Functions as Relations (1 / 2)

Consider: \( f(x) = x + 1, x \in \mathbb{Z} \)

**Definition:** Function
Example(s):

Function Terms (1 / 2)

Let \( f : X \rightarrow Y \) be a function. \( f(n) = p \ [ (n, p) \in f ] \).

- \( X \) is the \underline{domain} of \( f \)
- \( Y \) is the \underline{range} of \( f \)
- \( f \) \underline{maps} \( X \) to \( Y \)
- \( p \) is the \underline{image} of \( n \)
- \( n \) is the \underline{preimage} of \( p \)
- \( f \)'s \underline{range} is the set of all images of \( X \)'s elements

\textbf{Note:} A function's range need not equal its codomain.
Example(s):

\[ g = \{ (a, b) \mid b = a/2 \}, \quad a \in \{0, 2, 4, 8\}, \]
\[ b \in \{0, 1, 2, 3, 4, 5\} \]
Two Functions You Need To Know (1 / 4)

1. Floor \( \lfloor x \rfloor \)

**Definition:** Floor Function

**Example(s):**
Two Functions You Need To Know (2 / 4)

1. Floor \( \lfloor x \rfloor \) (cont.)

Using Floor for Rounding to the Nearest Integer

Two Functions You Need To Know (3 / 4)

2. Ceiling \( \lceil x \rceil \)

Definition: Ceiling Function

Example(s):
Two Functions You Need To Know (4 / 4)

2. Ceiling \( \lceil x \rceil \) (cont.)

Example(s):

Example: Type A UPC Code Check Digits

The check digit equals the image of this function:

\[
\begin{align*}
    s &= \text{Sum of digits in positions 1, 3, 5, 7, 9, & 11} \\
    t &= \text{Sum of digits in positions 2, 4, 6, 8, & 10} \\
    u &= 3s + t; \text{ the check digit is } (10 - u \% 10) \% 10.
\end{align*}
\]

Using the above sample:

\[
\begin{align*}
    s &= 39, \ t = 24, \text{ and } u = 3(39) + 24 = 141. \\
    \text{The check digit} &= (10 - 141 \% 10) \% 10 = 9.
\end{align*}
\]
Important Distinction: *Continuous* vs. *Discontinuous* Functions

Consider: \( f = \{(x, x + 1) \mid x \in \ldots \} \)

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How should the graph of our long-distance calling plan function look?

\[
\text{Cost(length)} = \begin{cases} 
50 \text{ cents} & \text{if length} \leq 10 \text{ minutes} \\
50 + 5 \cdot \lceil \text{length} - 10 \rceil \text{ cents} & \text{Otherwise}
\end{cases}
\]
Categories of Functions: Injective

**Definition:** Injective Functions  (a.k.a. One-to-one)

Example(s):

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Categories of Functions: Surjective

**Definition:** Surjective Functions  (a.k.a. Onto)

Example(s):

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Categories of Functions: Bijective

**Definition: Bijective Functions** (a.k.a. One-to-one Correspondence)

**Example(s):**

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Odds and Ends

**Definition: Functional Composition**

Let \( f : Y \rightarrow Z \) and \( g : X \rightarrow Y \). The composition of \( f \) and \( g \), denoted \( f \circ g \), is the function \( h = f(g(x)) \), where \( h : X \rightarrow Z \).

**Definition: Inverse Functions**

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Beyond Unary Functions

**Definition: Binary Functions**

**Example(s):**