Topic 4: Arguments

* The Logical Kind, Not The Talk Radio Kind.

Monty Python’s “The Argument Clinic”

Featuring:

- Michael Palin as “Man”
- Rita Davies as “Receptionist”
- Graham Chapman as “Mr. Barnard”
- John Cleese as “Mr. Vibrating”
- Eric Idle as “Complainer”
- Terry Jones as “Spreaders”

Definition: Argument
Inductive and Deductive Reasoning (1 / 3)

Definition: Inductive Argument

Definition: Deductive Argument

Inductive and Deductive Reasoning (2 / 3)

Example(s):
What type of argument is this?

3 is a prime number, 5 is a prime number, and 7 is a prime number. Therefore, all positive odd integers above 1 are prime numbers.

Structure of a Deductive Argument

\((p_1 \land p_2 \land \ldots \land p_n) \rightarrow q\)
Valid and Sound Arguments (1 / 2)

**Definition:** Valid Argument

Example(s):

Valid and Sound Arguments (2 / 2)

**Example(s):**

**Definition:** Sound Argument
Some Rules of Inference (1 / 2)

Learn these!

1. Addition

2. Simplification

3. Conjunction

4. Modus Ponens

Some Rules of Inference (2 / 2)

Learn these, too!

5. Modus Tollens

6. Hypothetical Syllogism

7. Disjunctive Syllogism

8. Resolution
Examples of Valid Arguments (1 / 4)

#1: You accidentally drop a pen. You know that the pen will fall if it is dropped. How do you know that the pen will fall?

Examples of Valid Arguments (2 / 4)

#2: If $191$ is divisible by $7$, then $191^2$ is divisible by $49$.

$191$ is divisible by $7$, so $191^2$ is divisible by $49$.

Is this argument valid?
Examples of Valid Arguments (3 / 4)

#3: If you email me a love note, I’ll send you flowers. If you don’t, I’ll study Discrete Math. If I study Discrete Math, I’ll do well on the quiz.

Can we conclude that, if I don’t send you flowers, I’ll do well on the quiz?

Examples of Valid Arguments (4 / 4)

#3: (cont.)

\[ p: \text{You email me a love note} \quad q: \text{I send you flowers} \]
\[ r: \text{I study Discrete Math} \quad s: \text{I do well on the quiz} \]
\[ p \rightarrow q \quad p \rightarrow r \quad r \rightarrow s \]
\[ \therefore \quad \neg q \rightarrow s \quad ??? \]
Rules of Inference for Predicates (1 / 3)

Four common rules that you need to know:

1. **Universal Instantiation**
   \[ \forall x \ P(x), \ x \in D / \therefore P(d) \text{ if } d \in D \]

2. **Universal Generalization**
   \[ P(d) \text{ for any } d \in D / \therefore \forall x \ P(x), \ x \in D \]

3. **Existential Instantiation**
   \[ \exists x \ P(x), \ x \in D / \therefore P(d) \text{ for some } d \in D \]

4. **Existential Generalization**
   \[ P(d) \text{ for some } d \in D / \therefore \exists x \ P(x), \ x \in D \]

Rules of Inference for Predicates (2 / 3)

**Example(s):**

Everyone taking CSc 144 has had a programming class.
Hugo is in CSc 144. **Has he had a programming class?**
Rules of Inference for Predicates (3 / 3)

Here’s a more general example using the same setup:

Someone’s taking CSc 144. Everyone in CSc 144 has had a programming class. Does there exist someone who both had a programming class and is taking CSc 144?

\[(1) \exists x \; C(x) \quad \text{(Given)}\]
\[(2) \forall x \; (C(x) \rightarrow P(x)) \quad \text{(Given)}\]

\[\therefore \exists x \; (P(x) \land C(x))\]

Fallacies (1 / 2)

Definition: Fallacy

Three classic types:

1. Affirming the Conclusion (or … Consequent)
2. Denying the Hypothesis (or ... Antecedent)

3. Begging the Question (a.k.a. Circular Reasoning)

Fallacies for Fun

1. Fallacy of Interrogation

2. ‘No True Scotsman’ Fallacy
Extra Slides

The remaining slides in this topic are some that I no longer cover in class. I won’t ask about them on a quiz or an exam, but they could be referenced on a homework or in SIs.

Specious Reasoning: The Bear Patrol (1 / 3)

Homer: Ah, not a bear in sight. The Bear Patrol must be working like a charm!

Lisa: That’s specious reasoning, Dad. [...] By your logic, I could claim that this rock keeps tigers away!

Homer: Oh . . . and how does it work?

Lisa: It doesn’t work. [...] It’s just a stupid rock. [...] But I don’t see any tigers around here, do you?

Homer: Lisa, I want to buy your rock.

From: The Simpsons, “Much Apu About Nothing”

(Season 7, Episode 151, Production Code 3F20)
Definition: Specious Reasoning

An unsupported or improperly constructed argument. (That is, an unsound or invalid argument.)

Question: Where is the error in Homer’s logic?

\[ b: \text{ There are bears in Springfield} \]
\[ w: \text{ The Bear Patrol is working} \]

First issue: Which of these is Homer’s argument?

\[
\begin{align*}
(1) & \quad \neg b \quad \text{(Given)} \\
(2) \quad \therefore w \quad \text{(???)}
\end{align*}
\]

\[
\begin{align*}
(1) & \quad w \quad \text{(Given)} \\
(2) \quad \therefore \neg b \quad \text{(???)}
\end{align*}
\]

The first seems most reasonable in context.

Question: Where is the error in Homer’s logic? (cont.)

Next, what is the missing piece of Homer’s argument?

\[
\begin{align*}
(1) & \quad \neg b \\
(2) \quad \neg b \rightarrow w \quad \leftarrow \text{this is what we’re trying to show!} \\
(3) \quad \therefore w \quad \text{(1, 2, Modus Ponens)}
\end{align*}
\]

OK, then, how about . . .

\[
\begin{align*}
(1) & \quad \neg b \\
(2) \quad w \rightarrow \neg b \quad \leftarrow \text{might sound good, but . . .} \\
(3) \quad \therefore w \quad \text{(1, 2, um . . . Abracadabra?)}
\end{align*}
\]

(The second form of Homer’s argument fails similarly.)