Sample First Midterm Exam — The Questions

Background and Advice

Discrete Math students seem to appreciate seeing what one of my exams looks like before they take the first one. I think that reducing anxiety is a good thing, so here's a sample first midterm for you to use to help prepare for the first midterm. I suggest that you treat this as if it were the actual exam. That is, **do not** look at it until you have studied all of the topics, and then do it in the same setting as we use for midterms: A time period of 50 minutes with no book, no notes, and no electronics. Only when you are finished (or out of time!) should you download and look at the sample answers, and compare them to your own. (Need a timer? Try typing `set timer 50 minutes` into Google.)

The actual exam will have a dedicated cover page with directions, rather than all of this background, and we'll let you know when you can open the exam and get started. On both the actual exam and this practice exam, the Page O' Logical Equivalences is provided on the last page. And, yes, you can tear it off of the exam to make it easier to use.

Please remember: What you see here is just a sample set of questions. On the actual exam, you will see different questions, some on different topics, phrased in different ways, and presented in different formats. If you are expecting the actual exam to be only a slight variation of this sample exam, you will be disappointed. Study *everything* we've covered!

Last note: This is the only sample exam I will provide this semester. After seeing this (and the real first midterm exam), you'll have a very good idea of what to expect on future exams.

1. (11 points) Math Review

   (a) What is the symbol for the set of positive integers?

   (b) True or False: 14 is a rational number.

   (c) Evaluate: $\sum_{w=-4}^{2} \frac{w}{2}$

   (d) Create an example that shows subtraction does not distribute over multiplication.

   (e) Simplify: $\log_2((\frac{8}{32})^2)$

2. (21 points) More Math Review!

   (a) What is 443_{10} in both binary and hexadecimal?

   (b) What is $12A_{11}$ in Base 7?

   (c) Simplify to a fraction of the form $\frac{a}{b}$: $1 + \frac{2}{3+\frac{4}{5}}$.

   (d) Let $A = \{b, e, f, g\}$, $B = \{a, c, d, f, g, h\}$, and $C = \{b, d, g\}$, over the universe $\mathcal{U} = \{a, b, c, d, e, f, g, h\}$. What are the results of the evaluations of the expressions $C - (A \cap B)$ and $(\overline{A} \cap C) \cup B$?

   (e) Evaluate: $\prod_{a=3}^{3} \sum_{b=10}^{12} (2a + b)$

   (f) Evaluate all three expressions: $7 \mid 15$, $7 \setminus 15$, and $7 \,\%\, 15$.

3. (3 points) Define: **Compound Proposition**

4. (6 points) Is $\overline{p \rightarrow q} \equiv p \wedge \overline{q}$? Construct a truth table to determine your answer (and don't forget to tell us what your answer is!).

5. (6 points) Without using a truth table, show that $\overline{p \vee q} \equiv \overline{p \vee (\overline{p} \wedge q)}$ Remember to justify each step.

6. (2 points) What must be done to prove $\forall x P(x), x \in D$ is true, for a given propositional function $P(x)$ and domain D?

7. (12 points) Convert the following English statements into logical notation in terms of the operators, symbols, and quantifiers used in class. Be sure to identify your propositions and predicates; domains are provided.

   (a) Someone is taller than somebody. (domain: people)

   (b) There is no free lunch. (domain: meals)

   (c) "The only consistent people are the dead." (domain: Creatures)

8. (14 points) Convert the following logical expressions to <u>conversational</u> English sentences. Assume that:

$$\begin{aligned} B(\alpha) &: \alpha \text{ is a hummingbird} \\ L(\alpha) &: \alpha \text{ is large} \\ H(\alpha) &: \alpha \text{ eats honey} \\ C(\alpha) &: \alpha \text{ is brightly colored} \end{aligned}$$

   (a) $\overline{L(Tweety)} \oplus C(Tweety)$

   (b) $\exists y(B(y) \wedge \overline{L(y)})$, $y \in$ Animals

   (c) $\forall z(\overline{H(z)} \rightarrow \overline{C(z)})$, $z \in$ Animals

   (d) $\forall w(\overline{L(w)} \vee \overline{H(w)})$, $w \in$ Birds

9. (12 points) Consider this English conditional proposition: Whenever I study logic, I am happy. Construct variations of it, in English, as instructed below.

   (a) Converse of the original, with "when" at the front.

   (b) Contrapositive of the original, in "if – then" form.

   (c) Inverse of the original, with "if" in the middle of the statement.

