Collected Definitions for Exam #3

This is the ‘official’ collection of need-to-know definitions for Exam #3. I can’t recall the last time I didn’t ask a definition question on an exam. To help you better prepare yourself for definition questions, I’ve assembled this list. My pledge to you: If I ask you for a definition on the exam, the term will come from this list. Note that this is not a complete list of the definitions given in class. You should know the others, too, but I won’t specifically ask you for their definitions on the exam.

(Continued from the Exam #2 Topic 7 definition list. If we ask you to define a Topic 7 term on Exam #3, it will come from this list.)

- **Identity matrices**, denoted $I_n$, are $n \times n$ matrices populated with 1 down the main diagonal (upper-left to lower-right) and with 0 elsewhere.
- The $n^{th}$ matrix power of an $m \times m$ matrix $A$, denoted $A^n$, is the matrix resulting from $n - 1$ successive matrix products of $A$. $A^1 = I_n$.
- The logical matrix product of an $m \times n$ 0–1 matrix $A$ and an $n \times l$ 0–1 matrix $B$ is an $m \times l$ 0–1 matrix $C = A \odot B$ in which $c_{ij} = \bigvee_{k=1}^{n}(a_{ik} \land b_{kj})$.
- The $r^{th}$ logical matrix power of an $m \times m$ 0–1 matrix $A$, denoted $A^r$, is the matrix resulting from $r - 1$ successive logical matrix products of $A$. $A^0 = I_m$.

Topic 8: Relations

- A (binary) relation from set $X$ to set $Y$ is a subset of the Cartesian Product of the domain $X$ and the codomain $Y$.
- A relation $R$ on set $A$ is reflexive if $(a, a) \in R$, $\forall a \in A$.
- A relation $R$ on set $A$ is symmetric if, whenever $(a, b) \in R$, then $(b, a) \in R$, for $a, b \in A$.
- A relation $R$ on set $A$ is antisymmetric if $(x, y) \in R$ and $x \neq y$, then $(y, x) \notin R$, $\forall x, y \in A$.
- A relation $R$ on set $A$ is transitive if, whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for $a, b, c \in A$.
- The inverse of a relation $R$ on set $A$, denoted $R^{-1}$, contains all of the ordered pairs of $R$ with their components exchanged. (That is, $R^{-1} = \{(b, a) \mid (a, b) \in R\}$.)
- Let $G$ be a relation from set $A$ to set $B$, and let $F$ be a relation from $B$ to set $C$. The composite of $F$ and $G$, denoted $F \circ G$, is the relation of ordered pairs $(a, c), a \in A, c \in C$, such that $b \in B$, $(a, b) \in G$, and $(b, c) \in F$.
- A relation $R$ on set $A$ is an equivalence relation if it is reflexive, symmetric, and transitive.
- The equivalence class of an equivalence relation $R$ on set $B$, and an element $b \in B$, is $\{c \mid c \in B \land (b, c) \in R\}$ and is denoted $[b]$. That is, the equivalence class is the set of all elements of the base relation equivalent to a given element as defined by the relation.
- A relation $R$ on set $A$ is a (reflexive/weak) partial order if it is reflexive, antisymmetric, and transitive.
- A relation $R$ on set $A$ is irreflexive if, for all members of $A$, $(a, a) \notin R$.
- A relation $R$ on set $A$ is an irreflexive (or strict) partial order if it is irreflexive, antisymmetric, and transitive.
- Let $R$ be a weak partial order on set $A$. $a$ and $b$ are said to be comparable if $a, b \in A$ and either $a \leq b$ or $b \leq a$ (that is, either $(a, b) \in R$ or $(b, a) \in R$).
- A weak partially-ordered relation $R$ on set $A$ is a total order if every pair of elements $a, b \in A$ are comparable.

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A function from set $X$ to set $Y$, denoted $f : X \to Y$, is a relation from $X$ to $Y$ such that $f(x)$ is defined $\forall x \in X$ and, for each $x \in X$, there is exactly one $(x, y) \in f$.

For each of the following, let $f : X \to Y$ be a function, and assume $f(n) = p$.

- $X$ is the domain of $f$; $Y$ is the codomain of $f$.
- $f$ maps $X$ to $Y$.
- $p$ is the image of $n$; $n$ is the pre-image of $p$.
- The range of $f$ is the set of all images of elements of $X$. (Note that the range need not equal the codomain.)

- The floor of a value $n$, denoted $\lfloor n \rfloor$, is the largest integer $\leq n$.
- The ceiling of a value $m$, denoted $\lceil m \rceil$, is the smallest integer $\geq m$.
- A function $f : X \to Y$ is injective (a.k.a. one-to-one) if, for each $y \in Y$, $f(x) = y$ for at most one member of $X$.
- A function $f : X \to Y$ is surjective (a.k.a. onto) if $f$’s range is $Y$ (the range = the codomain).
- A bijective function (a.k.a. a one-to-one correspondence) is both injective and surjective.
- The inverse of a bijective function $f$, denoted $f^{-1}$, is the relation $\{(y, x) \mid (x, y) \in f\}$.
- Let $f : Y \to Z$ and $g : X \to Y$. The composition of $f$ and $g$, denoted $f \circ g$, is the function $h = f(g(x))$, where $h : X \to Z$.
- A function $f : X \times Y \to Z$ (or $f(x, y) = z$) is a binary function.

There were no definitions in this topic!