, fix your errors!)
   (c) Let's generalize this problem: We have s other society members, and need c of them to join Lynn for the birthday dinner, with the driver being one of the c. Rewrite your expressions from parts (a) & (b) in terms of s and c.
   (d) Using an algebraic proof, prove that your two expressions from (c) are equal.
   (e) What you created in parts (a) & (b) was a specific case of a combinatorial proof. Write it out as a formal proof, but in terms of s and c, using the two generalized expressions you created in part (c). *Need help? I did a combinatorial proof in class. Rosen does another, of Corollary 1 on page 439.*