   (d) Same meaning as the original, but with "only if" in the middle of the statement.

10. (3 points) Fill in the Blanks! We know that there are at least three pairs of terms for p and q in the implication $p \to q$. Write in the missing name of each pair below.

   (a) p is the ____________________ and q is the consequent.

   (b) p is the ____________________ and q is the conclusion.

   (c) p is sufficient and q is ____________________.

11. (3 points) Define: **Logical Equivalence** (We covered two definitions in class; either is acceptable.)

12. (7 points) Miscellany!

   (a) Evaluate: $0010 \vee (1010 \wedge \overline{0111})$, with the operators having the normal meanings in logic.

   (b) If a truth table has five proposition labels, how many rows of truth values will the complete truth table have?

   (c) In the logical expression $a \oplus b \oplus c \oplus d$, which operator is evaluated first, the left–most, the center, or the right–most $\oplus$, and why?

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(McCann)

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The Page O' Logical Equivalences ("POLE")

Table I: Some Equivalences using AND ($\wedge$) and OR ($\vee$):

(a)	$p \wedge p \equiv p$ $p \vee p \equiv p$	Idempotent Laws
(b)	$p \wedge \mathbf{F} \equiv \mathbf{F}$ $p \vee \mathbf{T} \equiv \mathbf{T}$	Domination Laws
(c)	$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity Laws
(d)	$p \wedge q \equiv q \wedge p$ $p \vee q \equiv q \vee p$	Commutative Laws
(e)	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ $(p \vee q) \vee r \equiv p \vee (q \vee r)$	Associative Laws
(f)	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	Distributive Laws
(g)	$p \wedge (p \vee q) \equiv p$ $p \vee (p \wedge q) \equiv p$	Absorption Laws

Table II: Some More Equivalences (adding Negation ($\neg$)):

(a)	$\neg(\neg p) \equiv p$	Double Negation
(b)	$p \wedge \neg p \equiv \mathbf{F}$ $p \vee \neg p \equiv \mathbf{T}$	Negation Laws
(c)	$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's Laws

Table III: Still More Equivalences (adding Implication ($\rightarrow$)):

(a)	$p \rightarrow q \equiv \neg p \vee q$	Law of Implication
(b)	$p \rightarrow q \equiv \neg q \rightarrow \neg p$	Law of the Contrapositive
(c)	$\mathbf{T} \rightarrow p \equiv p$	"Law of the True Antecedent"
(d)	$p \rightarrow \mathbf{F} \equiv \neg p$	"Law of the False Consequent"
(e)	$p \rightarrow p \equiv \mathbf{T}$	Self-implication (a.k.a. Reflexivity)
(f)	$p \rightarrow q \equiv (p \wedge \neg q) \rightarrow \mathbf{F}$	Reductio Ad Absurdum
(g)	$\neg p \rightarrow q \equiv p \vee q$	
(h)	$\neg(p \rightarrow q) \equiv p \wedge \neg q$	
(i)	$\neg(p \rightarrow \neg q) \equiv p \wedge q$	
(j)	$(p \rightarrow q) \vee (q \rightarrow p) \equiv \mathbf{T}$	Totality
(k)	$(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$	Exportation Law (a.k.a. Currying)
(l)	$(p \wedge q) \rightarrow r \equiv (p \rightarrow r) \vee (q \rightarrow r)$	
(m)	$(p \vee q) \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$	
(n)	$p \rightarrow (q \wedge r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$	
(o)	$p \rightarrow (q \vee r) \equiv (p \rightarrow q) \vee (p \rightarrow r)$	
(p)	$p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \rightarrow r)$	Commutativity of Antecedents

Table IV: Yet More Equivalences (adding Exclusive OR ($\oplus$) and Biimplication ($\leftrightarrow$)):

(a)	$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$	Definition of Biimplication
(b)	$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$	
(c)	$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$	
(d)	$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$	Definition of Exclusive Or
(e)	$p \oplus q \equiv \neg(p \leftrightarrow q)$	
(f)	$p \oplus q \equiv p \leftrightarrow \neg q \equiv \neg p \leftrightarrow q$	

You may tear off this page if you wish!

Notes:
1. p, q, and r represent arbitrary logical expressions. They may represent equivalent expressions (e.g., if $p \equiv q$, then by Absorption $p \wedge (p \vee p) \equiv p$).
2. $\mathbf{T}$ and $\mathbf{F}$ represent the logical values True and False, respectively.
3. These tables show many of the common and/or useful logical equivalences; this is not an exhaustive collection!